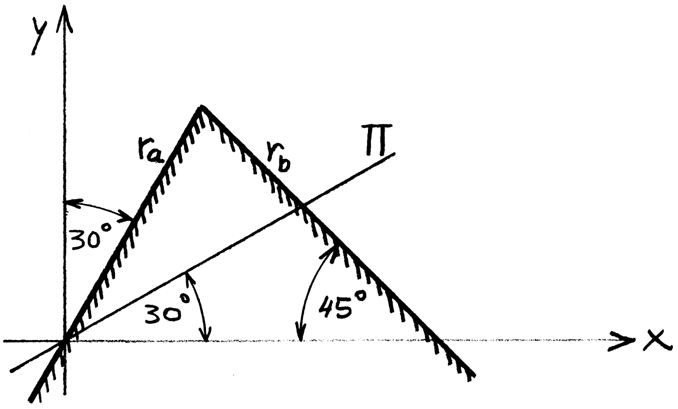
1. Na vogalu enakomerno debele homogene stene, v kateri vlada homogeno **ravninsko napetostno stanje**, izrežemo elementarni del, kakor je prikazano na skici.
   Na robu $\mathbf{r}_a$ deluje enakomerna zvezna obtežba $\vec{p}_a = q\,(-\vec{\mathbf{e}}_x + \sqrt{3}\,\vec{\mathbf{e}}_y)$, na robu $\mathbf{r}_b$ pa enakomerna zvezna obtežba $\vec{p}_b = -q\sqrt{2}\,(\vec{\mathbf{e}}_x + \vec{\mathbf{e}}_y)$.

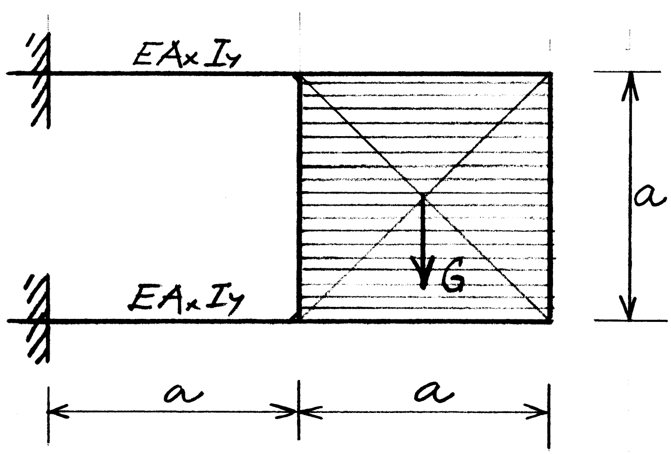
   a. Skiciraj obtežbo ter ustrezni zunanji normali robov obravnavane stene!
   b. Določi komponente tenzorja napetosti glede na kartezijski koordinatni sistem $(x,\ y,\ z)$!
   c. Določi rezultirajoči vektor napetosti v ravnini Π z njegovima komponentama v koordinatnem sistemu $(x,\ y,\ z)$ ter normalno in strižno napetost v tej ravnini!



2. Deformacijsko stanje nosilnega elementa v ravnini $(x,\ z)$ je opisano z matriko majhnih deformacij ε_{ij}. Točka $T_0(0,0)$ je nepomično vrtljivo podprta, v točki $T_1(400,0)$ pa je preprečen navpični pomik u_z. Določi oba pomika in zasuk točke $T_2(200,20)$!

$$[\varepsilon_{ij}] = 10^{-4} \begin{bmatrix} z(2x+1) & 0 & 0,5\,x^2 \\ 0 & 0 & 0 \\ 0,5\,x^2 & 0 & 400z \end{bmatrix}$$

3. Enakomerno debela homogena plošča teže G je togo pritrjena na dva enaka previsna nosilca. Določi zasuk plošče v ravnini $(x,\ z)$!



$$E = 200\,000\,MPa$$
$$A_x = 12\,cm^2$$
$$I_y = 400\,cm^4$$
$$G = 32\,kN$$
$$a = 2,4\,m$$

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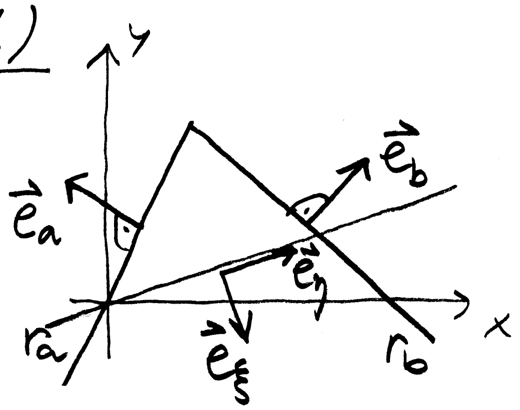
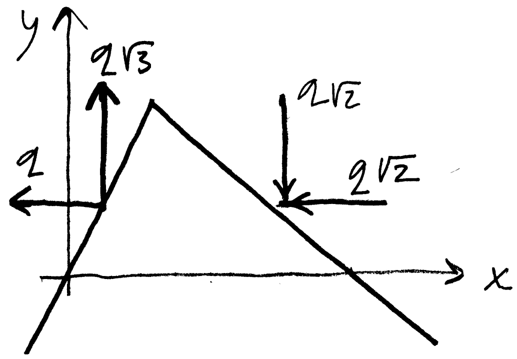
Ad 1.)

a)





b) Rot r_a: $\vec{e}_a = -\frac{\sqrt{3}}{2}\vec{e}_x + \frac{1}{2}\vec{e}_y$

$$p_{ax} = \sigma_{xx} e_{ax} + \sigma_{xy} e_{ay} \;\ldots\; -q = -\sigma_{xx}\frac{\sqrt{3}}{2} + \sigma_{xy}\cdot\frac{1}{2}$$
$$p_{ay} = \sigma_{xy} e_{ax} + \sigma_{yy} e_{ay} \;\ldots\; q\sqrt{3} = -\sigma_{xy}\frac{\sqrt{3}}{2} + \sigma_{yy}\cdot\frac{1}{2}$$

$$\sqrt{3}\,\sigma_{xx} - \sigma_{xy} = 2q \qquad \ldots (A)$$
$$\sigma_{yy} - \sqrt{3}\,\sigma_{xy} = 2q\sqrt{3} \qquad \ldots (B)$$

Rot r_b: $\vec{e}_b = \frac{\sqrt{2}}{2}\vec{e}_x + \frac{\sqrt{2}}{2}\vec{e}_y$

$$p_{bx} = \sigma_{xx} e_{bx} + \sigma_{xy} e_{by} \;\ldots\; -q\sqrt{2} = \sigma_{xx}\frac{\sqrt{2}}{2} + \sigma_{xy}\frac{\sqrt{2}}{2}$$
$$p_{by} = \sigma_{xy} e_{bx} + \sigma_{yy} e_{by} \;\ldots\; -q\sqrt{2} = \sigma_{xy}\frac{\sqrt{2}}{2} + \sigma_{yy}\frac{\sqrt{2}}{2}$$

$$\sigma_{xx} + \sigma_{xy} = -2q \longrightarrow \sigma_{xy} = -2q - \sigma_{xx}$$
$$\sigma_{yy} + \sigma_{xy} = -2q \longrightarrow \sigma_{xy} = -2q - \sigma_{yy}$$

$$-2q - \sigma_{xx} = -2q - \sigma_{yy} \longrightarrow \boxed{\sigma_{xx} = \sigma_{yy}}$$

$$(A) \longrightarrow \sqrt{3}\,\sigma_{xx} - (-2q - \sigma_{xx}) = 2q \longrightarrow \boxed{\begin{matrix}\sigma_{xx} = 0 \\ \sigma_{yy} = 0\end{matrix}}$$

$$\boxed{\sigma_{xy} = -2q}$$

$$\boxed{[\sigma_{ij}] = 2q\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

$$(B) \rightarrow 2q\sqrt{3} = 2q\sqrt{3}$$

✓

c) $\vec{e}_\xi = \frac{1}{2}\vec{e}_x - \frac{\sqrt{3}}{2}\vec{e}_y$; $\vec{e}_\eta = \frac{\sqrt{3}}{2}\vec{e}_x + \frac{1}{2}\vec{e}_y$

$$\begin{Bmatrix} \sigma_{\xi x} \\ \sigma_{\xi y} \\ \sigma_{\xi z} \end{Bmatrix} = \begin{bmatrix} 0 & -2q & 0 \\ -2q & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \\ 0 \end{Bmatrix} = \begin{Bmatrix} q\sqrt{3} \\ -q \\ 0 \end{Bmatrix}$$

$$\boxed{\vec{\sigma}_\xi = q\sqrt{3}\,\vec{e}_x - q\,\vec{e}_y}$$

$$\sigma_{\xi\xi} = \vec{\sigma}_\xi \vec{e}_\xi \longrightarrow \boxed{\sigma_{\xi\xi} = q\sqrt{3}}$$

$$\sigma_{\xi\eta} = \vec{\sigma}_\xi \vec{e}_\eta \longrightarrow \boxed{\sigma_{\xi\eta} = q}$$

Ad 2.)

$$\boxed{\begin{aligned} u_y &= 0 \\ \omega_x &= \omega_z = 0 \end{aligned}}$$

$$\vec{\omega} = \vec{\omega}_0 + \int_{x_0}^{x} (\vec{\nabla} \times \vec{\varepsilon}_x)_{z=z_0}\, dx + \int_{z_0}^{z} (\vec{\nabla} \times \vec{\varepsilon}_z)\, dz$$

$$\vec{u} = \vec{u}_0 + \int_{x_0}^{x} (\vec{\varepsilon}_x + \vec{\omega}_x)_{z=z_0}\, dx + \int_{z_0}^{z} (\vec{\varepsilon}_z + \vec{\omega}_z)\, dz$$

$$\omega_y = \vec{\omega}\,\vec{e}_y = \omega_y^0 + \int_0^x \vec{e}_y (\vec{\nabla} \times \vec{\varepsilon}_x)_{z=0}\, dx + \int_0^z \vec{e}_y (\vec{\nabla} \times \vec{\varepsilon}_z)\, dz$$

$$\vec{e}_y(\vec{\nabla} \times \vec{\varepsilon}_x) = \begin{vmatrix} 0 & 1 & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \varepsilon_{xx} & 0 & \varepsilon_{xz} \end{vmatrix} = \frac{\partial \varepsilon_{xx}}{\partial z} - \frac{\partial \varepsilon_{xz}}{\partial x} = 10^{-4}(x+1)$$

$$\vec{e}_y(\vec{\nabla} \times \vec{\varepsilon}_z) = \begin{vmatrix} 0 & 1 & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \varepsilon_{zx} & 0 & \varepsilon_{zz} \end{vmatrix} = \frac{\partial \varepsilon_{zx}}{\partial z} - \frac{\partial \varepsilon_{zz}}{\partial x} = 0$$

$$\omega_y = \omega_y^0 + 10^{-4}\int_0^x (x+1)\, dx \longrightarrow \boxed{\omega_y = \omega_y^0 + 10^{-4}\left(\frac{x^2}{2} + x\right)}$$

$$\boxed{\omega_y = \omega_{zx} = -\omega_{xz}}$$

$$u_x = u_x^0 + \int_0^x (\varepsilon_{xx})_{z=0}\, dx + \int_0^z (\varepsilon_{zx} + \omega_{zx})\, dz$$

$$u_x = \int_0^z \left[0{,}5 \cdot 10^{-4} x^2 + \omega_y^0 + 10^{-4}\left(\frac{x^2}{2} + x\right)\right] dz$$

$$u_x = z\,\omega_y^0 + 10^{-4} \int_0^z (x^2 + x)\, dz$$

$$\boxed{u_x = z\left[\omega_y^0 + 10^{-4}(x^2 + x)\right]}$$

$$u_z = u_z^0 + \int_0^x (\varepsilon_{zx} + \omega_{zx})_{z=0}\, dx + \int_0^z \varepsilon_{zz}\, dz$$

$$u_z = \int_0^x \left[0{,}5 \cdot 10^{-4} x^2 - \omega_y^0 - 10^{-4}\left(\frac{x^2}{2} + x\right)\right] dx +$$

$$+ \int_0^z 400 \cdot 10^{-4} z\, dz$$

$$\boxed{u_z = -x\,\omega_y^0 + 10^{-4}\left(-\frac{x^2}{2} + 200 z^2\right)}$$

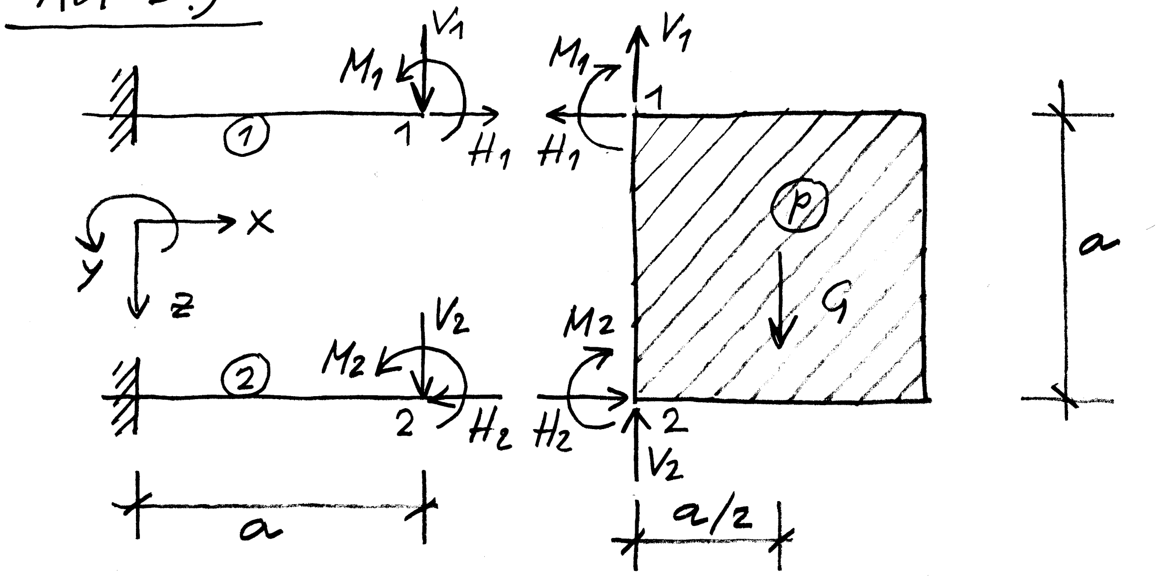
$$T_1 (x = 400,\ z = 0) \longrightarrow u_z = 0$$

$$-400\,\omega_y^0 - 10^{-4}\,\frac{400^2}{2} = 0 \longrightarrow \boxed{\omega_y^0 = -200 \cdot 10^{-4}}$$

$$\boxed{\begin{aligned} u_x &= 10^{-4} z (x^2 + x - 200) \\ u_z &= 10^{-4}\left(-\frac{x^2}{2} + 200x + 200 z^2\right) \\ \omega_y &= 10^{-4}\left(\frac{x^2}{2} + x - 200\right) \end{aligned}}$$

$$T_2 (200,\ 20) \longrightarrow \boxed{\begin{aligned} u_x &= 80 \\ u_z &= 10 \\ \omega_y &= 2 \end{aligned}} \cdot 10^{-4}$$

Ad 3.)



(p) : $\sum Z = 0 \quad \ldots \quad \boxed{V_1 + V_2 = G}$ $\quad\ldots$ (A)

$\sum X = 0 \quad \ldots \quad \boxed{H_1 = H_2}$ $\quad\ldots$ (B)

$\sum M^1 = 0 \quad \ldots \quad \boxed{H_2 a - M_1 - M_2 = G \frac{a}{2}}$ $\quad\ldots$ (C)

$$u_1 = H_1 \frac{a}{EA_x}$$

$$w_1 = V_1 \frac{a^3}{3EI_y} - M_1 \frac{a^2}{2EI_y}$$

$$\omega_1 = -V_1 \frac{a^2}{2EI_y} + M_1 \frac{a}{EI_y}$$

$$u_2 = -H_2 \frac{a}{EA_x}$$

$$w_2 = V_2 \frac{a^3}{3EI_y} - M_1 \frac{a^2}{2EI_y}$$

$$\omega_2 = -V_2 \frac{a^2}{2EI_y} + M_1 \frac{a}{EI_y}$$

$$\vec{u}_2 = \vec{u}_1 + \vec{\omega}_1 \times \vec{r}_2$$

$$\vec{\omega}_2 = \vec{\omega}_1 \longrightarrow \boxed{\omega_2 = \omega_1} \ldots (D)$$

$$\vec{u}_1 = u_1 \vec{e}_x + w_1 \vec{e}_z$$

$$\vec{\omega}_1 = \omega_1 \vec{e}_y$$

$$\vec{r}_2 = a \vec{e}_z$$

$$u_2 \vec{e}_x + w_2 \vec{e}_z = (u_1 + a\,\omega_1)\vec{e}_x + w_1 \vec{e}_z$$

$\boxed{w_2 = w_1}$ … (E) $\qquad$ $\boxed{u_2 = u_1 + a\omega_1}$ … (F)

$$(E) \longrightarrow \quad V_1 \frac{a^3}{3} - M_1 \frac{a^2}{2} = V_2 \frac{a^3}{3} - M_2 \frac{a^2}{2}$$

$$(D) \longrightarrow \quad V_1 \frac{a^2}{2} - M_1 a = V_2 \frac{a^2}{2} - M_2 a$$

$$\boxed{\begin{matrix} V_1 = V_2 \\ M_1 = M_2 \end{matrix}}$$

$$(A) \longrightarrow \quad \boxed{V_1 = V_2 = \frac{G}{2}} \quad \ldots (K)$$

$$(F),(B),(K) \rightarrow -H_1 \frac{a}{EA_x} = H_1 \frac{a}{EA_x} - \frac{G}{2}\frac{a^3}{2EI_y} + M_1 \frac{a^2}{EI_y}$$

$$\boxed{H_1 \frac{2}{A_x} = G \frac{a^2}{4I_y} - M_1 \frac{a}{I_y}}$$

$$(C),(B) \longrightarrow \quad \boxed{H_1 = -\frac{G}{2} + M_1 \frac{2}{a}}$$

$$\boxed{M_1 = G \frac{a(a^2 A_x - 4I_y)}{4(a^2 A_x + 4I_y)}} \longrightarrow \boxed{M_1 = M_2 = 1911{,}13 \text{ kNcm}}$$

$$\boxed{\begin{matrix} H_1 = H_2 = 31{,}926 \text{ kN} \\ V_1 = V_2 = 16{,}0 \text{ kN} \end{matrix}}$$

$$\omega_y = \omega_1 = \omega_2 = \frac{a}{EI_y}\left(-V_1 \frac{a}{2} + M_1\right)$$

$$\boxed{\omega_y = -2{,}534 \cdot 10^{-4}}$$