MTT IZPIT 18. 9. 1990

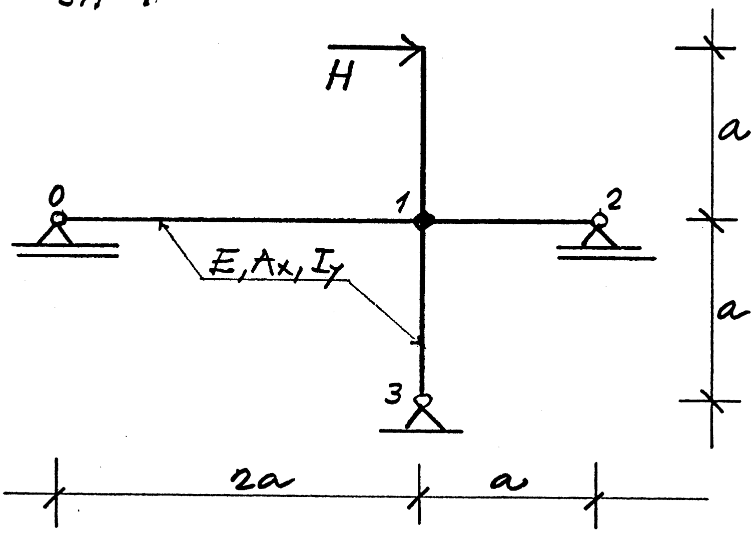
1. Rešitev mehanskega problema trdnega telesa je matrika majhnih deformacij ε_{ij}. V točki $T_0(0,0,0)$ je telo togo vpeto. V točki $T(2,2,0)$ določi:
   a) velikosti in smeri glavnih normalnih napetosti
   b) deformirano koordinatno bazo

$$[\varepsilon_{ij}] = 10^{-3} \cdot \begin{bmatrix} 10x & y^2 & 0 \\ y^2 & 4xy & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$2\mu = 16\,000\ kN/cm^2$

$\lambda = 8000\ kN/cm^2$

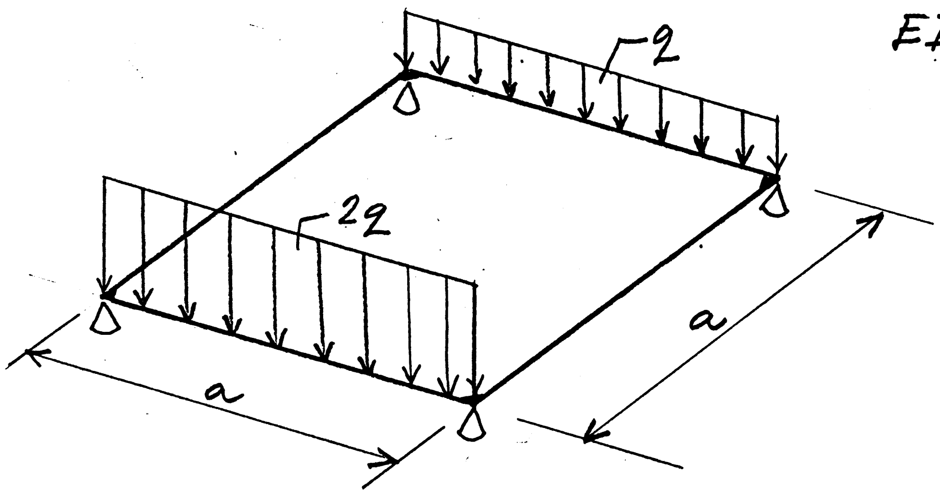
2. Določi pomike točke 2 ter skiciraj potek notranjih sil!



$E = 21000\ kN/cm^2$
$A_x = 114\ cm^2$
$I_y = 5870\ cm^4$
$a = 5\ m$
$H = 10\ kN$

Vpliv osne podajnosti stebra 1-3 lahko zanemariš.

3. Določi in skiciraj notranje sile!



$EI_y = GI_x$

MTT IZPIT 26. 1. 1990

Ad 1.

a)
$$[\varepsilon_{ij}]_T = 10^{-3}\begin{bmatrix} 20 & 4 & 0 \\ 4 & 16 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0{,}020 & 0{,}004 & 0 \\ 0{,}004 & 0{,}016 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\varepsilon_{11,22} = 10^{-3}\left(\frac{20+16}{2} \pm \sqrt{\left(\frac{20-16}{2}\right)^2 + 4^2}\right)$$

$\varepsilon_{11} = 22{,}47 \cdot 10^{-3}$	$\varepsilon_{22} = 13{,}53 \cdot 10^{-3}$	$\varepsilon_{33} = 0$

$$I_1^{\varepsilon} = (20+16)\cdot 10^{-3} = 36 \cdot 10^{-3}$$

$$\sigma_{11} = 2\mu\,\varepsilon_{11} + \lambda I_1^{\varepsilon} \longrightarrow \sigma_{11} = 648\ kN/cm^2$$
$$\sigma_{22} = 2\mu\,\varepsilon_{22} + \lambda I_1^{\varepsilon} \longrightarrow \sigma_{22} = 504\ kN/cm^2$$
$$\sigma_{33} = \lambda I_1^{\varepsilon} \longrightarrow \sigma_{33} = 288\ kN/cm^2$$

$$tg\, 2\alpha_\sigma = tg\, 2\alpha_\varepsilon = \frac{2\cdot 4}{20-16} = 2 \longrightarrow \boxed{\alpha_\sigma = 31{,}7^\circ}$$

b)
$$\omega_z = \omega_z(T_0) + \int_0^x \vec{e}_z\,(\vec{\nabla}\times\vec{\varepsilon}_x)_{y=0}\,dx + \int_0^y \vec{e}_z\,(\vec{\nabla}\times\vec{\varepsilon}_y)\,dy$$

$$\vec{e}_z(\vec{\nabla}\times\vec{\varepsilon}_x) = \begin{vmatrix} 0 & 0 & 1 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \varepsilon_{xx} & \varepsilon_{xy} & 0 \end{vmatrix} = \frac{\partial \varepsilon_{xy}}{\partial x} - \frac{\partial \varepsilon_{xx}}{\partial y} = 0$$

$$\vec{e}_z(\vec{\nabla}\times\vec{\varepsilon}_y) = \begin{vmatrix} 0 & 0 & 1 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \end{vmatrix} = \frac{\partial \varepsilon_{yy}}{\partial x} - \frac{\partial \varepsilon_{xy}}{\partial y} = 2y\cdot 10^{-3}$$

$$\omega_z = 0 + 10^{-3}\int_0^y 2y\,dy \longrightarrow \boxed{\omega_z = 10^{-3}y^2 = \omega_{xy}}$$

$$u_x = u_x(T_0) + \int_0^x (\varepsilon_{xx})_{y=0}\, dx + \int_0^y (\varepsilon_{yx} + \omega_{yx})\, dy$$

$$u_x = 0 + 10^{-3}\left(\int_0^x 10x\, dx + \int_0^y (y^2 - y^2)\, dy\right)$$

$$\boxed{u_x = 5x^2 \cdot 10^{-3}}$$

$$u_y = u_y(T_0) + \int_0^x (\varepsilon_{xy} + \omega_{xy})_{y=0}\, dx + \int_0^y \varepsilon_{yy}\, dy$$

$$u_y = 0 + 10^{-3} \int_0^x (y^2 + y^2)_{y=0}\, dx + \int_0^y 4xy\, dy$$

$$\boxed{u_y = 2xy^2 \cdot 10^{-3}}$$

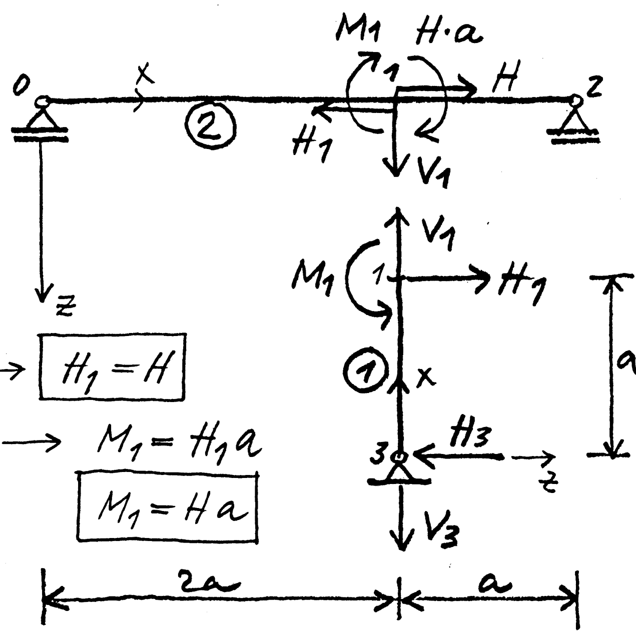
$$\vec{e}_x{}' = \vec{e}_x + \frac{\partial \vec{u}}{\partial x} = \vec{e}_x + 10^{-3}(10x\, \vec{e}_x + 2y^2\, \vec{e}_y)$$

$$\vec{e}_y{}' = \vec{e}_y + \frac{\partial \vec{u}}{\partial y} = \vec{e}_y + 10^{-3}(0 + 4xy\, \vec{e}_y)$$

V točki $T(2,2,0)$:

$$\boxed{\begin{aligned} \vec{e}_x{}' &= 1{,}020\, \vec{e}_x + 0{,}080\, \vec{e}_y \\ \vec{e}_y{}' &= 1{,}016\, \vec{e}_y \\ \vec{e}_z{}' &= \vec{e}_z \end{aligned}}$$

Ad 2.)



$$\boxed{\begin{aligned} u_z^{2}(1) &= -u_x^{1}(1) \\ \omega_y^{2}(1) &= \omega_y^{1}(1) \end{aligned}}$$

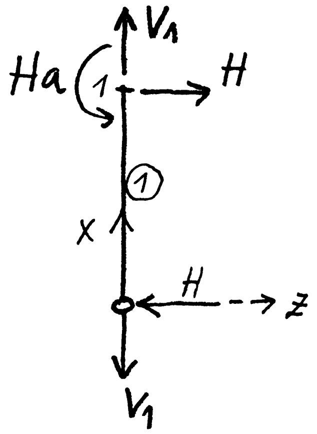
② $\sum X = 0 \longrightarrow \boxed{H_1 = H}$

① $\sum M^3 = 0 \longrightarrow M_1 = H_1 a$

$\boxed{V_3 = V_1}$ $\boxed{M_1 = H a}$

$\boxed{H_3 = H}$

①



$$M_y = Hx = -w'' E I_y$$

$$E I_y w' = -H\frac{x^2}{2} + C_1$$

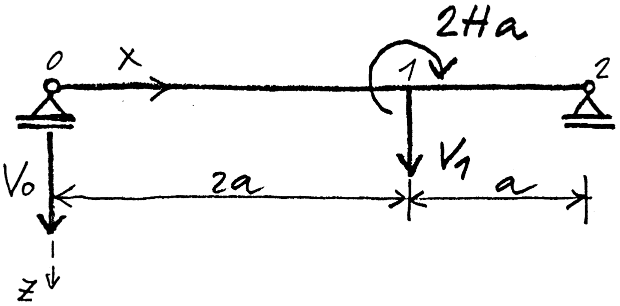
$$E I_y w = -H\frac{x^3}{6} + C_1 x + C_2$$

$$x=0 \rightarrow w=0 \rightarrow C_2 = 0$$

$$x = a :$$

$$\boxed{\begin{aligned} w(a) &= u_z^{①}(1) = -\frac{Ha^3}{6EI_y} + \frac{C_1 a}{EI_y} \\ \omega_y(a) &= \omega_y^{①}(1) = \frac{Ha^2}{2EI_y} - \frac{C_1}{EI_y} \\ u_x^{①}(1) &= \frac{V_1 a}{EA_x} \end{aligned}}$$

②



$$\boxed{V_0 = \frac{2H}{3} - \frac{V_1}{3}}$$

$$M_y = -V_0 x - V_1 \langle x-2a\rangle + 2Ha\langle x-2a\rangle^0 = -EI_y w''$$

$$EI_y w'' = \frac{2H}{3}x - \frac{V_1}{3}x + V_1\langle x-2a\rangle - 2Ha\langle x-2a\rangle^0$$

$$EI_y w'' = \frac{2H}{3}\left(x - 3a\langle x-2a\rangle^0\right) - \frac{V_1}{3}\left(x - 3\langle x-2a\rangle\right)$$

$$EI_y w' = \frac{2H}{3}\left(\frac{x^2}{2} - 3a\langle x-2a\rangle\right) - \frac{V_1}{6}\left(x^2 - 3\langle x-2a\rangle^2\right) + C_3$$

$$EI_y w = \frac{2H}{3}\left(\frac{x^3}{6} - \frac{3a}{2}\langle x-2a\rangle^2\right) - \frac{V_1}{6}\left(\frac{x^3}{3} - \langle x-2a\rangle^3\right) + C_3 x + C_4$$

$$x=0 \; ; \; w=0 \rightarrow \boxed{C_4 = 0}$$

$$x = 3a \; ; \; w = 0$$

$$\frac{2Ha^3}{3}\left(\frac{27}{6} - \frac{3}{2}\right) - \frac{V_1 a^3}{6}\left(\frac{27}{3} - 1\right) + 3C_3 a = 0$$

$$\boxed{C_3 = -\frac{2Ha^2}{3} + \frac{4V_1 a^2}{9}}$$

Pomik in zasuk v točki 1 $(x=2a)$:

$$EI_y\,\omega_y(1) = -\frac{2H}{3}\cdot\frac{4a^2}{2} + \frac{V_1}{6}\cdot 4a^2 + \frac{2Ha^2}{3} - \frac{4V_1a^2}{9}$$

$$\boxed{\omega_y^{②}(1) = -\frac{2Ha^2}{3EI_y} + \frac{2V_1a^2}{9EI_y}}$$

$$EI_y\,w(1) = \frac{2H}{3}\cdot\frac{8a^3}{6} - \frac{V_1}{6}\cdot\frac{8a^3}{3} + 2a\left(-\frac{2Ha^2}{3} + \frac{4V_1a^2}{9}\right)$$

$$\boxed{u_z^{②}(1) = -\frac{4Ha^3}{9EI_y} + \frac{4V_1a^3}{9EI_y}}$$

$$\boxed{u_z^{②}(1) = -u_x^{①}(1)} \longrightarrow -\frac{4Ha^3}{9EI_y} + \frac{4V_1a^3}{9EI_y} = -\frac{V_1a}{EA_x}$$

$$V_1\left(\frac{4a^2}{9I_y} + \frac{1}{A_x}\right) = \frac{4Ha^2}{9I_y} \longrightarrow \boxed{V_1 = H\,\frac{4a^2A_x}{4a^2A_x + 9I_y}}$$

$$V_1 = H\,\frac{4\cdot 500^2\cdot 114}{4\cdot 500^2\cdot 114 + 9\cdot 5870} \doteq H \longrightarrow \boxed{V_1 = H}$$

$$\boxed{\omega_y^{②}(1) = \omega_y^{①}(1)} \longrightarrow -\frac{2Ha^2}{3EI_y} + \frac{2V_1a^2}{9EI_y} = \frac{Ha^2}{2EI_y} - \frac{C_1}{EI_y}$$

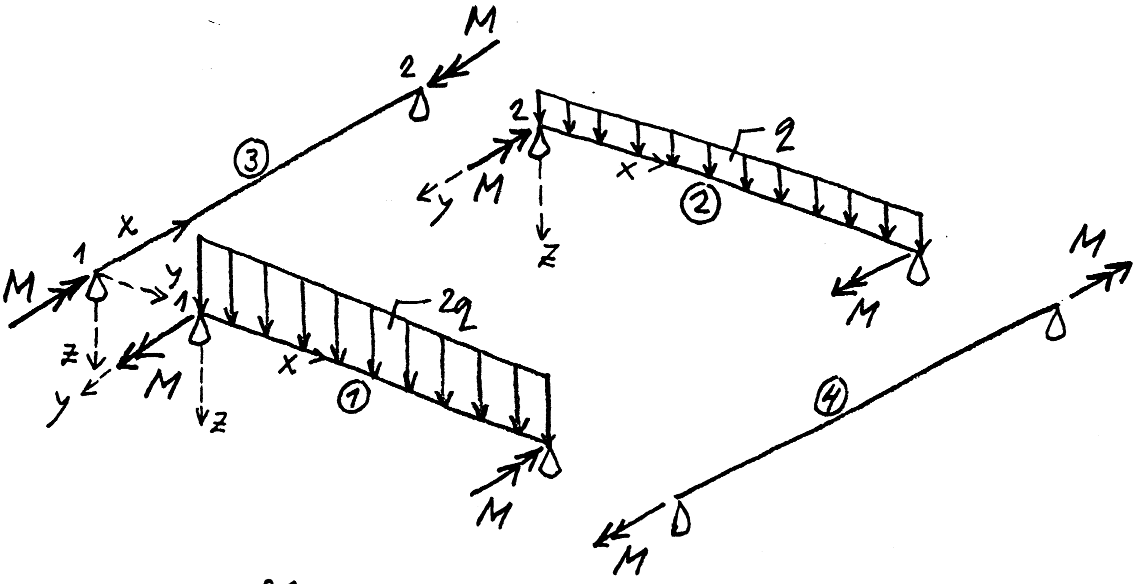
$$\boxed{C_1 = \frac{17Ha^2}{18}}$$

$$u_x(2) = u_z^{①}(1) = -\frac{Ha^3}{6EI_y} + C_1a = \frac{Ha^3}{EI_y}\left(-\frac{1}{6} + \frac{17}{18}\right)$$

$$\boxed{u_x(2) = \frac{7Ha^3}{9EI_y}} \qquad u_x(2) = \frac{7\cdot 10\cdot 500^3}{9\cdot 21000\cdot 5870}$$

$$\boxed{u_x(2) = 7{,}89\ \text{cm}}$$

$$V_0 = \frac{2Ha}{3a} - H\frac{a}{3a} \longrightarrow \boxed{V_0 = \frac{H}{3}}$$



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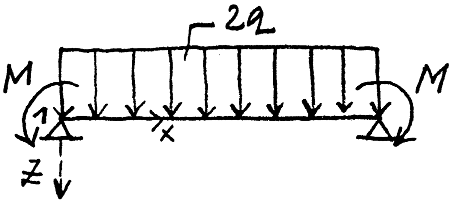
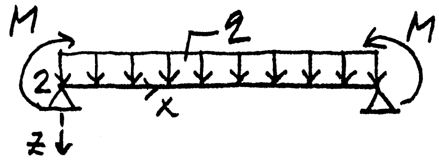


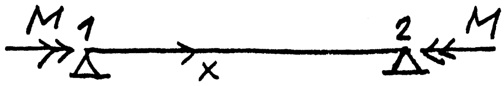
$$\omega_y^{①}(1) = \frac{Ma}{2EI_y} - \frac{2qa^3}{24EI_y}$$

②



$$\omega_y^{②}(2) = -\frac{Ma}{2EI_y} - \frac{qa^3}{24EI_y}$$

③



$$M_x = -M$$

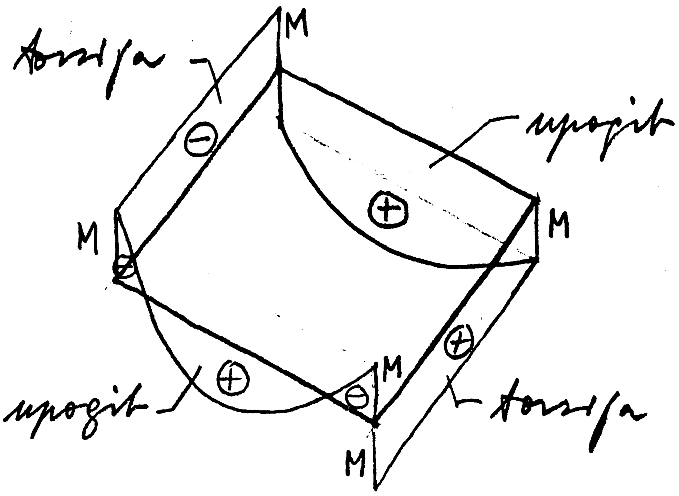
$$\frac{d\omega_x}{dx} = \frac{M_x}{GI_x} = -\frac{M}{GI_x} \longrightarrow \omega_x^{③} = -\frac{Mx}{GI_x} + C_1$$

$$x=0 \ \ldots \ \omega_x^{③}(1) = -\omega_y^{①}(1) \longrightarrow C_1 = -\omega_y^{①}(1)$$

$$x=a \ \ldots \ \omega_x^{③}(2) = -\omega_y^{①}(1) - \frac{Ma}{GI_x} = -\omega_y^{②}(2)$$

$$\frac{Ma}{2EI_y} - \frac{2qa^3}{24EI_y} + \frac{Ma}{GI_x} = -\frac{Ma}{2EI_y} - \frac{qa^3}{24EI_y}$$

$$\boxed{M = \frac{qa^2}{48}}$$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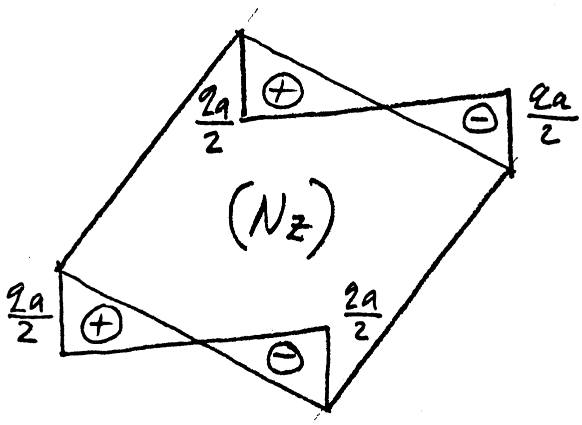
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