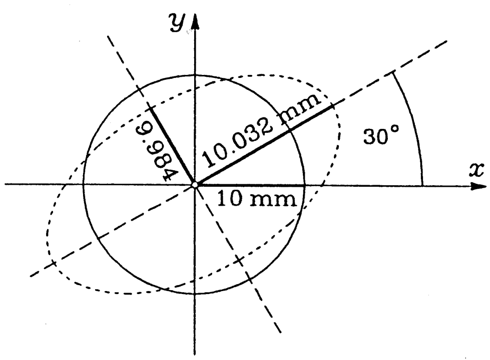
1. Na neobteženo pločevino debeline $\delta = 1\,\mathrm{mm}$ narišemo krog s polmerom $r = 10\,\mathrm{mm}$. Pod vplivom obtežbe, ki deluje v ravnini pločevine, ostane pločevina ravna, krog pa se spremeni v pravilno elipso, katere daljša os je nagnjena za 30^o glede na os x.
   a. Določi glavne linearne deformacije ter novo debelino pločevine!
   b. Določi napetosti glede na koordinatni sistem (x, y, z)!
   c. Določi ravnine in velikosti največjih strižnih napetosti!



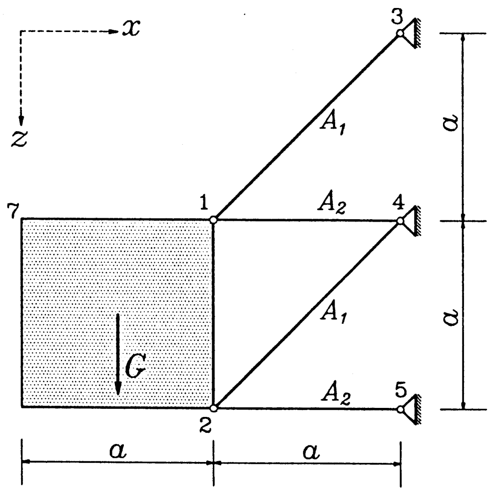
$E = 2.1 \cdot 10^5\,\mathrm{MPa}$

$\nu = 0.3$

$a' = 10{,}032\,\mathrm{mm}$

$b' = 9{,}984\,\mathrm{mm}$

2. Absolutno toga, enakomerno debela homogena stena teže G je podprta, kot kaže skica. Določi osne sile v palicah in vektor pomika točke 7 v odvisnosti od teže G!



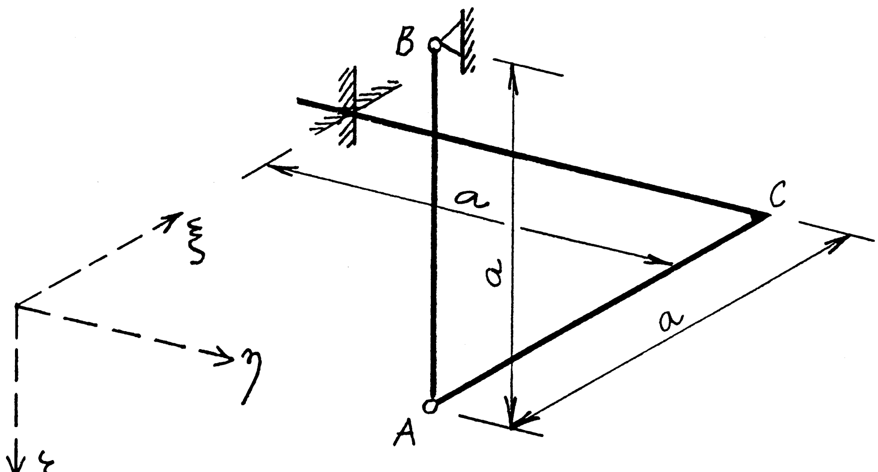
$E = 200\,000\,\mathrm{MPa}$

$a = 2\,\mathrm{m}$

$A_1 = 10\sqrt{2}\,\mathrm{cm}^2$

$A_2 = 10\,\mathrm{cm}^2$

3. Določi osno silo v palici AB, če konstrukcijo ohladimo za $\Delta T = 90\,\mathrm{K}$!



$EI_y = 50 \cdot 10^6\,\mathrm{kNcm}^2$

$GI_x = 50 \cdot 10^6\,\mathrm{kNcm}^2$

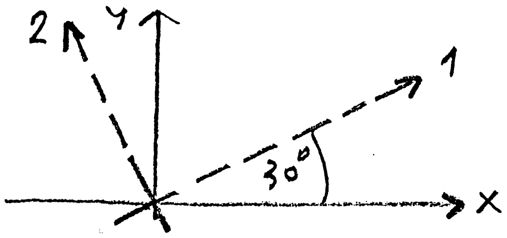
$EA_x = 50 \cdot 10^4\,\mathrm{kN}$

$\alpha_T = 1.2 \cdot 10^{-5}/\mathrm{K}$

$a = 2\,\mathrm{m}$

Ad 1.)

a)



$$\boxed{\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0}$$

$$\varepsilon_{11} = \frac{10{,}032 - 10}{10} \longrightarrow \boxed{\varepsilon_{11} = 0{,}0032}$$

$$\varepsilon_{22} = \frac{9{,}984 - 10}{10} \longrightarrow \boxed{\varepsilon_{22} = -0{,}0016}$$

$$\sigma_{33} = \sigma_{zz} = 2\mu\, \varepsilon_{33} + \lambda(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) = 0$$

$$\varepsilon_{33} = -\frac{\lambda}{2\mu + \lambda}(\varepsilon_{11} + \varepsilon_{22}) \longrightarrow \boxed{\varepsilon_{33} = -0{,}00069}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} = 121\,154 \text{ MPa}$$

$$\mu = \frac{E}{2(1+\nu)} = 80\,769 \text{ MPa}$$

$$\delta' = \delta(1 + \varepsilon_{33}) \longrightarrow \boxed{\delta' = 0{,}9993 \text{ mm}}$$

b)

$$e_{x1} = \frac{\sqrt{3}}{2} \qquad e_{x2} = -\frac{1}{2}$$

$$e_{y1} = \frac{1}{2} \qquad e_{y2} = \frac{\sqrt{3}}{2}$$

$$\vec{e}_1 = \frac{1}{2}(\sqrt{3}\,\vec{e}_x + \vec{e}_y)$$

$$\vec{e}_2 = \frac{1}{2}(-\vec{e}_x + \sqrt{3}\,\vec{e}_y)$$

$$\vec{e}_3 \equiv \vec{e}_z$$

$$\varepsilon_{xx} = \varepsilon_{11} e_{x1}^2 + \varepsilon_{22} e_{x2}^2 \longrightarrow \varepsilon_{xx} = 0{,}0020$$

$$\varepsilon_{yy} = \varepsilon_{11} e_{y1}^2 + \varepsilon_{22} e_{y2}^2 \longrightarrow \varepsilon_{yy} = -0{,}0004$$

$$\varepsilon_{xy} = \varepsilon_{11} e_{x1} e_{y1} + \varepsilon_{22} e_{x2} e_{y2} \longrightarrow \varepsilon_{xy} = 0{,}00208$$

$$\varepsilon_{zz} = -0{,}00069$$

$$I_1^{\varepsilon} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \rightarrow \boxed{I_1^{\varepsilon} = 0{,}00091}$$

$$\sigma_{xx} = 2\mu\, \varepsilon_{xx} + \lambda I_1^{\varepsilon} \longrightarrow \sigma_{xx} = 434 \text{ MPa}$$

$$\sigma_{yy} = 2\mu\, \varepsilon_{yy} + \lambda I_1^{\varepsilon} \longrightarrow \sigma_{yy} = 46 \text{ MPa}$$

$$\sigma_{xy} = 2\mu\, \varepsilon_{xy} \longrightarrow \sigma_{xy} = 336 \text{ MPa}$$

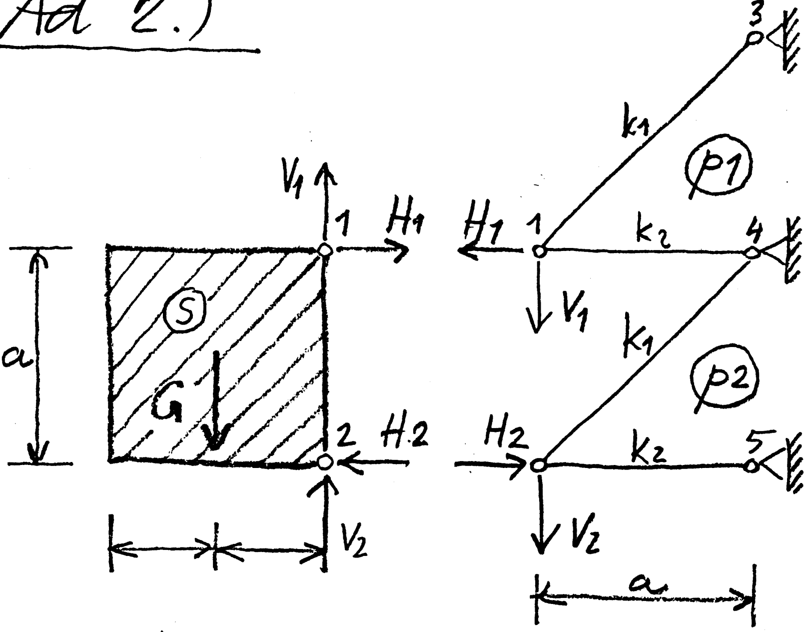
c) $$\operatorname{tg} 2\alpha_\tau = -\frac{\sigma_{xx}-\sigma_{yy}}{2\sigma_{xy}} = -0{,}577 \longrightarrow 2\alpha_\tau = -30^\circ$$

$$\boxed{\alpha_\tau = -15^\circ}$$

$$\tau_{1,2} = \pm\sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$$

$$\boxed{\tau_{1,2} = \pm\ 388 \text{ MPa}}$$

Ad 2.)



$$k_1 = \frac{EA_1}{a\sqrt{2}}$$

$$k_1 = 1000 \text{ kN/cm}$$

$$k_2 = \frac{EA_2}{a}$$

$$k_2 = 1000 \text{ kN/cm}$$

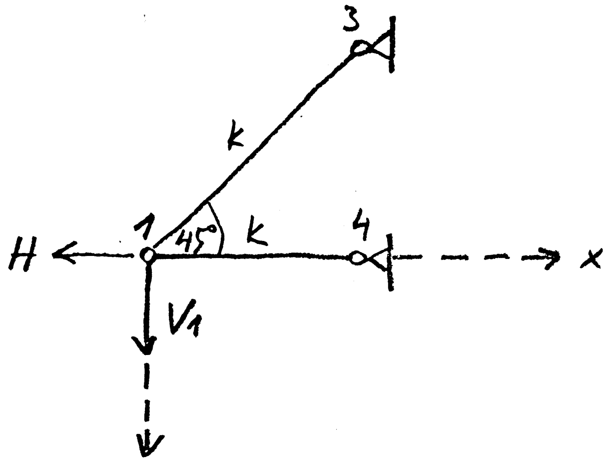
$$\boxed{k_1 = k_2 = k}$$

$$H_1 = H_2 = H$$

$$V_1 + V_2 = G$$

$$H_1 a = G\frac{a}{2} \rightarrow \boxed{H = \frac{G}{2}}$$

p1



$$[K_{14}] = k \begin{array}{c|cc} & 1 & 0 \\ \hline 1 & 1 & 0 \\ 0 & 0 & 0 \end{array} \longrightarrow [K_{14}] = \begin{bmatrix} k & 0 \\ 0 & 0 \end{bmatrix}$$

$$[K_{13}] = k \begin{array}{c|cc} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \hline \frac{\sqrt{2}}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{1}{2} \end{array} \longrightarrow [K_{13}] = \begin{bmatrix} k/2 & -k/2 \\ -k/2 & k/2 \end{bmatrix}$$

$$[K_{11}] = -[K_{13}] - [K_{14}] \longrightarrow [K_{11}] = \begin{bmatrix} -3k/2 & k/2 \\ k/2 & -k/2 \end{bmatrix}$$

$$\left.\begin{aligned} -\frac{3k}{2} u_1 + \frac{k}{2} w_1 - H &= 0 \\ \frac{k}{2} u_1 - \frac{k}{2} w_1 + V_1 &= 0 \end{aligned}\right\} \rightarrow \begin{aligned} -3u_1 + w_1 &= \frac{2}{k} H \\ u_1 - w_1 &= -\frac{2}{k} V_1 \quad \oplus \\ \hline -2u_1 &= \frac{2}{k}(H - V_1) \end{aligned}$$

$$\boxed{\begin{aligned} u_1 &= \frac{1}{k}(V_1 - H) \\ w_1 &= \frac{1}{k}(3V_1 - H) \end{aligned}}$$

p2 :

$$\boxed{\begin{aligned} u_2 &= \frac{1}{k}(V_2 + H) \\ w_2 &= \frac{1}{k}(3V_2 + H) \end{aligned}}$$

5 $\boxed{\vec{u}_2 = \vec{u}_1 + \vec{\omega}_1 \times \vec{r}_2}$

$$\vec{u}_1 = u_1 \vec{e}_x + w_1 \vec{e}_z$$
$$\vec{\omega}_1 = \omega_1 \vec{e}_y \quad , \quad \vec{r}_1 = a \vec{e}_z$$

$$\vec{u}_2 = u_2 \vec{e}_x + w_2 \vec{e}_z = u_1 \vec{e}_x + w_1 \vec{e}_z + a\omega_1 \vec{e}_x$$

$$\boxed{u_2 = u_1 + a\omega_1} \qquad \boxed{w_2 = w_1}$$

$$\frac{1}{k}(V_2 + H) = \frac{1}{k}(V_1 - H) + a\omega_1$$

$$\omega_1 = \frac{1}{ak}(V_2 - V_1 + 2H)$$

$$\frac{1}{k}(3V_2 + H) = \frac{1}{k}(3V_1 - H) \longrightarrow 3(V_1 - V_2) = 2H = G$$
$$V_1 + V_2 = G$$

$$\boxed{V_1 = \frac{2}{3} G} \quad \boxed{V_2 = \frac{1}{3} G}$$
$$\boxed{H = \frac{G}{2}}$$

$$\boxed{\omega_1 = \frac{2}{3ak} G}$$

$$\vec{r}_7 = -a \vec{e}_x$$

$u_1 = \frac{1}{6k} G$	$w_1 = \frac{3}{2k} G$
$u_2 = \frac{5}{6k} G$	$w_2 = \frac{3}{2k} G$

$$\vec{u}_7 = \vec{u}_1 + \vec{\omega}_1 \times \vec{r}_7 = \frac{G}{6k} \vec{e}_x + \frac{3G}{2k} \vec{e}_z + \frac{2G}{3k} \vec{e}_z$$

$$\boxed{\vec{u}_7 = \frac{G}{k}\left(\frac{1}{6} \vec{e}_x + \frac{13}{6} \vec{e}_z\right)}$$

$$\boxed{\vec{u}_7 = G\left(0{,}1667 \vec{e}_x + 2{,}1667 \vec{e}_z\right) \cdot 10^{-3}}$$

$$N_{13} = k\left[(u_3 - u_1)\cdot\frac{\sqrt{2}}{2} - (w_3 - w_1)\cdot\frac{\sqrt{2}}{2}\right]$$

$$\boxed{N_{13} = \frac{2\sqrt{2}}{3}G}$$

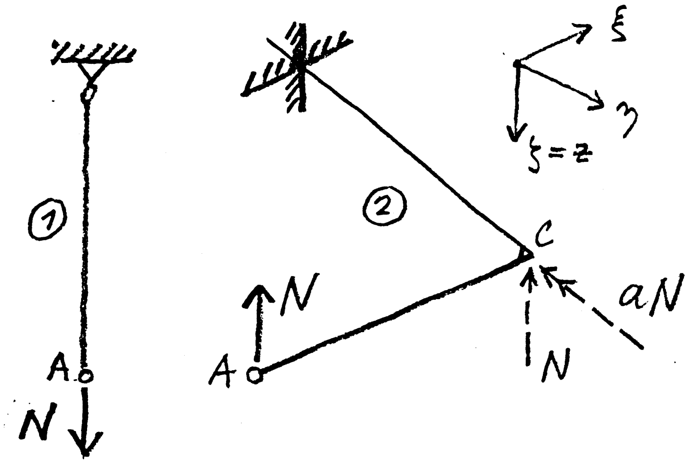
$$N_{14} = k\,(u_4 - u_1)\cdot 1 \rightarrow \boxed{N_{14} = -\frac{1}{6}G}$$

$$N_{24} = k\left[(u_4 - u_2)\frac{\sqrt{2}}{2} - (w_4 - w_2)\cdot\frac{\sqrt{2}}{2}\right]$$

$$\boxed{N_{24} = \frac{\sqrt{2}}{3}G}$$

$$N_{25} = k\,(u_5 - u_2)\cdot 1 \rightarrow \boxed{N_{25} = -\frac{5}{6}G}$$

Ad 3.)



$$\boxed{u_\xi^{①}(A) = u_\xi^{②}(A)}$$

①: $u_\xi^{①}(A) = +N\frac{a}{EA_x} + a\alpha_T\Delta T$

②: $u_z^{②}(A) = -N\frac{a^3}{3EI_y} - aN\frac{a}{GI_x}\cdot a - N\frac{a^3}{3EI_y}$

$$u_z^{②}(A) = -N\frac{2a^3}{3EI_y} - N\frac{a^3}{GI_x}$$

$$\rightarrow -N\left(\frac{2a^3}{3EI_y} + \frac{a^3}{GI_x} + \frac{a}{EA_x}\right) = a\alpha_T\Delta T$$

$$N\left(\frac{2a^2}{3EI_y}+\frac{a^2}{GI_x}+\frac{1}{EA_x}\right)=-\alpha_T\,\Delta T$$

$$\boxed{N=0{,}809\ kN}$$