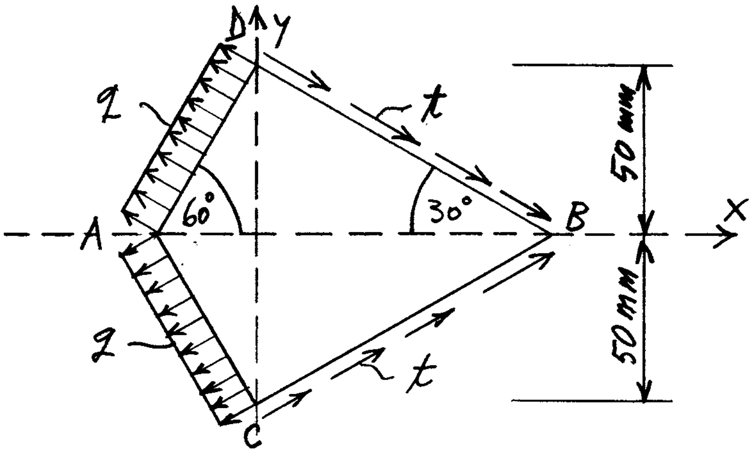
1. V elementu enakomerno debele stene vlada homogeno napetostno stanje. Robova $\overline{AD}$ in $\overline{AC}$ sta obteženа z normalno, robova $\overline{BD}$ in $\overline{BC}$ pa s tangencialno obtežbo, kot kaže skica.
   a. Določi velikost tangencialne obtežbe t tako, da bo element v ravnotežju!
   b. Določi velikosti in smeri glavnih normalnih deformacij !
   c. Določi novi dolžini diagonal $\overline{AB}$ in $\overline{CD}$!



$a = 50\,\mathrm{mm}$

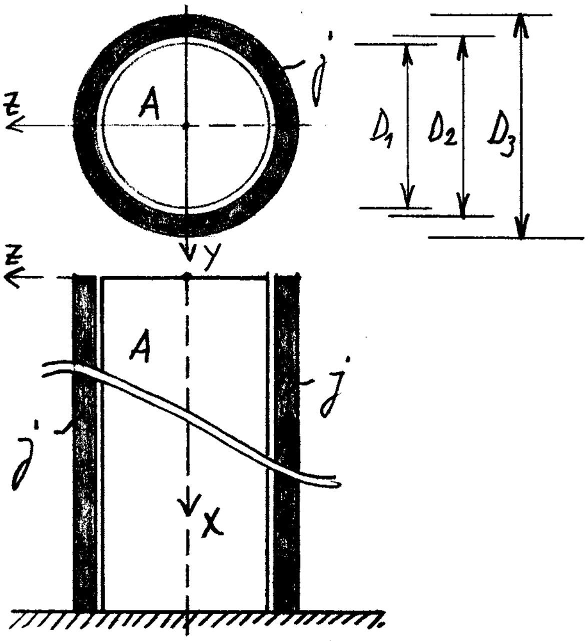
$q = 120\,\mathrm{MPa}$

$E = 200\,000\,\mathrm{MPa}$

$\nu = 0.3$

NI rešitve !
le : $q = t = 0$!

2. Aluminijast valj premera $D_1 = 20\,\mathrm{cm}$ centrično postavimo v jekleno cev z notranjim premerom $D_2 = 20.04\,\mathrm{cm}$ in zunanjim premerom $D_3 = 20.44\,\mathrm{cm}$.



   a. Za koliko moramo segreti aluminijasti valj, da se dotakne notranje stene cevi? Pri tem predpostavimo, da ni prevajanja toplote po vmesnem zračnem prostoru.
   b. V nadaljevanju valj in cev hkrati segrevamo. Določi napetosti v valju in cevi, ko se temperatura valja in cevi izenači pri $110^o\,\mathrm{C}$!
   c. Reši nalogo a. ob predpostavki, da valj in cev segrevamo hkrati!

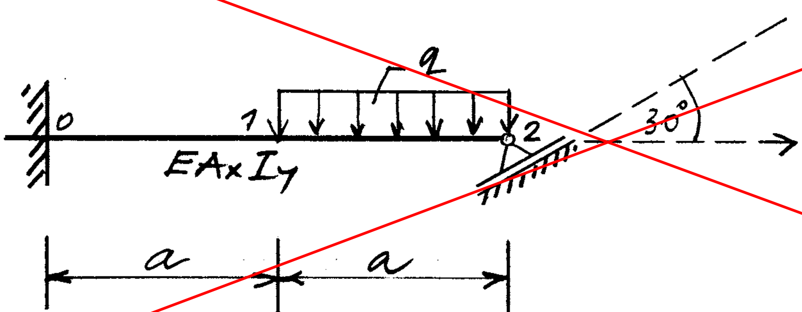
$E_j = 22\,000\,\mathrm{kN/cm^2}$ $\nu_j = 0.3$

$\alpha_j = 1.2 \cdot 10^{-5}/\mathrm{K}$

$E_a = 7\,000\,\mathrm{kN/cm^2}$ $\nu_a = 0.34$

$\alpha_a = 2.353 \cdot 10^{-5}/\mathrm{K}$

3. ~~Določi vektor pomika točke 2 glede na koordinatni sistem (x, z)! Določi in skiciraj diagrame notranjih sil!~~



$q = 20\,\mathrm{kN/m}$

$E = 20\,000\,\mathrm{kN/cm^2}$

$A_x = 40\,\mathrm{cm^2}$

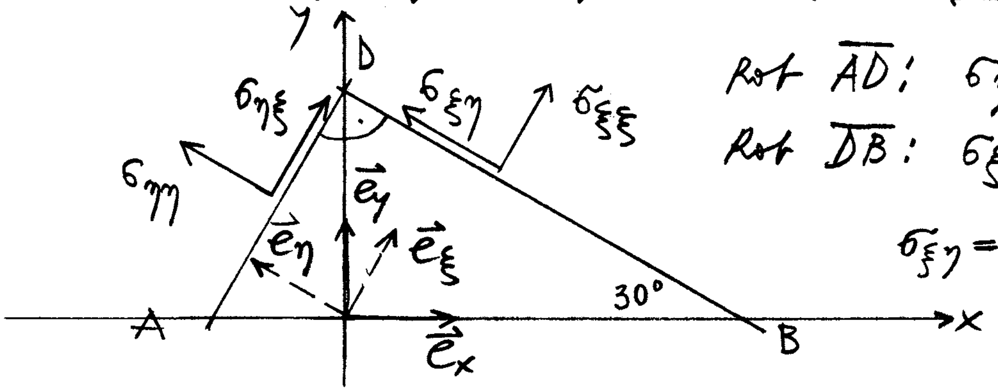
$I_y = 3000\,\mathrm{cm^4}$

$a = 2,4\,m$

MTT IZPIT 16. 9. 1996

Ad 1.)

Naloga je rešljiva le pri $\boxed{t = q = 0}$!



Rob $\overline{AD}$: $\sigma_{\eta\xi} = 0$

Rob $\overline{DB}$: $\sigma_{\xi\eta} = -t$

$\sigma_{\xi\eta} = \sigma_{\eta\xi} \rightarrow t = 0$

Ad 2.)

$$\varepsilon^v_{yy} = \varepsilon^v_{zz} = \frac{D_2 - D_1}{D_1} = \alpha_a \Delta T_1$$

a)

$$\boxed{\Delta T_1 = \frac{D_2 - D_1}{D_1 \alpha_a}} \longrightarrow \boxed{\Delta T_1 = 85\,^\circ C}$$

b) $\Delta T_2 = \Delta T - \Delta T_1 = 110^\circ - 85^\circ \longrightarrow \boxed{\Delta T_2 = 25\,^\circ C}$

$$\delta = \frac{1}{2}(20{,}44 - 20{,}04) \longrightarrow \delta = 0{,}2 \text{ cm}$$

$$\varepsilon^a_{yy} = \frac{1}{E_a}\left(\sigma^a_{yy} - \nu_a \sigma^a_{zz}\right) + \alpha_a \Delta T_2$$

$$\boxed{\varepsilon^a_{yy} = -\frac{q}{E_a}(1 - \nu_a) + \alpha_a \Delta T_2}$$

$$\varepsilon^j_{ss} = \frac{1}{E_j}\left(\sigma^j_{ss} - \nu_j \sigma^j_{rr}\right) + \alpha_j \Delta T$$

$$\boxed{\varepsilon^j_{ss} = \frac{q}{E_j}\left(\frac{D_2}{2\delta} + \nu_j\right) + \alpha_j \Delta T}$$

$$\sigma^a_{yy} = \sigma^a_{zz} = -q$$

$$\sigma^j_{rr} = -q$$

$$\sigma^j_{ss} = \frac{q D_2}{2\delta}$$

$$\sigma^a_{xx} = \sigma^j_{xx} = 0$$

$$\boxed{\varepsilon^a_{yy} = \varepsilon^j_{ss}} \longrightarrow q = \frac{\alpha_a \Delta T_2 - \alpha_j \Delta T}{\frac{1}{E_j}\left(\frac{D_2}{2\delta} + \nu_j\right) + \frac{1}{E_a}(1 - \nu_a)}$$

$q < 0 \longrightarrow$ Po izenačenju temperatur pri 110°C se valj in cev ne dotikata!

2) $$\varepsilon_{yy}^{a\,\prime} = \frac{D' - D_1}{D_1} = \alpha_a \Delta T' \longrightarrow D' = D_1(1 + \alpha_a \Delta T')$$

$$\varepsilon_{ss}^{j\,\prime} = \frac{D' - D_2}{D_2} = \alpha_j \Delta T' \longrightarrow D' = D_2(1 + \alpha_j \Delta T')$$

$$D_1 + D_1 \alpha_a \Delta T' = D_2 + D_2 \alpha_j \Delta T'$$

$$\boxed{\Delta T' = \frac{D_2 - D_1}{\alpha_a D_1 - \alpha_j D_2}} \longrightarrow \boxed{\Delta T' = 173{,}8^\circ}$$

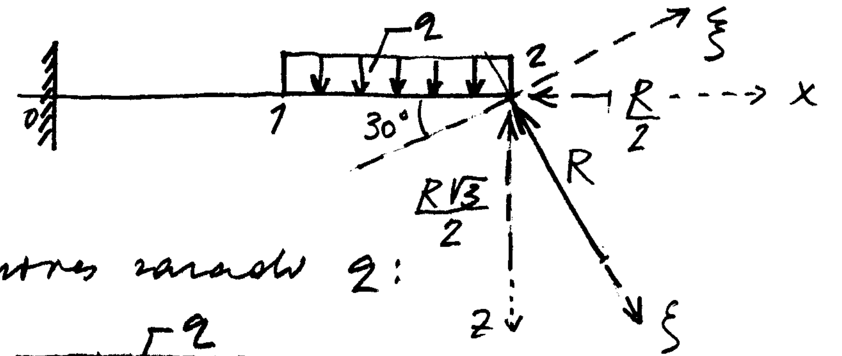
Kontrola: $D' = 20(1 + 2{,}353 \cdot 10^{-5} \cdot 173{,}8^\circ)$

$D' = 20{,}0818$ cm

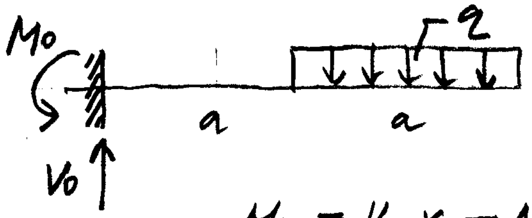
$D' = 20{,}04(1 + 1{,}2 \cdot 10^{-5} \cdot 173{,}8^\circ)$

$D' = 20{,}0818$ cm ✓

Ad 3.)



Tč. 2: – przez zarazów q:



$$V_0 = qa, \quad M_0 = \frac{3qa^2}{2}$$

$$M_y = V_0 x - M_0 - \frac{q}{2}\langle x - a\rangle^2$$

$$M_y = qax - \frac{3qa^2}{2} - \frac{q}{2}\langle x - a\rangle^2 = -EJ_y w''$$

$$EJ_y w'' = \frac{q}{2}\left(-2ax + 3a^2 + \langle x - a\rangle^2\right)$$

$$EI_y w' = \frac{q}{2}\left(-ax^2 + 3a^2x + \frac{1}{3}\langle x-a\rangle^3\right) + C_1$$

$$EI_y w = \frac{q}{2}\left(-\frac{ax^3}{3} + \frac{3a^2}{2}x^2 + \frac{1}{12}\langle x-a\rangle^4\right) + C_1 x + C_2$$

$$x = 0 \rightarrow w = 0,\; w' = 0 \longrightarrow C_1 = C_2 = 0$$

$$x = 2a \rightarrow EI_y w = \frac{qa^4}{2}\left(-\frac{8}{3} + \frac{12}{2} + \frac{1}{12}\right)$$

$$\boxed{w_2(q) = \frac{41qa^4}{24EI_y}}$$

- poves zaradi R: $\quad w_2(R) = -\frac{R\sqrt{3}}{2} \cdot \frac{(2a)^3}{3EI_y}$

$$\boxed{w_2(R) = -R\,\frac{4\sqrt{3}\,a^3}{3EI_y}}$$

tč. 2: $\quad u_x = -\frac{R}{2} \cdot \frac{2a}{EA_x} \rightarrow \boxed{u_x = -R\,\frac{a}{EA_x}}$

$$\boxed{u_z = \frac{41qa^4}{24EI_y} - R\,\frac{4\sqrt{3}\,a^3}{3EI_y}}$$

$$u_x = -R\,\frac{240}{20000 \cdot 40} \rightarrow \boxed{u_x = -0{,}0003\,R}$$

$$u_z = \frac{41 \cdot 0{,}20 \cdot 240^4}{24 \cdot 20000 \cdot 3000} - R\,\frac{4\sqrt{3} \cdot 240^3}{3 \cdot 20000 \cdot 3000}$$

$$\boxed{u_z = 18{,}8928 - 0{,}5321\,R}$$

$$u_\xi = u_x e_{\xi x} + u_z e_{\xi z} \qquad e_{\xi x} = \frac{\sqrt{3}}{2} \qquad e_{\xi z} = -\frac{1}{2}$$

$$u_\zeta = u_x e_{\zeta x} + u_z e_{\zeta z} = 0 \qquad e_{\zeta x} = \frac{1}{2} \qquad e_{\zeta z} = \frac{\sqrt{3}}{2}$$

$$-0{,}0003\,R \cdot \frac{1}{2} + (18{,}8928 - 0{,}5321\,R)\frac{\sqrt{3}}{2} = 0$$

$$\boxed{R = 35{,}495\ kN}$$

Ad. 2:

$$u_x = -0{,}0003 \cdot 35{,}495 \rightarrow \boxed{u_x = -0{,}0106 \text{ cm}}$$

$$u_z = 18{,}8928 - 0{,}5321 \cdot 35{,}495$$

$$\boxed{u_z = 0{,}0061 \text{ cm}}$$

$$V_0 = qa - R\frac{\sqrt{3}}{2} = 0{,}20 \cdot 240 - 35{,}495 \cdot \frac{\sqrt{3}}{2}$$

$$\boxed{V_0 = 17{,}26 \text{ kN}} \longrightarrow \boxed{N_z^0 = 17{,}26 \text{ kN}}$$

$$M_0 = \frac{3qa^2}{2} - \frac{R\sqrt{3}}{2} \cdot 2a = \frac{3 \cdot 0{,}20 \cdot 240^2}{2} - 35{,}495 \cdot \sqrt{3} \cdot 240$$

$$\boxed{M_0 = 2525 \text{ kNcm}} \longrightarrow \boxed{M_y^0 = -2525 \text{ kNcm}}$$

$$M_y^1 = \frac{R\sqrt{3}}{2} a - \frac{qa^2}{2} \longrightarrow \boxed{M_y^1 = -5{,}75 \text{ kNcm}}$$

$$N_z^1 = -\frac{R\sqrt{3}}{2} + qa \longrightarrow \boxed{N_z^1 = 17{,}26 \text{ kN}}$$

$$N_z^2 = -R\frac{\sqrt{3}}{2} \qquad \boxed{N_z^2 = -30{,}74 \text{ kN}}$$