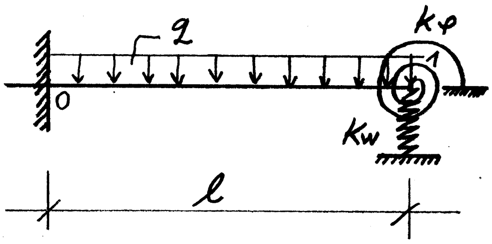
MTT IZPIT 27. 3. 1989

1. Rešitev mehanskega problema trdnega telesa je matrika majhnih deformacij ε_{ij}. V točki $T(2,2,0)$ določi:
   a) velikosti glavnih normalnih napetosti
   b) deformirano materialno koordinatno bazo

$$[\varepsilon_{ij}] = 10^{-3} \cdot \begin{bmatrix} 10x & y^2 & 0 \\ y^2 & 4xy & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$2\mu = 16\,000\ kN/cm^2$
$\lambda = 8\,000\ kN/cm^2$

2. Nosilec je v točki 1 elastično podprt. Konstanta linearne vzmeti je k_w, konstanta spiralne vzmeti pa je k_φ. Določi pomike in zasuke obeh podpor!



$l = 4\ m$
$q = 0{,}1\ MN/m$
$EI_y = 4\ MNm^2$
$k_w = 1\ MN/m$
$k_\varphi = 0{,}05\ MNm/rad$

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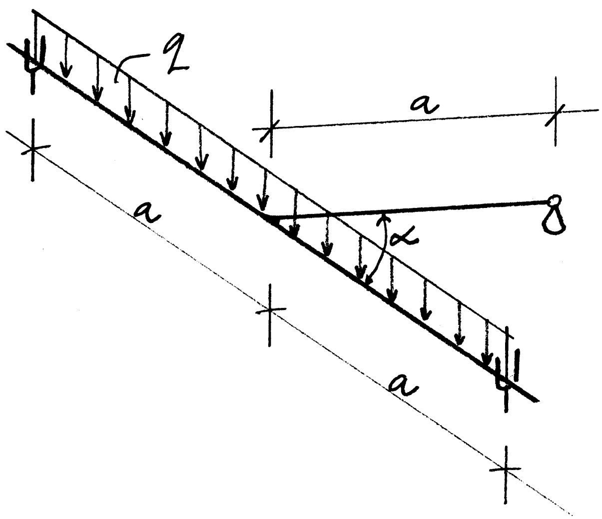
3. Določi in skiciraj notranje sile!



$\alpha = 60°$
$EI_y = G I_x$

Ad 1.

a) $[\varepsilon_{ij}]_T = 10^{-3}\begin{bmatrix} 20 & 4 & 0 \\ 4 & 16 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0{,}020 & 0{,}004 & 0 \\ 0{,}004 & 0{,}016 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\varepsilon_{11,22} = 10^{-3}\left(\frac{20+16}{2} \pm \sqrt{\left(\frac{20-16}{2}\right)^2 + 4^2}\right)$$

$$\boxed{\varepsilon_{11} = 22{,}47 \cdot 10^{-3}} \qquad \boxed{\varepsilon_{22} = 13{,}53 \cdot 10^{-3}}$$

$$\sigma_{11} = 2\mu\, \varepsilon_{11} + \lambda(\varepsilon_{11} + \varepsilon_{22}) \rightarrow \quad \sigma_{11} = 648 \text{ kN/cm}^2$$

$$\sigma_{22} = 2\mu\, \varepsilon_{22} + \lambda(\varepsilon_{11} + \varepsilon_{22}) \rightarrow \quad \sigma_{22} = 504 \text{ kN/cm}^2$$

$$\sigma_{33} = \lambda(\varepsilon_{11} + \varepsilon_{22}) \rightarrow \quad \sigma_{33} = 288 \text{ kN/cm}^2$$

b) za referenčno točko izberem $T_0(0,0,0)$:

$$\omega_z = \omega_z(T_0) + \int_0^x \vec{e}_z(\vec{\nabla}\times\vec{\varepsilon}_x)_{y=0}\,dx + \int_0^y \vec{e}_z(\vec{\nabla}\times\vec{\varepsilon}_y)\,dy$$

$$\vec{e}_z(\vec{\nabla}\times\vec{\varepsilon}_x) = \begin{vmatrix} 0 & 0 & 1 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \varepsilon_{xx} & \varepsilon_{xy} & 0 \end{vmatrix} = \varepsilon_{xy,x} - \varepsilon_{xx,y} = 0$$

$$\vec{e}_z(\vec{\nabla}\times\vec{\varepsilon}_y) = \begin{vmatrix} 0 & 0 & 1 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \end{vmatrix} = \varepsilon_{yy,x} - \varepsilon_{xy,y} = 2y \cdot 10^{-3}$$

$$\omega_z = \omega_z(T_0) + 10^{-3}\cdot\int_0^y 2y\,dy = \underline{\underline{\omega_z(T_0) + 10^{-3}\cdot y^2}}$$

$$u_x = u_x(T_0) + \int_0^x \varepsilon_{xx}\Big|_{y=0} dx + \int_0^y (\varepsilon_{yx} - \omega_z)\, dy$$

$$u_x = u_x(T_0) + 10^{-3}\left[\int_0^x 10x\, dx + \int_0^y (y^2 - \omega_z^0 \cdot 10^3 - y^2)\, dy\right]$$

$$\boxed{u_x = u_x(T_0) + 5x^2 \cdot 10^{-3} - \omega_z^0 \cdot y}$$

$$u_y = u_y(T_0) + \int_0^x (\varepsilon_{xy} + \omega_z)_{y=0}\, dx + \int_0^y \varepsilon_{yy}\, dy$$

$$u_y = u_y(T_0) + 10^{-3}\left[\int_0^x (y^2 + \omega_z^0 \cdot 10^3 + y^2)_{y=0}\, dx + \int_0^y 4xy\, dy\right]$$

$$\boxed{u_y = u_y(T_0) + \omega_z^0 \cdot x + \cancel{4}\,2xy^2 \cdot 10^{-3}}$$

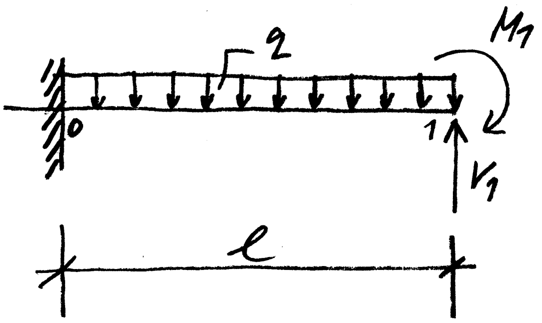
$$\vec{e}_x{}' = \vec{e}_x + \frac{\partial \vec{u}}{\partial x} = \vec{e}_x + 10x \cdot 10^{-3}\vec{e}_x + (\omega_z^0 + 2y^2 \cdot 10^{-3})\vec{e}_y$$

$$\vec{e}_y{}' = \vec{e}_y + \frac{\partial \vec{u}}{\partial y} = \vec{e}_y - \omega_z^0 \vec{e}_x + 4xy \cdot 10^{-3} \vec{e}_y$$

V točki $T(2,2,0)$:

$$\boxed{\begin{aligned} \vec{e}_x{}' &= 1{,}020\, \vec{e}_x + (\omega_z^0 + 0{,}080)\, \vec{e}_y \\ \vec{e}_y{}' &= -\omega_z^0\, \vec{e}_x + 1{,}016\, \vec{e}_y \end{aligned}}$$

Ad 2.



$$\boxed{\begin{aligned} M_1 &= k_\varphi \cdot \omega_1 \\ V_1 &= k_w \cdot w_1 \end{aligned}}$$

$$w_1 = \frac{q l^4}{8EI_y} - k_w w_1 \frac{l^3}{3EI_y} + k_\varphi \omega_1 \frac{l^2}{2EI_y}$$

$$\omega_1 = -\frac{q l^3}{6EI_y} + k_w w_1 \frac{l^2}{2EI_y} - k_\varphi \omega_1 \frac{l}{EI_y}$$

$$w_1 \left(1 + \frac{k_w l^3}{3EI_y}\right) - \omega_1 \frac{k_\varphi l^2}{2EI_y} = \frac{q l^4}{8EI_y}$$

$$w_1 \frac{k_w l^2}{2EI_y} - \omega_1 \left(1 + \frac{k_\varphi l}{EI_y}\right) = \frac{q l^3}{6EI_y}$$

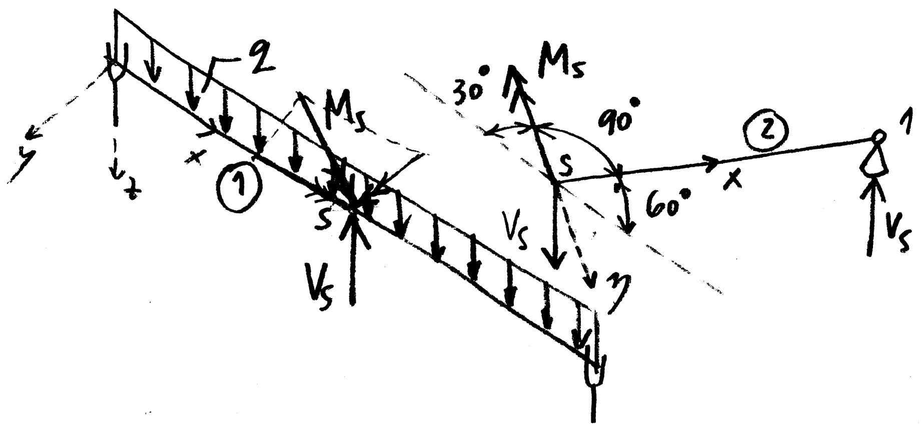
$$\begin{bmatrix} 6{,}333 & -0{,}1 \\ 2 & -1{,}05 \end{bmatrix} \begin{bmatrix} w_1 \\ \omega_1 \end{bmatrix} = \begin{bmatrix} 0{,}8 \\ 0{,}2667 \end{bmatrix}$$

$D_{et} = -6{,}45$

$D_1 = -0{,}8133 \longrightarrow$ $\boxed{w_1 = 0{,}126 \ m}$

$D_2 = -0{,}0889 \longrightarrow$ $\boxed{\omega_1 = 0{,}0138 \ rd}$

Ad 3.



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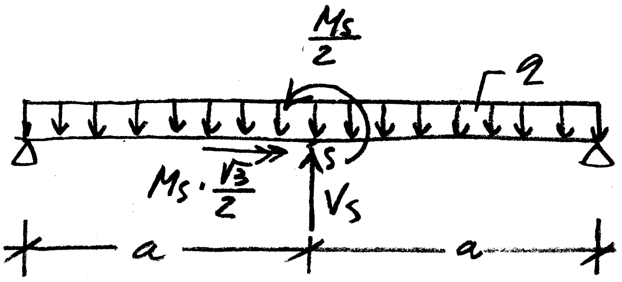


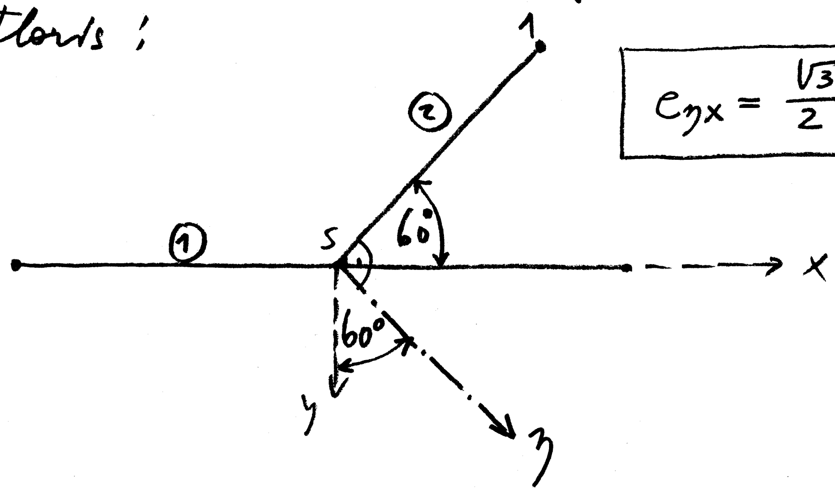
$$w_S = \frac{5q(2a)^4}{384 E I_y} - \frac{V_S (2a)^3}{48 E I_y}$$

$$\boxed{w_S = \frac{5qa^4}{24 E I_y} - \frac{V_S a^3}{6 E I_y}}$$

$$\boxed{\omega_y^S = \frac{M_S a}{12 E I_y}}$$

$$\boxed{\omega_x^S = \frac{M_S \sqrt{3}\, a}{4 G I_x}}$$

Tloris:



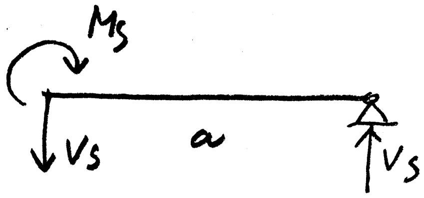
$e_{\eta x} = \frac{\sqrt{3}}{2}$	$e_{\eta y} = \frac{1}{2}$

$$\omega_\eta^{②}(s) = \omega_x^S e_{\eta x} + \omega_y^S e_{\eta y} = \frac{M_S \sqrt{3}\, a}{4 G I_x} \cdot \frac{\sqrt{3}}{2} + \frac{M_S a}{12 E I_y} \cdot \frac{1}{2}$$

$$\omega_\eta^{②}(s) = \frac{M_S a}{E I_y}\left(\frac{3}{8} + \frac{1}{24}\right) \rightarrow \boxed{\omega_\eta^{②}(s) = \frac{5 M_S a}{12 E I_y}}$$

$$w_1 = w_S - a\, \omega_\eta^{②}(s) - V_S \cdot \frac{a^3}{3 E I_y} = 0 \quad !$$

②



$$\boxed{V_S = \frac{M_S}{a}}$$

$$w_1 = \frac{5qa^4}{24EI_y} - \frac{M_s a^2}{6EI_y} - \frac{5M_s a^2}{12EI_y} - \frac{M_s a^2}{3EI_y} = 0$$

$$M_s\left(\frac{1}{6} + \frac{5}{12} + \frac{1}{3}\right) = \frac{5qa^2}{24} \rightarrow \boxed{M_s = \frac{5qa^2}{22}}$$

$$\boxed{V_s = \frac{5qa}{22}}$$