MTT IZPIT 3. 6. 1992

1. Napetostno stanje telesa je v točki T opisano s komponentami σ_{ij} tenzorja napetosti glede na koordinatni sistem (x, y, z).

a) določi rezutirajoči vektor napetosti v ravnini, katere normala $\vec{e}_\xi$ oklepa enake kote z osmi x, y, z in dokaži, da je rezultirajoča napetost pravokotna na to ravnino!

$$[\sigma_{ij}] = \begin{bmatrix} q & 2q & 2q \\ 2q & q & 2q \\ 2q & 2q & q \end{bmatrix}$$

b) določi velikosti in smeri glavnih normalnih napetosti

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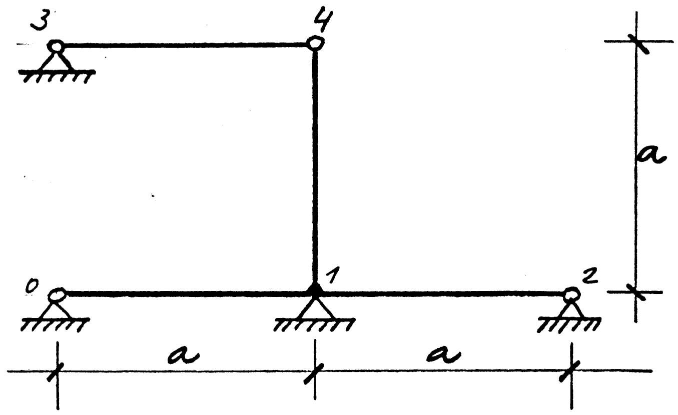
* 2. Določi in skiciraj notranje sile, ki nastopijo v prikazani konstrukciji, če jo segrejemo za $\Delta T = 40\,K$!



$E = 20\,000\ kN/cm^2$

$a = 3\ m$

$A_x = 10\ cm^2$

$I_y = 1000\ cm^4$

$\alpha_T = 1{,}25 \cdot 10^{-5}/K$

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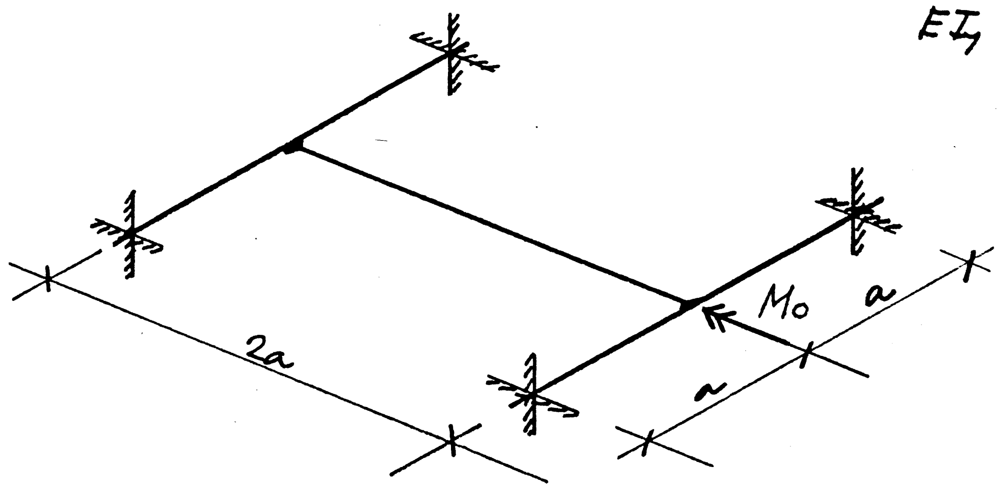
3. Določi in skiciraj notranje sile!



$EI_y = GI_x$

MTT IZPIT 15. 5. 1989

Ad 1.

a) $e_{\xi x} = e_{\xi y} = e_{\xi z} \longrightarrow 3e_{\xi x}^2 = 1 \longrightarrow e_{\xi x} = \frac{1}{\sqrt{3}}$

$$\vec{e}_\xi = \frac{1}{\sqrt{3}}(\vec{e}_x + \vec{e}_y + \vec{e}_z)$$

$$\begin{Bmatrix} \sigma_{\xi x} \\ \sigma_{\xi y} \\ \sigma_{\xi z} \end{Bmatrix} = \begin{bmatrix} q & 2q & 2q \\ 2q & q & 2q \\ 2q & 2q & q \end{bmatrix} \cdot \frac{1}{\sqrt{3}} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \frac{5}{\sqrt{3}} \begin{Bmatrix} q \\ q \\ q \end{Bmatrix}$$

$$\boxed{\vec{\sigma}_\xi = \frac{5q}{\sqrt{3}}(\vec{e}_x + \vec{e}_y + \vec{e}_z)}$$

$$\sigma_{\xi\xi} = \vec{\sigma}_\xi \, \vec{e}_\xi = \frac{5q}{3}(1+1+1) \longrightarrow \boxed{\sigma_{\xi\xi} = 5q}$$

$$\boxed{\vec{\sigma}_\xi = \sigma_{\xi\xi} \, \vec{e}_\xi}$$

b) $\boxed{\sigma_{\xi\xi} \equiv \sigma_{33} = 5q}$

$$I_1 = 3q$$

$$I_2 = 3 \begin{vmatrix} q & 2q \\ 2q & q \end{vmatrix} = -9q^2$$

$$I_3 = q(q^2 - 4q^2) - 2q(2q^2 - 4q^2) + 2q(4q^2 - 2q^2)$$

$$I_3 = 5q^3$$

$$\boxed{\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0} \longrightarrow$$

$$\sigma^3 - 3q\sigma^2 - 9q^2\sigma - 5q^3 = 0$$

$$(\sigma^3 - 3q\sigma^2 - 9q^2\sigma - 5q^3) : (\sigma - 5q) = \sigma^2 + 2q\sigma + q^2$$

$$\underline{-\sigma^3 + 5q\sigma^2}$$

$$2q\sigma^2 - 9q^2\sigma$$

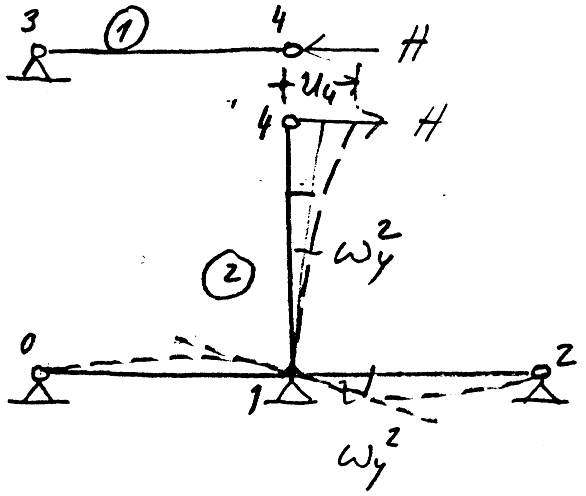
$$\underline{-2q\sigma^2 + 10q^2\sigma}$$

$$q^2\sigma - 5q^3 \checkmark$$

$$\sigma^2 + 2q\sigma + q^2 = 0 \longrightarrow \boxed{\sigma_{11} = \sigma_{22} = -q}$$

Glavna smer $\vec{e}_3 \equiv \vec{e}_\zeta$, glavni smeri $\vec{e}_1$ in $\vec{e}_2$ sta pravokotni na $\vec{e}_\zeta$; sicer pa poljubni, ker je $\sigma_{11} = \sigma_{22}$!

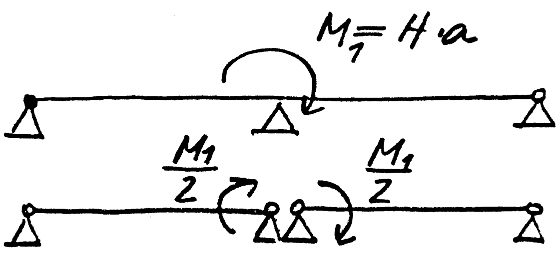
Ad 2.



$$\boxed{u_4^{(1)} = u_4^{(2)}}$$

(1) : $u_4^{(1)} = -\dfrac{Ha}{EA_x} + a\,\alpha_T\,\Delta T$

(2) : $u_4^{(2)} = -a\,\omega_y^2 + \dfrac{Ha^3}{3EI_y}$



$$\omega_y^2 = -\frac{Ha}{2} \cdot \frac{a}{3EI_y}$$

$$\omega_y^2 = -\frac{Ha^2}{6EI_y}$$

$$u_4^{(2)} = \frac{Ha^3}{6EI_y} + \frac{Ha^3}{3EI_y} \rightarrow \underline{\underline{u_4^{(2)} = \frac{Ha^3}{2EI_y}}}$$

$$-\frac{Ha}{EA_x} + a\alpha_T\Delta T = \frac{Ha^3}{2EI_y}$$

$$H\left(\frac{a}{EA_x} + \frac{a^3}{2EI_y}\right) = a\alpha_T\Delta T$$

$$H\,\frac{2I_y + a^2A_x}{2A_xI_y} = E\alpha_T\Delta T \longrightarrow \boxed{H = \frac{2A_xI_yE\alpha_T\Delta T}{2I_y + a^2A_x}}$$

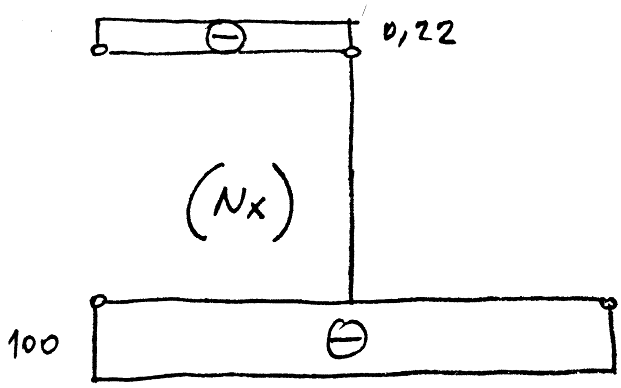
$$H = \frac{2\cdot10\cdot1000\cdot20000\cdot1{,}25\cdot10^{-5}\cdot40}{2\cdot1000 + 300^2\cdot10}$$

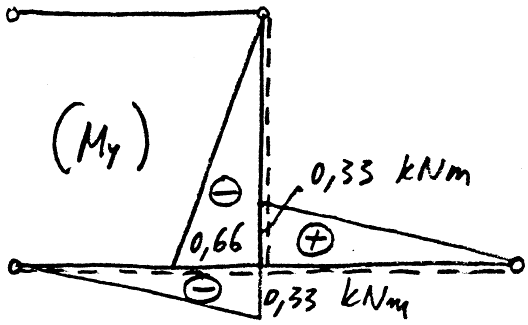
$$\boxed{H = 0{,}22\ kN}$$

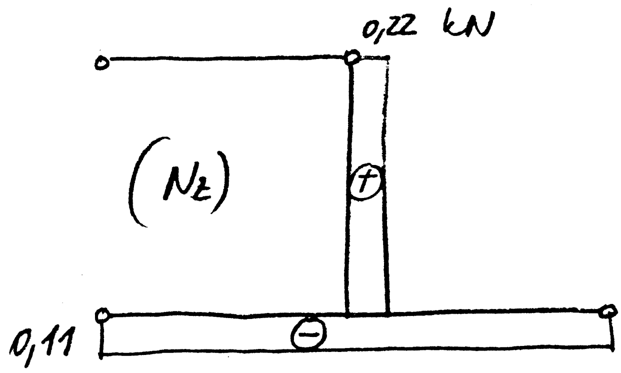
Nosilec 0-2 :

$$-\frac{Na}{EA_x} + a\alpha_T\Delta T = 0 \longrightarrow \boxed{N = EA_x\alpha_T\Delta T}$$

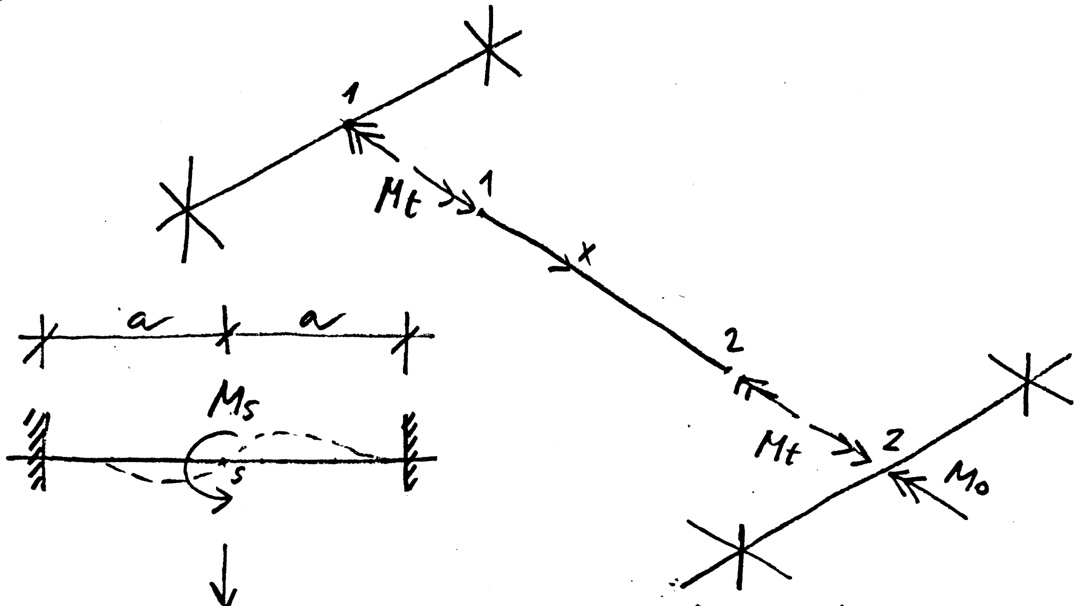
$$\boxed{N = 100\ kN}$$

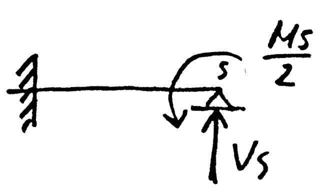






Ad 3.





$$w_s = -V_s \frac{a^3}{3EI_y} - \frac{M_s}{2} \cdot \frac{a^2}{2EI_y} = 0$$

$$\boxed{V_s = -\frac{3M_s}{4a}}$$

$$\omega_s = V_s \frac{a^2}{2EI_y} + \frac{M_s}{2} \frac{a}{EI_y} \longrightarrow \boxed{\omega_s = \frac{M_s a}{8EI_y}}$$

$$\boxed{\omega_1 = -\frac{M_t a}{8EI_y}} \qquad \boxed{\omega_2 = (M_t - M_0) \frac{a}{8EI_y}}$$

$$\omega_2 = \omega_1 - M_t \frac{2a}{GI_x} = -M_t \frac{a}{8EI_y} - M_t \frac{2a}{GI_x}$$

$$-M_t \left(\frac{a}{8EI_y} + \frac{2a}{EI_y} \right) = (M_t - M_0) \frac{a}{8EI_y}$$

$$\boxed{M_t = \frac{M_0}{18}}$$