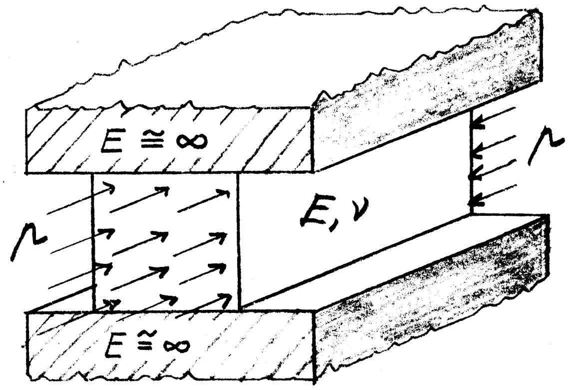
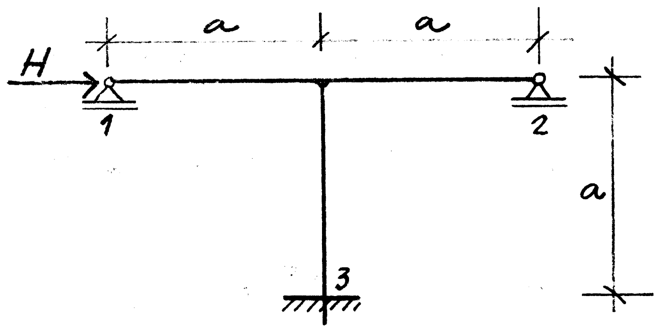
MTT IZPIT 27. 1. 1986

1. Jeklena prizma je brez trenja vstavljena med dve absolutno togi plošči. Po osnovnih ploskvah jo enakomerno obtežimo s pravokotno obtežbo p.
   a) določi napetosti v prizmi ter ustrezno specifično spremembo prostornine!
   b) kakšne so napetosti in specifična sprememba prostornine, če segrejemo prizmo za $90°$ K?
   c) določi spremembo temperature, pri kateri se bo prizma tesno vendar brez napetosti dotikala obeh togih plošč!



$p = 160$ MPa
$E = 2 \cdot 10^5$ MPa
$\nu = 0{,}3$
$\alpha = 1{,}25 \times 10^{-5} / °K$

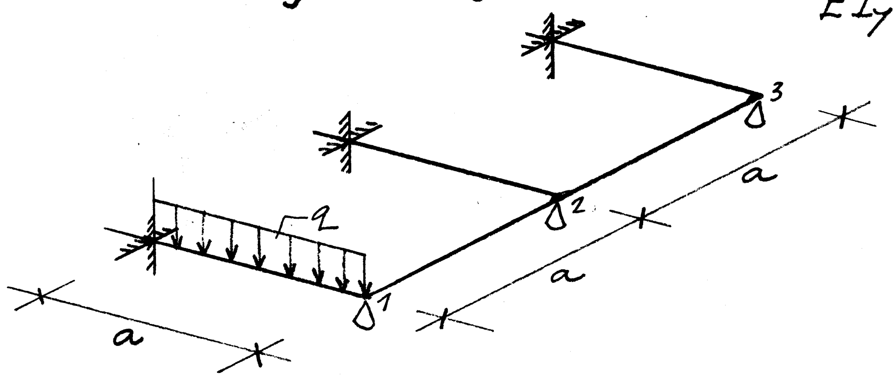
2. Določi pomike podpore 1 ter skiciraj notranje sile!



$a = 4\,m$
$E = 200\,000$ MPa
$A_x = 0{,}01\ m^2$
$I_y = 0{,}0002\ m^4$
$H = 0{,}8$ MN

3. Določi in skiciraj notranje sile!

$EI_y = GI_x$



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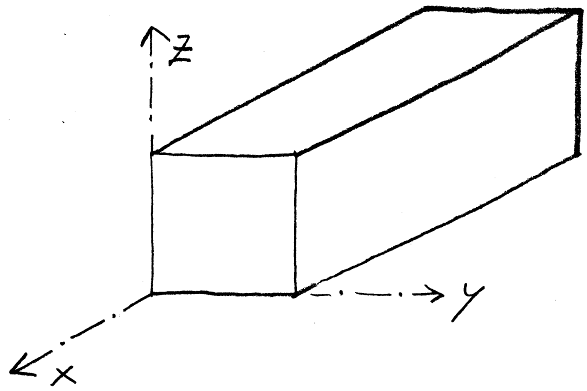
Ad 1.)



$$\boxed{\sigma_{xx} = -p = -160\ MPa}$$

$$\boxed{\sigma_{yy} = \sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0}$$

$$\boxed{\varepsilon_{zz} = 0}$$

a) $\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right] = \frac{1}{E}(\sigma_{zz} + \nu p) = 0$

$\boxed{\sigma_{zz} = -\nu p}$ $\boxed{\sigma_{zz} = -48\ MPa}$

$I_1^\sigma = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = -p - \nu p = -(1+\nu)p$

$\boxed{I_1^\sigma = -208\ MPa}$

$\varepsilon_V = I_1^\varepsilon = \frac{1-2\nu}{E} I_1^\sigma$ $\boxed{\varepsilon_V = -0{,}000416}$

b) $\varepsilon_{zz} = \frac{1}{E}(\sigma_{zz} + \nu p) + \alpha \Delta T = 0$

$\sigma_{zz} = -(E\alpha\Delta T + \nu p)$ $\boxed{\sigma_{zz} = -273\ MPa}$

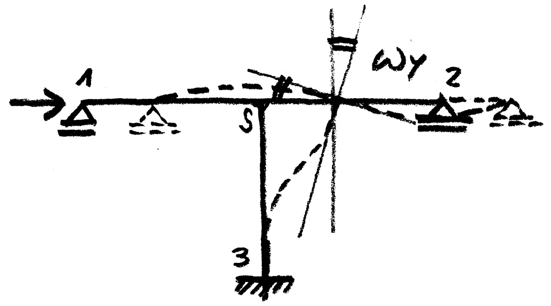
$I_1^\sigma = \sigma_{xx} + \sigma_{zz}$ $\boxed{I_1^\sigma = -433\ MPa}$

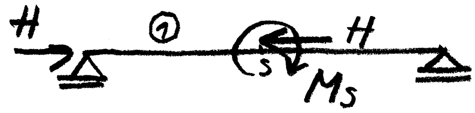
$\varepsilon_V = \frac{1-2\nu}{E} I_1^\sigma + 3\alpha\Delta T$ $\boxed{\varepsilon_V = 0{,}004241}$

c) $\sigma_{zz} = -(E\alpha\Delta T + \nu p) = 0$

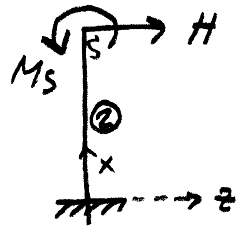
$\Delta T = -\frac{\nu p}{E\alpha}$ $\boxed{\Delta T = -19{,}2°K}$

Ad 2.)

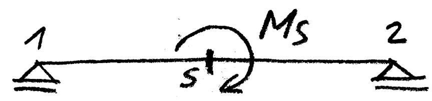




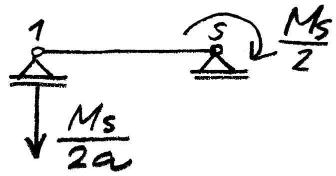
s : $\boxed{\omega_y^{(1)} = \omega_y^{(2)}}$



(1)



$\longrightarrow$



$$EI_y w'' = -M_y = \frac{M_S}{2a} x$$

$$EI_y w' = \frac{M_S}{4a} x^2 + C_1$$

$$EI_y w = \frac{M_S}{12a} x^3 + C_1 x + C_2$$

$x = 0 \; \ldots \quad w = 0 \longrightarrow C_2 = 0$

$x = a \; \ldots \quad w = 0 \qquad \frac{M_S a^2}{12} + C_1 a = 0 \longrightarrow \boxed{C_1 = -\frac{M_S a}{12}}$

$$\omega_y^{(1)}(x=a) = -\frac{M_S a}{EI_y}\left(\frac{1}{4} - \frac{1}{12}\right) \; \ldots \; \boxed{\omega_y^{(1)}(x=a) = -\frac{M_S a}{6EI_y}}$$

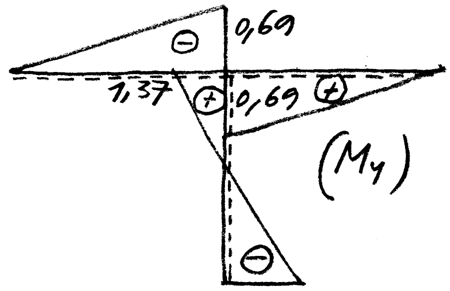
(2) $\boxed{\omega_y^{(2)}(S) = \frac{M_S a}{EI_y} - \frac{H a^2}{2EI_y}}$

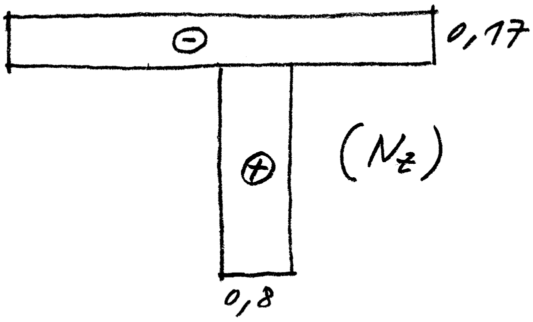
$$s: \; \omega_y^{(1)} = \omega_y^{(2)} \; \ldots \; -\frac{M_S a}{6EI_y} = \frac{M_S a}{EI_y} - \frac{H a^2}{2EI_y}$$

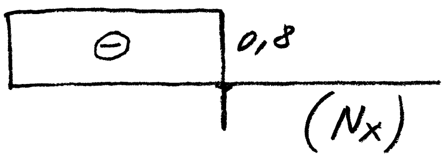
$\boxed{M_S = \frac{3Ha}{7}}$ $\qquad$ $\boxed{M_S = 1{,}37 \text{ MNm}}$

$\boxed{w_1 = 0}$ $$u_1 = H\frac{a^3}{3EI_y} - M_S\frac{a^2}{2EI_y} + H\frac{a}{EA_x}$$

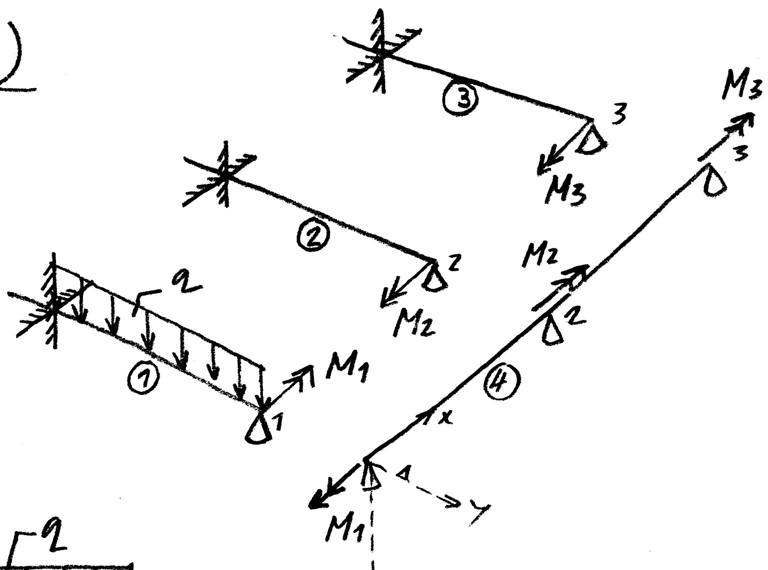
$$\boxed{u_1 = H\cdot a\cdot\frac{5A_x\cdot a + 42 I_y}{42\, EA_x I_y}}$$ $$\boxed{u_1 = 0{,}15398\ m}$$

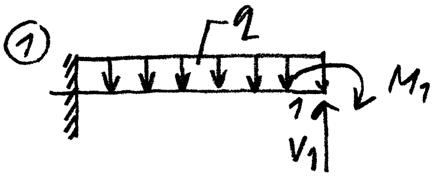






Ad 3)





$\boxed{w_1 = 0\ !}$

$$\frac{q a^4}{8EI_y} - \frac{V_1 a^3}{3EI_y} + \frac{M_1 a^2}{2EI_y} = 0$$

$$\boxed{V_1 = \frac{3qa}{8} + \frac{3M_1}{2a}}$$

$$\omega_y^{(1)}(1) = \frac{-q a^3}{6EI_y} + V_1 \frac{a^2}{2EI_y} - M_1 \frac{a}{EI_y}$$

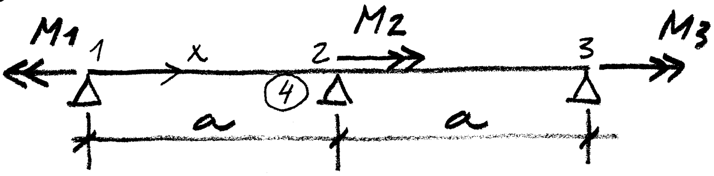
$$\boxed{\omega_y^{(1)}(1) = \frac{q a^3}{48 E I_y} - \frac{M_1 a}{4EI_y}}$$

$$\boxed{\omega_y^{(2)}(2) = \frac{M_2 a}{4EI_y}}$$

$$\boxed{\omega_y^{(3)}(3) = \frac{M_3 a}{4EI_y}}$$

$$\boxed{M_3 = M_1 - M_2}$$

4



$$\boxed{\begin{aligned} \omega_x^{(4)}(1) &= -\omega_y^{(1)}(1) \\ \omega_x^{(4)}(2) &= -\omega_y^{(2)}(2) \\ \omega_x^{(4)}(3) &= -\omega_y^{(3)}(3) \end{aligned}}$$

$$M_x = M_1 - M_2 \langle x-a \rangle^0 = G I_x \frac{d\omega_x}{dx}$$

$$\omega_x = \left(M_1 x - M_2 \langle x-a \rangle + C_1\right) \cdot \frac{1}{EI_y}$$

$$x = 0 \;\ldots\; \omega_x = -\frac{q a^3}{48 E I_y} + M_1 \frac{a}{4EI_y} = C_1$$

$$\omega_x = \frac{1}{EI_y}\left[M_1\left(x + \frac{a}{4}\right) - M_2 \langle x-a \rangle - \frac{q a^3}{48}\right]$$

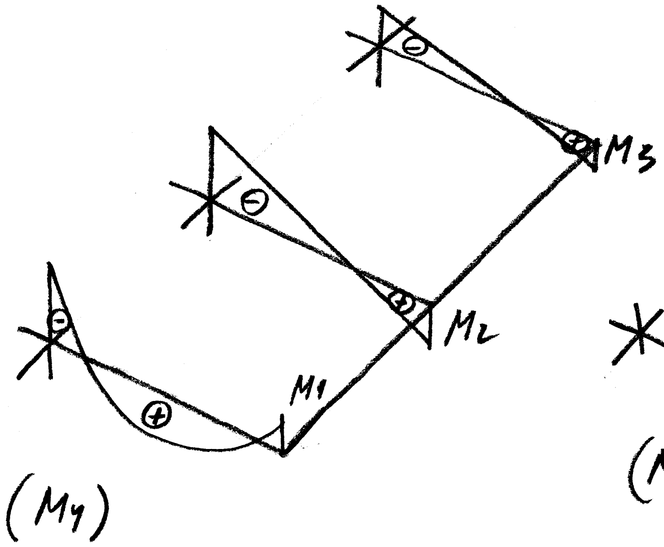
$$x = a \;\ldots\; \frac{1}{EI_y}\left[M_1 \frac{5a}{4} - \frac{q a^3}{48}\right] = -M_2 \frac{a}{4EI_y}$$

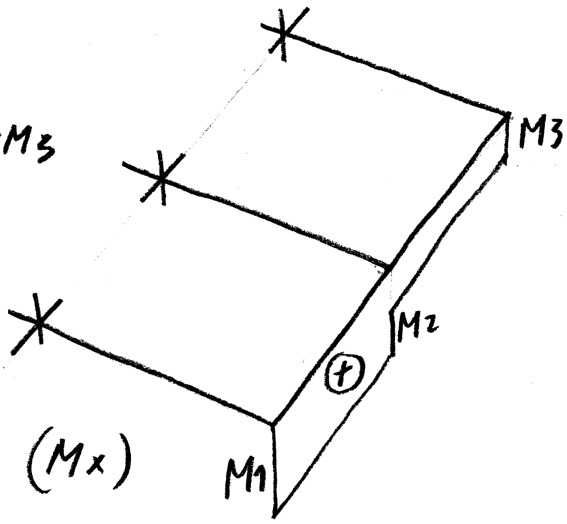
$$x = 2a \;\ldots\; \frac{1}{EI_y}\left[M_1 \frac{9a}{4} - M_2 \cdot a - \frac{q a^3}{48}\right] = -M_3 \frac{a}{4EI_y}$$

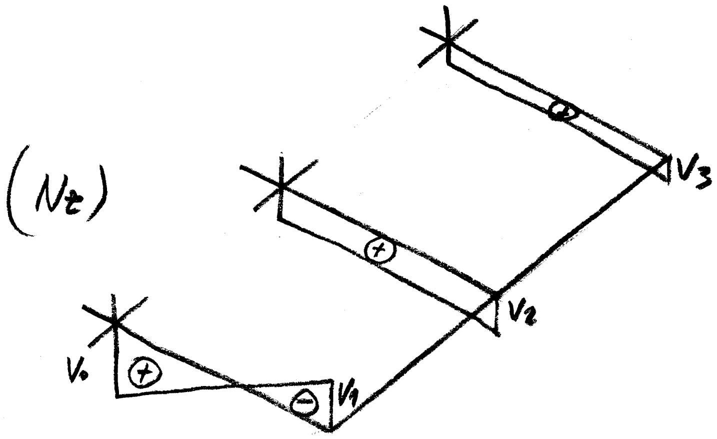
$$M_1 \cdot \frac{5}{4} + M_2 \cdot \frac{1}{4} = \frac{q a^2}{48} \;\ldots\; 60 M_1 + 12 M_2 = q a^2$$

$$M_1 \cdot \frac{13}{4} - M_2 \cdot \frac{5}{4} = \frac{q a^2}{48} \;\ldots\; 120 M_1 - 60 M_2 = q a^2$$

$$\boxed{M_1 = \frac{q a^2}{70}} \qquad \boxed{M_2 = \frac{q a^2}{84}} \qquad \boxed{M_3 = \frac{q a^2}{420}}$$


M3
M2
M1
(My)


M3
M2
M1
(Mx)


(Nz)
V3
V2
V0
V1