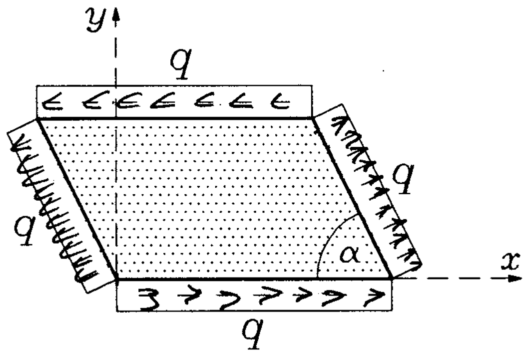
1. Na element enakomerno debele stene deluje zunanja obtežba kot kaže skica. Ob predpostavki, da vlada v elementu homogeno ravninsko napetostno stanje, določi kot α, pri katerem je element v ravnotežju! Določi velikosti in smeri glavnih linearnih deformacij v tem primeru ter jih označi na skici!



$E = 200\,000\,\mathrm{MPa}$

$\nu = 0.25$

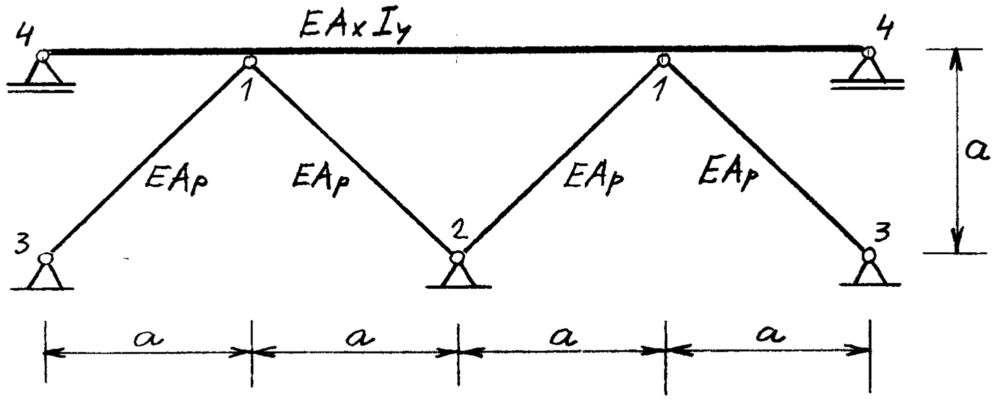
30%

2. Deformacijsko stanje telesa je podano s komponentami ε_{ij} tenzorja majhnih deformacij. V točki $T_0(0,0,0)$ je telo nepomično vrtljivo podprto, v točki $T_1(0,3,-18)$ pa je preprečen pomik v smeri z. Preveri, če so zasuki in pomiki enolični! Določi zasuke in pomike točke $T_2(0,1,1)$!

$$[\varepsilon_{ij}] = 10^{-3}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 4(2y+z) & 3y^2+2y+z \\ 0 & 3y^2+2y+z & 3 \end{bmatrix}$$

35%

3. Določi osno silo, ki nastopi v palici $\overline{13}$, če celotno konstrukcijo segrejemo za 40 K! Nasvet: upoštevaj simetrijo!



$E = 210\,000\,\mathrm{MPa}$

$\alpha_T = 1.25 \cdot 10^{-5}/\mathrm{K}$

$a = 2.4\,\mathrm{m}$

$A_p = 10\,\mathrm{cm}^2$

$A_x = 40\,\mathrm{cm}^2$

$I_y = 3060\,\mathrm{cm}^4$

V pomoč: *Ravnotežne enačbe paličja v vozlišču i :*

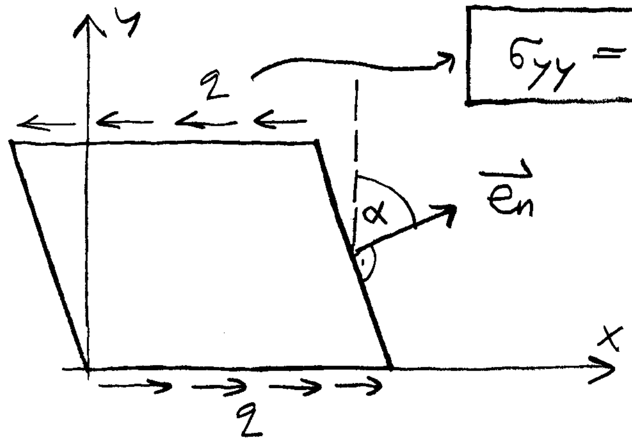
$$\sum_{j=1}^{n} [K_{ij}] \left\{ \begin{matrix} u_j - u_i \\ w_j - w_i \end{matrix} \right\} = -\left\{ \begin{matrix} P_i^x \\ P_i^z \end{matrix} \right\} + \sum_{j=1}^{n} \Theta_{ij} \left\{ \begin{matrix} \cos\alpha_{ij} \\ \cos\gamma_{ij} \end{matrix} \right\}$$

$$\Theta_{ij} = k_{ij} l_{ij} \alpha_{ij}^T \Delta T_{ij}$$

35%

MTT IZPIT 14.2.96

Ad 1.)



$$\boxed{\sigma_{yy} = 0, \quad \sigma_{yx} = \sigma_{xy} = q}$$

$e_{nx} = \sin\alpha$

$e_{ny} = \cos\alpha$

$e_{nz} = 0$

$$\vec{e}_n = \sin\alpha\, \vec{e}_x + \cos\alpha\, \vec{e}_y \qquad \vec{p}_n = q\, \vec{e}_n$$

$$\vec{p}_n = q \sin\alpha\, \vec{e}_x + q\cos\alpha\, \vec{e}_y = \vec{\sigma}_x e_{nx} + \vec{\sigma}_y e_{ny}$$

$$p_{nx} = q\sin\alpha = \sigma_{xx} e_{nx} + \sigma_{yx} e_{ny} = \sigma_{xx}\sin\alpha + q\cos\alpha$$

$$p_{ny} = q\cos\alpha = \sigma_{xy} e_{nx} + \sigma_{yy} e_{ny} = q\sin\alpha$$

$$\rightarrow \cos\alpha = \sin\alpha \longrightarrow \operatorname{tg}\alpha = 1 \longrightarrow \boxed{\alpha = 45^\circ}$$

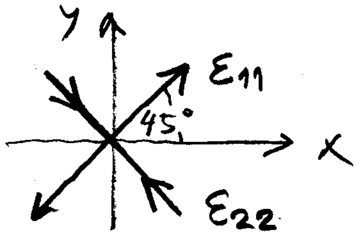
$$\sin\alpha = \cos\alpha = \frac{\sqrt{2}}{2} \rightarrow q\frac{\sqrt{2}}{2} = \sigma_{xx}\frac{\sqrt{2}}{2} + q\frac{\sqrt{2}}{2}$$

$$\boxed{\sigma_{xx} = 0}$$

$$[\sigma_{ij}] = \begin{bmatrix} 0 & q & 0 \\ q & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \sigma_{11,22} = \pm\sqrt{\sigma_{xy}^2} = \pm q$$

$$\boxed{\sigma_{11} = q \quad , \quad \sigma_{22} = -q}$$

$$\operatorname{tg} 2\alpha_\sigma = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} = \infty \longrightarrow 2\alpha_\sigma = 90^\circ \rightarrow \boxed{\alpha_\sigma = 45^\circ}$$



$$\boxed{\begin{aligned} \varepsilon_{11} &= \frac{1+\nu}{E}\, q \\ \varepsilon_{22} &= -\frac{1+\nu}{E}\, q \\ \varepsilon_{33} &= 0 \end{aligned}}$$

$$\boxed{I_1^\sigma = 0}$$

Ad 2.)

$$\vec{\varepsilon}_x = \vec{0}$$
$$\vec{\varepsilon}_y = \varepsilon_{yy}\,\vec{e}_y + \varepsilon_{yz}\,\vec{e}_z$$
$$\vec{\varepsilon}_z = \varepsilon_{zy}\,\vec{e}_y + \varepsilon_{zz}\,\vec{e}_z$$

$$K_{xx} = \frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} - 2\,\frac{\partial^2 \varepsilon_{yz}}{\partial y\,\partial z} = 0 \longrightarrow \quad 0 = 0 \ \checkmark$$

$$\vec{\omega} = \vec{\omega}_0 + \int_{y_0}^{y} (\vec{\nabla}\times\vec{\varepsilon}_y)_{z=z_0}\,dy + \int_{z_0}^{z} (\vec{\nabla}\times\vec{\varepsilon}_z)\,dz$$

$$\omega_x = \vec{e}_x\cdot\vec{\omega} = \omega_x^0 + \int_0^y \vec{e}_x(\vec{\nabla}\times\vec{\varepsilon}_y)_{z=0}\,dy + \int_0^z \vec{e}_x(\vec{\nabla}\times\vec{\varepsilon}_z)\,dz$$

$$\vec{e}_x(\vec{\nabla}\times\vec{\varepsilon}_y) = \begin{vmatrix} 1 & 0 & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \varepsilon_{yy} & \varepsilon_{yz} \end{vmatrix} = \frac{\partial \varepsilon_{yz}}{\partial y} - \frac{\partial \varepsilon_{yy}}{\partial z} = 10^{-3}(6y-2)$$

$$\vec{e}_x(\vec{\nabla}\times\vec{\varepsilon}_z) = \begin{vmatrix} 1 & 0 & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \varepsilon_{zy} & \varepsilon_{zz} \end{vmatrix} = \frac{\partial \varepsilon_{zz}}{\partial y} - \frac{\partial \varepsilon_{zy}}{\partial z} = -1\cdot 10^{-3}$$

$$\omega_x = \omega_x^0 + 10^{-3}\left\{\int_0^y (6y-2)_{z=0}\,dy - \int_0^z dz\right\}$$

$$\boxed{\omega_x = \omega_x^0 + 10^{-3}(3y^2 - 2y - z)}$$

$$\vec{u} = \vec{u}_0 + \int_{y_0}^{y} (\vec{\varepsilon}_y + \vec{\omega}_y)_{z=z_0}\,dy + \int_{z_0}^{z} (\vec{\varepsilon}_z + \vec{\omega}_z)\,dz$$

$$u_y = \vec{u}\cdot\vec{e}_y = u_y^0 + \int_0^y (\varepsilon_{yy} + \cancel{\omega_{yy}})_{z=0}\,dy + \int_0^z (\varepsilon_{zy} + \overbrace{\omega_{zy}}^{-\omega_x})\,dz$$

$$u_y = 10^{-3}\left\{\int_0^y (8y+4z)_{z=0}\,dy + \int_0^z (3y^2 + 2y + z - 10^3\omega_x^0 - \right.$$
$$\left. -3y^2 + 2y + z)\,dz\right.$$

$$\boxed{u_y = -z\,\omega_x^0 + 10^{-3}(4y^2 + 4yz + z^2)}$$

$$u_z = \vec{u}\cdot\vec{e}_z = u_z^0 + \int_0^y (\varepsilon_{yz} + \omega_{yz})_{z=0}\, dy + \int_0^z (\varepsilon_{zz} + \omega_{zz})\, dz$$

$$u_z = 10^{-3}\left\{ \int_0^y (3y^2 + 2y + z + 3y^2 - 2y - z + 10^3 \omega_x^0)_{z=0}\, dy + \int_0^z 3\, dz \right\}$$

$$\boxed{u_z = y\,\omega_x^0 + 10^{-3}(2y^3 + 3z)}$$

$$T_1(0,3,-18) \longrightarrow u_z = 0 \longrightarrow 3\omega_x^0 + 10^{-3}(2\cdot 3^3 - 3\cdot 18) = 0$$

$$\boxed{\omega_x^0 = 0} \longrightarrow \boxed{\begin{aligned} \omega_x &= 10^{-3}(3y^2 - 2y - z) \\ u_y &= 10^{-3}(4y^2 + 4yz + z^2) \\ u_z &= 10^{-3}(2y^3 + 3z) \end{aligned}}$$

$$\vec{e}_x{}' = \vec{e}_x + \frac{\partial \vec{u}}{\partial x} \longrightarrow \vec{e}_x{}' = \vec{e}_x$$

$$\vec{e}_y{}' = \vec{e}_y + \frac{\partial \vec{u}}{\partial y} = \vec{e}_y + 10^{-3}\left[(8y + 4z)\vec{e}_y + 6y^2\vec{e}_z\right]$$

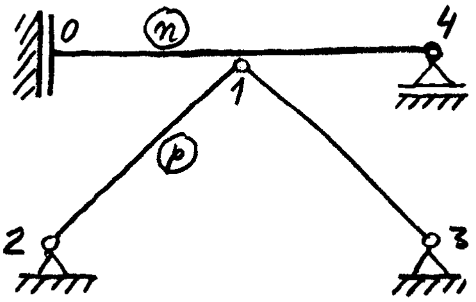
$$\vec{e}_z{}' = \vec{e}_z + \frac{\partial \vec{u}}{\partial z} = \vec{e}_z + 10^{-3}\left[(4y + 2z)\vec{e}_y + 3\vec{e}_z\right]$$

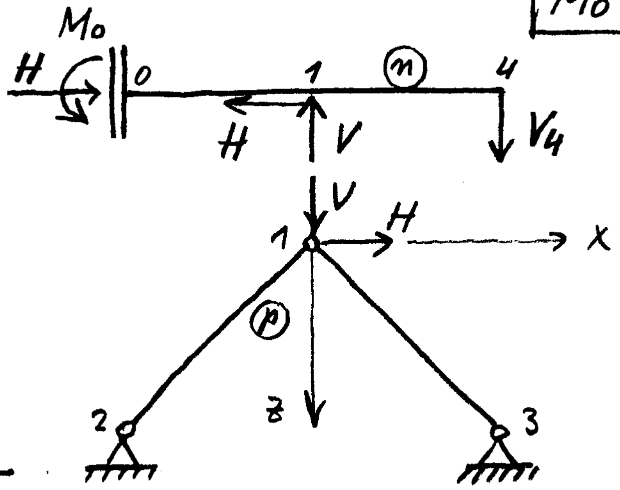
$$T_2(0,1,1) \longrightarrow \boxed{\begin{aligned} \vec{e}_x{}' &= \vec{e}_x \\ \vec{e}_y{}' &= 1{,}012\,\vec{e}_y + 0{,}006\,\vec{e}_z \\ \vec{e}_z{}' &= 0{,}006\,\vec{e}_y + 1{,}003\,\vec{e}_z \end{aligned}}$$

$$T_2(0,1,1) \longrightarrow \boxed{\begin{aligned} \omega_x &= 0 \\ u_y &= 0{,}009 \\ u_z &= 0{,}005 \end{aligned}}$$

Ad 3.)

$\boxed{M_0 = Va}$





(n): $u_1^{(n)} = -H\frac{a}{EA_x} + a\alpha_T \Delta T$

$$\boxed{u_1^{(n)} = -0,000285\; H + 0,12}$$

$$M_y = -M_0 + V\langle x-a\rangle^1 = -V(a-\langle x-a\rangle^1) = -EI_y w''$$

$$EI_y w' = V\left(ax - \frac{1}{2}\langle x-a\rangle^2\right) + C_1$$

$$EI_y w = V\left(a\frac{x^2}{2} - \frac{1}{6}\langle x-a\rangle^3\right) + C_1 x + C_2$$

$x=0 \;\dots\; w'=0 \longrightarrow$ $\boxed{C_1 = 0}$

$x=2a \;\dots\; w=0 \longrightarrow$ $\boxed{C_2 = -V\frac{11a^3}{6}}$

$$w = V\frac{a^3}{6EI_y}\left[3\left(\frac{x}{a}\right)^2 - \frac{1}{a^3}\langle x-a\rangle^3 - 11\right]$$

$x=a \longrightarrow \boxed{w_1^{(n)} = -V\frac{4a^3}{3EI_y}} \rightarrow \boxed{w_1^{(n)} = -0,287\,V}$

(p): $k_{12} = k_{13} = \frac{EA_p}{a\sqrt{2}} \longrightarrow \boxed{k_{12} = k_{13} = 619 \text{ kN/cm}}$

$$[K_{12}] = \begin{array}{c|cc} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \hline -\frac{\sqrt{2}}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{1}{2} \end{array}\; k_{12} \longrightarrow [K_{12}] = \begin{bmatrix} 309 & -309 \\ -309 & 309 \end{bmatrix}$$

$$[K_{13}] = \begin{array}{c|cc} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \hline \frac{\sqrt{2}}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{\sqrt{2}}{2} & \frac{1}{2} & \frac{1}{2} \end{array}\; k_{13} \longrightarrow [K_{13}] = \begin{bmatrix} 309 & 309 \\ 309 & 309 \end{bmatrix}$$

$$[K_{11}] = -([K_{12}]+[K_{13}]) \rightarrow [K_{11}] = \begin{bmatrix} -619 & 0 \\ 0 & -619 \end{bmatrix}$$

$$\theta_{12} = \theta_{13} = 619 \cdot 240\sqrt{2} \cdot 1{,}25 \cdot 10^{-5} \cdot 40$$

$$\boxed{\theta_{12} = \theta_{13} = 105 \ kN}$$

$$1: \begin{bmatrix} -619 & 0 \\ 0 & -619 \end{bmatrix} \begin{Bmatrix} u_1 \\ w_1 \end{Bmatrix} = \begin{Bmatrix} -H \\ -V \end{Bmatrix} + 105 \begin{Bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{Bmatrix} + 105 \begin{Bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{Bmatrix}$$

$$\begin{bmatrix} -619 & 0 \\ 0 & -619 \end{bmatrix} \begin{Bmatrix} u_1 \\ w_1 \end{Bmatrix} = \begin{Bmatrix} -H \\ -V + 148{,}5 \end{Bmatrix}$$

$$\boxed{\begin{aligned} u_1^{(P)} &= 0{,}001616 \ H \\ w_1^{(P)} &= 0{,}001616 \ V - 0{,}240 \end{aligned}}$$

$$\boxed{\begin{aligned} u_1^{(n)} &= u_1^{(P)} \\ w_1^{(n)} &= w_1^{(P)} \end{aligned}} \quad \begin{aligned} &\rightarrow -0{,}000285 H + 0{,}12 = 0{,}001616 H \\ &\rightarrow -0{,}287 V = 0{,}001616 V - 0{,}240 \end{aligned}$$

$H = 63{,}093$ kN	$V = 0{,}832$ kN

$$u_1 = 0{,}001616 \cdot 63{,}093 \quad \rightarrow \quad \boxed{u_1 = 0{,}102 \ cm}$$

$$w_1 = -0{,}287 \cdot 0{,}832 \quad \rightarrow \quad \boxed{w_1 = -0{,}239 \ cm}$$

$$N_{ij} = k_{ij}\left[(u_j - u_i)\cos\alpha_{ij} + (w_j - w_i)\cos\gamma_{ij}\right] - \theta_{ij}$$

$$N_{13} = 619\left[-0{,}102 \cdot \frac{\sqrt{2}}{2} + 0{,}239 \cdot \frac{\sqrt{2}}{2}\right] - 105$$

$$\boxed{N_{13} = -45{,}202 \ kN}$$