1. Napetostno stanje telesa je v točki T opisano s komponentami $\sigma_{ij}$ tenzorja napetosti glede na koordinatni sistem $(x, y, z)$.
   a. Določi rezultirajoči vektor napetosti v ravnini, katere normala oklepa enake kote z osmi $x$, $y$, $z$ in dokaži, da je rezultirajoča napetost pravokotna na to ravnino!
   b. Določi velikosti in smeri glavnih normalnih napetosti!

$$[\sigma_{ij}]_T = \begin{bmatrix} q & 2q & 2q \\ 2q & q & 2q \\ 2q & 2q & q \end{bmatrix}$$

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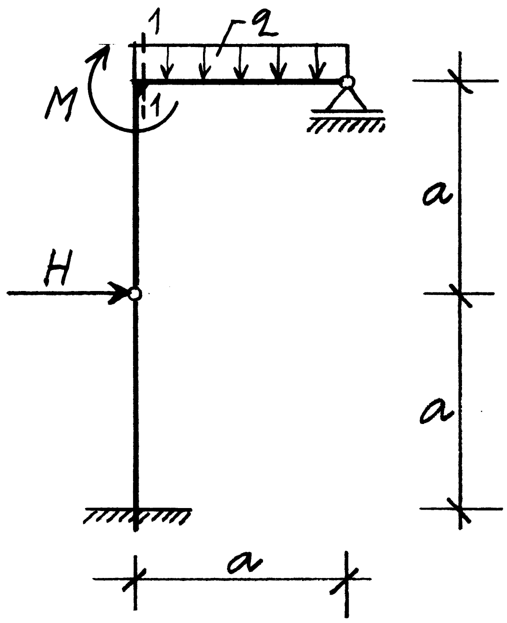
2. Z izrekom o virtualnem delu določi vse reakcije ter upogibni moment in prečno silo v prerezu 1–1!

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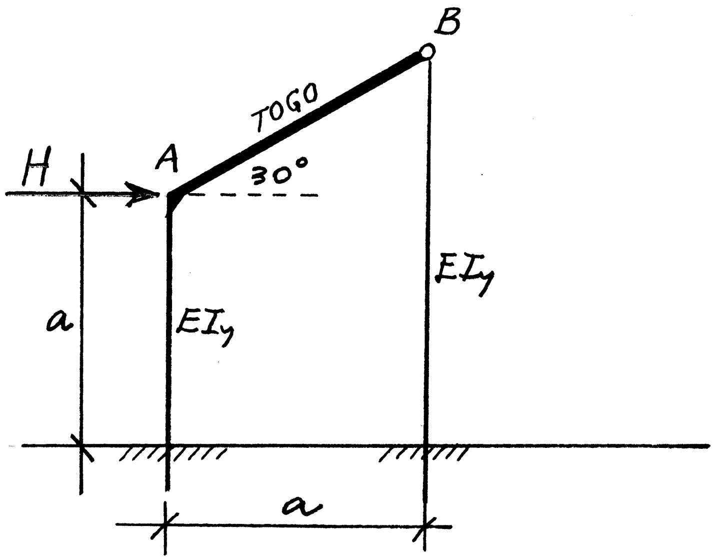
3. Določi vektor pomika vozlišča B ter notranje sile v togi prečki AB! Vpliv osnih sil na pomike stebrov lahko zanemariš.




$E = 20\,000\,kN/cm^2$

$I_y = 4\,000\,cm^4$

$a = 4\,m$

$H = 40\,kN$

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## Ad 1.

a) $e_{\xi x} = e_{\xi y} = e_{\xi z} \longrightarrow 3e_{\xi x}^2 = 1 \longrightarrow e_{\xi x} = \frac{1}{\sqrt{3}}$

$$\vec{e}_\xi = \frac{1}{\sqrt{3}}(\vec{e}_x + \vec{e}_y + \vec{e}_z)$$

$$\begin{Bmatrix} \sigma_{\xi x} \\ \sigma_{\xi y} \\ \sigma_{\xi z} \end{Bmatrix} = \begin{bmatrix} q & 2q & 2q \\ 2q & q & 2q \\ 2q & 2q & q \end{bmatrix} \cdot \frac{1}{\sqrt{3}} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \frac{5}{\sqrt{3}} \begin{Bmatrix} q \\ q \\ q \end{Bmatrix}$$

$$\boxed{\vec{\sigma}_\xi = \frac{5q}{\sqrt{3}}(\vec{e}_x + \vec{e}_y + \vec{e}_z)}$$

$$\sigma_{\xi\xi} = \vec{\sigma}_\xi \, \vec{e}_\xi = \frac{5q}{3}(1+1+1) \longrightarrow \boxed{\sigma_{\xi\xi} = 5q}$$

$$\boxed{\vec{\sigma}_\xi = \sigma_{\xi\xi} \, \vec{e}_\xi}$$

b) $\boxed{\sigma_{\xi\xi} \equiv \sigma_{33} = 5q}$

$$I_1 = 3q$$

$$I_2 = 3\begin{vmatrix} q & 2q \\ 2q & q \end{vmatrix} = -9q^2$$

$$I_3 = q(q^2 - 4q^2) - 2q(2q^2 - 4q^2) + 2q(4q^2 - 2q^2)$$

$$I_3 = 5q^3$$

$$\boxed{\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0} \longrightarrow$$

$$\sigma^3 - 3q\sigma^2 - 9q^2\sigma - 5q^3 = 0$$

$$(\sigma^3 - 3q\sigma^2 - 9q^2\sigma - 5q^3) : (\sigma - 5q) = \sigma^2 + 2q\sigma + q^2$$

$$\underline{-\sigma^3 + 5q\sigma^2}$$

$$2q\sigma^2 - 9q^2\sigma$$

$$\underline{-2q\sigma^2 + 10q^2\sigma}$$

$$q^2\sigma - 5q^3 \checkmark$$

$$\sigma^2 + 2q\sigma + q^2 = 0 \longrightarrow \boxed{\sigma_{11} = \sigma_{22} = -q}$$

Glavna smer $\vec{e}_3 \equiv \vec{e}_\zeta$, glavni smeri $\vec{e}_1$ in $\vec{e}_2$ sta pravokotni na $\vec{e}_3$, sicer pa poljubni, ker je $\sigma_{11} = \sigma_{22}$.

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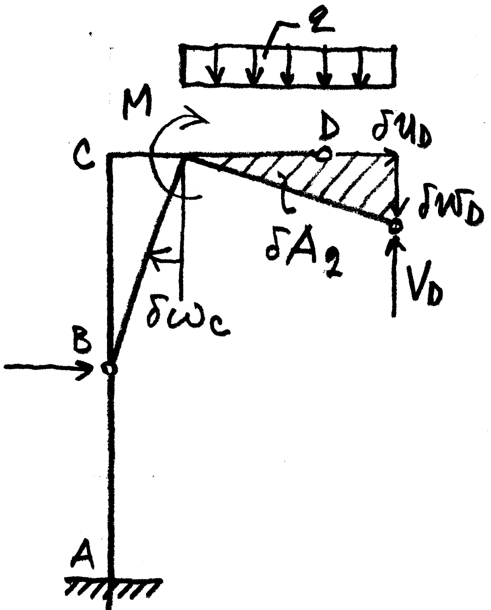
Ad 2.)



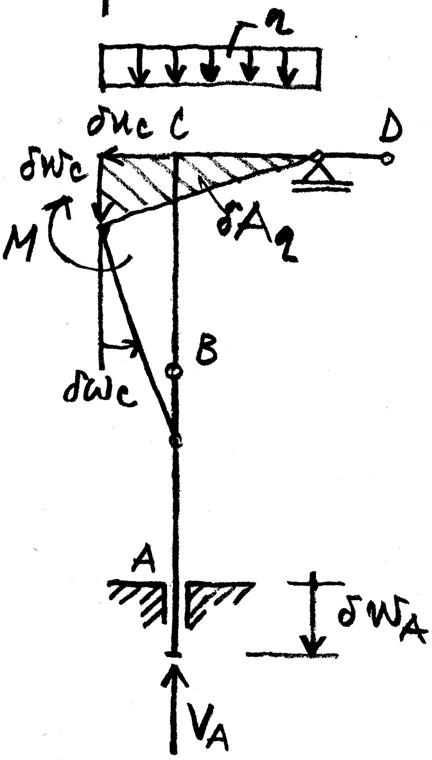

$$\delta W = -\delta w_D V_D + \delta\omega_C M + \delta A_q \cdot q = 0$$

$$\delta\omega_C = \frac{\delta w_D}{a}, \quad \delta A_q = \delta w_D \cdot \frac{a}{2}$$

$$-\delta w_D V_D + \frac{\delta w_D}{a} M + \delta w_D \frac{a}{2} q = 0$$

$$\delta w_D \underbrace{\left(-V_D + \frac{M}{a} + \frac{qa}{2}\right)}_{0} = 0$$

$$\boxed{V_D = \frac{M}{a} + \frac{qa}{2}}$$



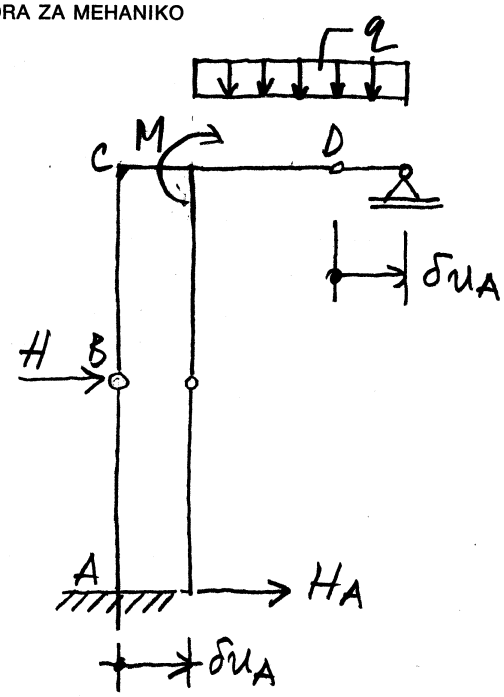

$$\delta W = -\delta w_A V_A - \delta\omega_C M + \delta A_q \cdot q = 0$$

$$\delta\omega_C = \frac{1}{a}\delta w_A, \quad \delta A_q = \delta w_A \cdot \frac{a}{2}$$

$$-\delta w_A \cdot V_A - \frac{\delta w_A}{a} M + \delta w_A \cdot \frac{a}{2} \cdot q = 0$$

$$\delta w_A \left(-V_A - \frac{M}{a} + \frac{qa}{2}\right) = 0$$

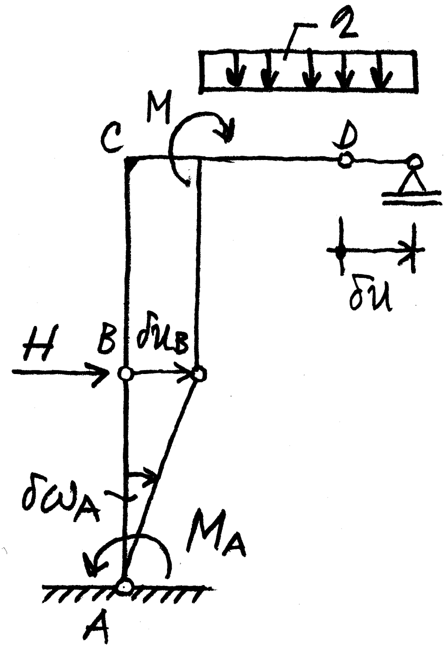
$$\boxed{V_A = -\frac{M}{a} + \frac{qa}{2}}$$

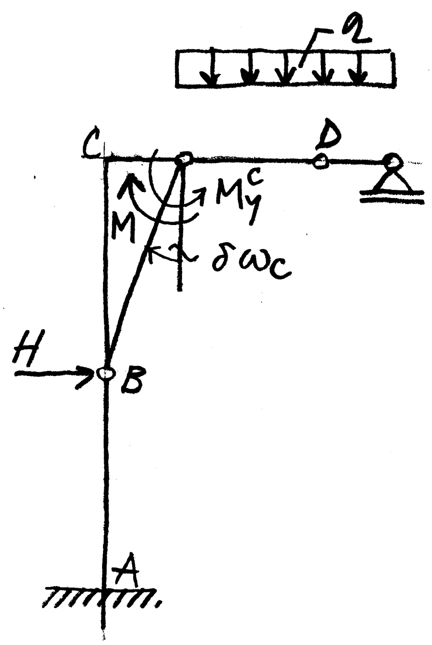



$$\delta W = \delta u_A H_A + \delta u_A H = 0$$

$$\delta u_A (H_A + H) = 0$$

$$\boxed{H_A = -H}$$




$$\delta W = -\delta \omega_A M_A + \delta u_B H = 0$$

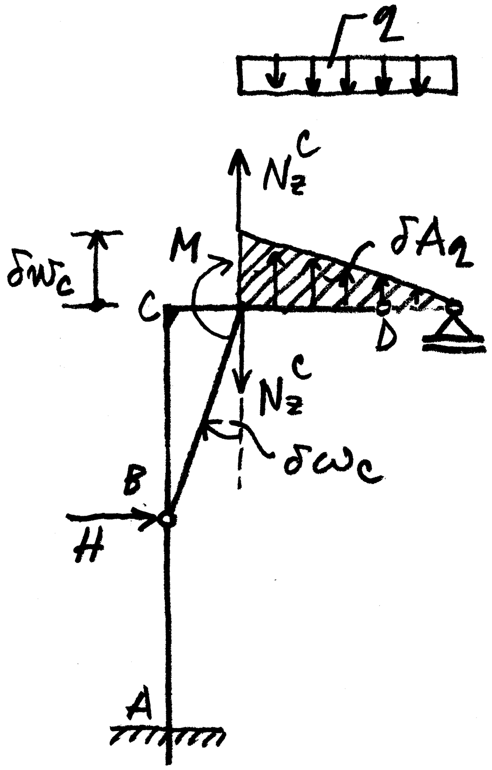
$$\delta u_B = a\, \delta \omega_A$$

$$\delta \omega_A (-M_A + aH) = 0$$

$$\boxed{M_A = aH}$$

$$\delta W = \delta \omega_C (M - M_y^C) = 0$$

$$\boxed{M_y^C = M}$$




$$\delta W = \delta w_C N_z^C - \delta A_q \cdot q + \delta\omega_C M = 0$$

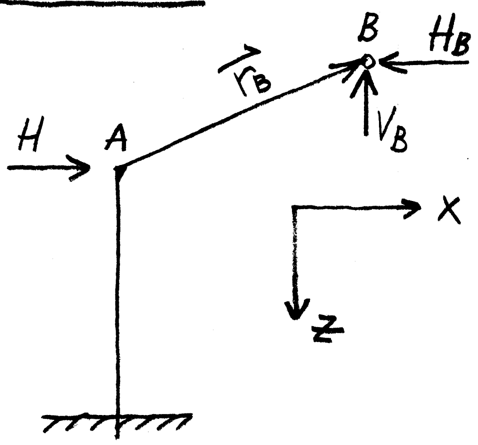
$$\delta A_q = \delta w_C \cdot \frac{a}{2}$$

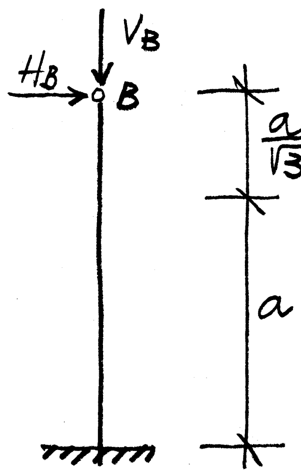
$$\delta\omega_C = \frac{\delta w_C}{a}$$

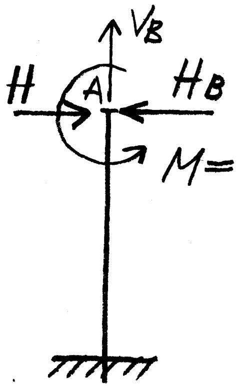
$$\delta w_C \left( N_z^C - \frac{qa}{2} + \frac{M}{a} \right) = 0$$

$$\boxed{N_z^C = \frac{qa}{2} - \frac{M}{a}}$$

## Ad 3.)




$$M = a V_B + \frac{a}{\sqrt{3}} H_B$$

$$\vec{r}_B = a\,\vec{e}_x - \frac{a}{\sqrt{3}}\,\vec{e}_z$$

$$A: \quad u_x^A = (H - H_B)\frac{a^3}{3EI_y} - \left(aV_B + \frac{a}{\sqrt{3}}H_B\right)\frac{a^2}{2EI_y}$$

$$\omega_y^A = -(H - H_B)\frac{a^2}{2EI_y} + \left(aV_B + \frac{a}{\sqrt{3}}H_B\right)\frac{a}{EI_y}$$

$$B: \quad u_x^B = H_B \frac{a^3\left(1+\frac{1}{\sqrt{3}}\right)^3}{3EI_y}$$

$$\vec{u}_B = \vec{u}_A + \vec{\omega}_A \times \vec{r}_B = u_x^A \vec{e}_x + \omega_y^A \vec{e}_y \times \left(a\vec{e}_x - \frac{a}{\sqrt{3}}\vec{e}_z\right)$$

$$u_x^B \vec{e}_x = \left(u_x^A - \omega_y^A \frac{a}{\sqrt{3}}\right)\vec{e}_x - \omega_y^A a \vec{e}_z$$

$$\omega_y^A a = 0 \longrightarrow \boxed{\omega_y^A = 0}$$

$$u_x^B = u_x^A - \omega_y^A \frac{a}{\sqrt{3}} \longrightarrow \boxed{u_x^B = u_x^A}$$

$$\omega_y^A = -H\frac{a^2}{2EI_y} + H_B\frac{a^2}{2EI_y} + V_B\frac{a^2}{EI_y} + H_B\frac{a^2}{\sqrt{3}EI_y} = 0$$

$$-H\cdot\frac{1}{2} + H_B\cdot\frac{1}{2} + V_B + H_B\frac{1}{\sqrt{3}} = 0$$

$$\boxed{H_B\left(2+\sqrt{3}\right) + V_B\cdot 2\sqrt{3} = H\cdot\sqrt{3}}$$

$$u_x^A = u_x^B \longrightarrow$$

$$H\frac{a^3}{3EI_y} - H_B\frac{a^3}{3EI_y} - V_B\frac{a^3}{2EI_y} - H_B\frac{a^3}{2\sqrt{3}EI_y} = H_B\frac{a^3\left(1+\frac{1}{\sqrt{3}}\right)^3}{3EI_y}$$

$$H\cdot\frac{1}{3} - H_B\left[\frac{1}{3} + \frac{1}{2\sqrt{3}} + \frac{1}{3}\left(1+\frac{1}{\sqrt{3}}\right)^3\right] - V_B\cdot\frac{1}{2} = 0$$

$$\boxed{H_B\left[2\sqrt{3} + 3 + 2\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^3\right] + V_B\cdot 3\sqrt{3} = H\cdot 2\sqrt{3}}$$

$$\begin{bmatrix} 3.7321 & 3.4641 \\ 20.0590 & 5.1962 \end{bmatrix} \cdot \begin{bmatrix} H_B \\ V_B \end{bmatrix} = H \begin{bmatrix} 1,7321 \\ 3,4641 \end{bmatrix}$$

$$D = -50,0940$$
$$D_1 = -3\ H$$
$$D_2 = -21,8150\ H$$

$$\boxed{\begin{aligned} H_B &= 0,060\ H \\ V_B &= 0,435\ H \end{aligned}}$$

$$\boxed{H_B = 2,395\ kN} \qquad \boxed{V_B = 17,419\ kN}$$

$$u_x^B = 2,395 \cdot \frac{400^3}{3 \cdot 20000 \cdot 4000}\left(1 + \frac{1}{\sqrt{3}}\right)^3$$

$$\boxed{u_x^B = 2,507\ cm}$$