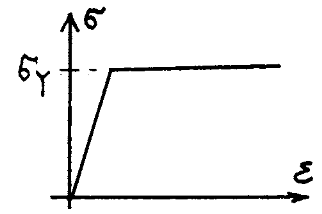
MEHANIKA TRDNIH TELES **IZPIT 10. 2. 1997**

1. Napetostno stanje delca $\mathcal{D}$ je opisano s komponentami σ_{ij} tenzorja napetosti glede na koordinatni sistem (x, y, z).

   a. Dokaži, da je ravnina, π_ζ, katere normala $\boldsymbol{e}_\zeta$ oklepa enake kote z osmi x, y, z, ena od glavnih ravnin podanega napetostnega stanja! Določi preostali dve glavni ravnini in vse glavne normalne napetosti!

   b. Določi specifično spremembo prostornine, ki pripada obravnavanemu delcu D (modul elastičnosti E in koeficient prečne kontrakcije ν sta podani vrednosti) !

   c. Obravnavano telo je narejeno iz bilinearnega elastično–plastičnega materiala. Pri enoosnem napetostnem stanju je prehod iz elastičnega v plastično stanje določen z mejo plastičnega tečenja σ_Y. Upoštevajoč Misesov pogoj tečenja določi napetost q_Y, pri kateri pri podanem prostorskem napetostnem stanju delca D nastopijo prve plastične deformacije! Napetost q_Y izrazi v odvisnosti od σ_Y !



$$[\sigma_{ij}] = \begin{bmatrix} q & 2q & 2q \\ 2q & q & 2q \\ 2q & 2q & q \end{bmatrix}$$

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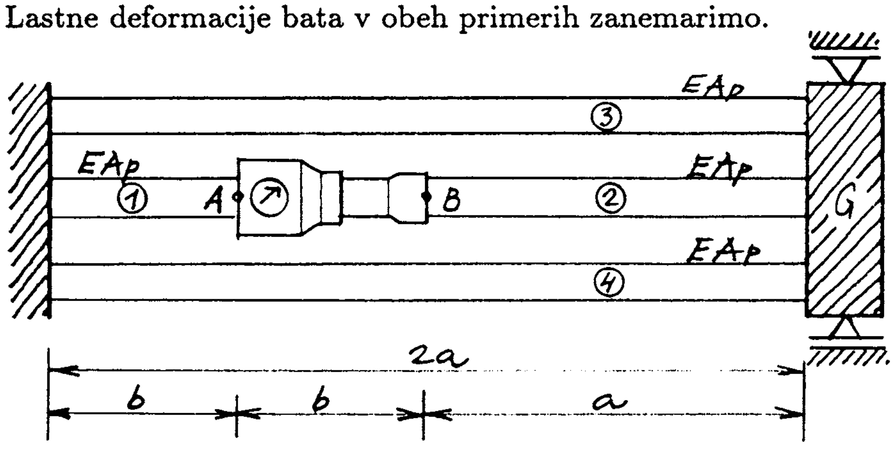
2. Med palici 1 in 2 vstavimo bat hidravlične naprave. V zaëtku med batom in palicama ni napetosti.

   a. Določi potrebno silo v batu, pri kateri se razdalja med točkama A in B poveča za $\delta = 2\,\mathrm{mm}$! Kolikšne so tedaj napetosti v palicah?

   b. Določi potrebno silo v batu, pri kateri se razdalja med točkama A in B ne spremeni, če palici 1 in 2 segrejemo za $\Delta T = 60$ K, palici 3 in 4 pa obdržita začetno temperaturo! Določi premik toge vezne grede G!

   Lastne deformacije bata v obeh primerih zanemarimo.



$$E = 200\,000\,\mathrm{MPa}$$
$$\alpha_T = 1.25 \cdot 10^{-5}/\mathrm{K}$$
$$a = 2\,\mathrm{m}$$
$$b = 1\,\mathrm{m}$$
$$A_p = 10\,\mathrm{cm}^2$$

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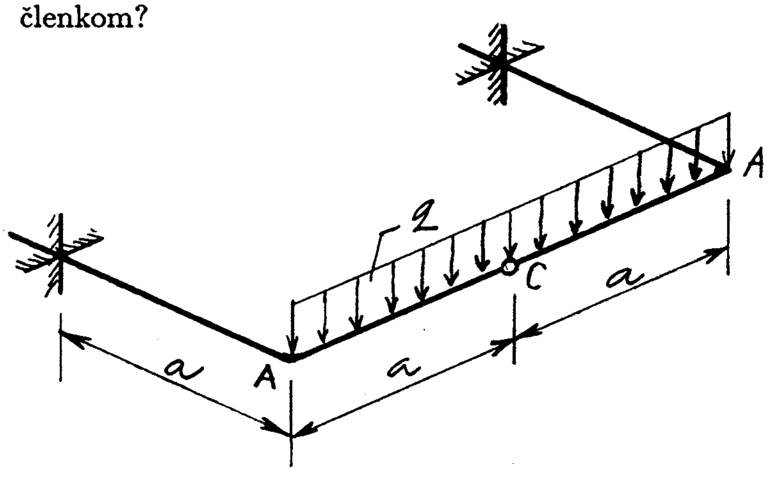
3. Določi navpični pomik točke C! Kako se spremeni ta pomik, če nosilec v točki C ni prekinjen s členkom?



$$q = 0.1\,\mathrm{MN/m}$$
$$EI_y = GI_x = 100\,\mathrm{MNm}^2$$
$$a = 3\,\mathrm{m}$$

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Ad 1.

a) $e_{\xi x} = e_{\xi y} = e_{\xi z} \longrightarrow 3e_{\xi x}^2 = 1 \longrightarrow e_{\xi x} = \frac{1}{\sqrt{3}}$

$$\vec{e}_\xi = \frac{1}{\sqrt{3}}(\vec{e}_x + \vec{e}_y + \vec{e}_z)$$

$$\begin{Bmatrix} \sigma_{\xi x} \\ \sigma_{\xi y} \\ \sigma_{\xi z} \end{Bmatrix} = \begin{bmatrix} q & 2q & 2q \\ 2q & q & 2q \\ 2q & 2q & q \end{bmatrix} \cdot \frac{1}{\sqrt{3}} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \frac{5}{\sqrt{3}} \begin{Bmatrix} q \\ q \\ q \end{Bmatrix}$$

$$\boxed{\vec{\sigma}_\xi = \frac{5q}{\sqrt{3}}(\vec{e}_x + \vec{e}_y + \vec{e}_z)}$$

$$\sigma_{\xi\xi} = \vec{\sigma}_\xi \, \vec{e}_\xi = \frac{5q}{3}(1+1+1) \longrightarrow \boxed{\sigma_{\xi\xi} = 5q}$$

$$\boxed{\vec{\sigma}_\xi = \sigma_{\xi\xi} \, \vec{e}_\xi}$$

b) $\boxed{\sigma_{\xi\xi} = \sigma_{33} = 5q}$

$$I_1 = 3q$$

$$I_2 = 3 \begin{vmatrix} q & 2q \\ 2q & q \end{vmatrix} = -9q^2$$

$$I_3 = q(q^2 - 4q^2) - 2q(2q^2 - 4q^2) + 2q(4q^2 - 2q^2)$$

$$I_3 = 5q^3$$

$$\boxed{\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0} \longrightarrow$$

$$\sigma^3 - 3q\sigma^2 - 9q^2\sigma - 5q^3 = 0$$

$$(\sigma^3 - 3q\sigma^2 - 9q^2\sigma - 5q^3) : (\sigma - 5q) = \sigma^2 + 2q\sigma + q^2$$

$$\underline{-\sigma^3 + 5q\sigma^2}$$

$$2q\sigma^2 - 9q^2\sigma$$

$$\underline{-2q\sigma^2 + 10q^2\sigma}$$

$$q^2\sigma - 5q^3 \checkmark$$

$$\sigma^2 + 2q\sigma + q^2 = 0 \longrightarrow \boxed{\sigma_{11} = \sigma_{22} = -q}$$

Glavna smer $\vec{e}_3 \equiv \vec{e}_\xi$, glavni smeri $\vec{e}_1$ in $\vec{e}_2$ sta pravokotni na $\vec{e}_\xi$, sicer pa poljubni, ker je $\sigma_{11} = \sigma_{22}$!

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b) $\boxed{I_1^\sigma = 3q}$ $\qquad I_1^\varepsilon \cong \varepsilon_V = \frac{1-2\nu}{E} I_1^\sigma$

$$\boxed{\varepsilon_V = \frac{3(1-2\nu)}{E} q}$$

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c) Mises: $\quad (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 = 6 k_M^2$

$$\sigma_{22} = \sigma_{33} = 0 \longrightarrow 2\sigma_{11}^2 = 6 k_M^2 \rightarrow \sigma_{11}^2 = 3 k_M^2$$

$$\sigma_{11} = \sigma_Y \longrightarrow \boxed{k_M = \frac{1}{\sqrt{3}} \sigma_Y}$$

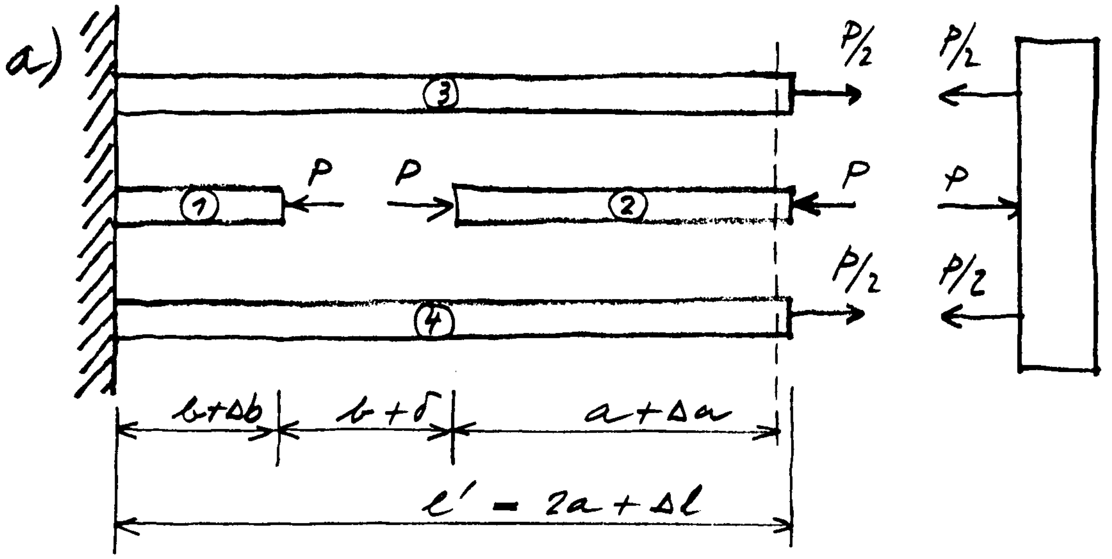
$$\boxed{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 = 2\sigma_Y^2}$$

$$\boxed{\sigma_{11} = \sigma_{22} = -q, \quad \sigma_{33} = 5q \quad :}$$

$$0 + 36q^2 + 36q^2 = 2\sigma_Y^2 \longrightarrow \boxed{q_Y = \frac{1}{6}\sigma_Y}$$

Ad 2.)

a)



$$l' = 2a + \frac{P}{2}\cdot\frac{2a}{EA_p} = \left(b - P\frac{b}{EA_p}\right) + \left(a - P\frac{a}{EA_p}\right) + (b+\delta)$$

$$P\frac{1}{EA_p}(a+b+a) = b + a + b + \delta - 2a$$

$$2b + a = 2a \longrightarrow \boxed{P = \frac{\delta EA_p}{2a+b}} \longrightarrow \boxed{P = 80\ kN}$$

b) $$l' = 2a + \frac{P}{2}\frac{2a}{EA_p} =$$

$$= \left(b - P\frac{b}{EA_p} + b\alpha_T\Delta T\right) + \left(a - P\frac{a}{EA_p} + a\alpha_T\Delta T\right) +$$

$$+ b$$

$$\frac{P}{EA_p}(2a+b) = 2a - 2a + (a+b)\alpha_T\Delta T$$

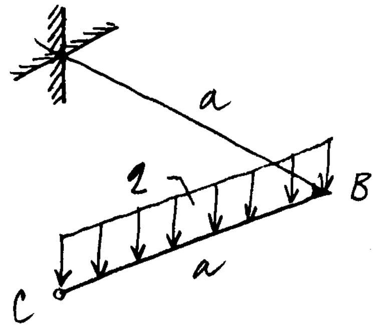
$$\boxed{P = \frac{a+b}{2a+b}EA_p\alpha_T\Delta T} \longrightarrow \boxed{P = 90\ kN}$$

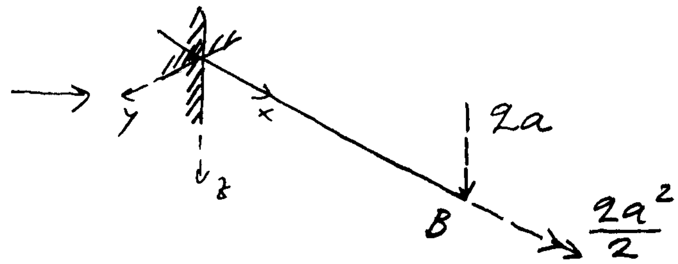
$$\boxed{u_G = P\frac{a}{EA_p}} \longrightarrow \boxed{u_G = 0{,}09\ cm}$$

a) Napjatosti v prutech :

①, ② $\rightarrow \boxed{\sigma = -8\ kN/cm^2}$ ③, ④ $\rightarrow \boxed{\sigma = 4\ kN/cm^2}$

Ad 3.) a) členek v točki C ;





$$\omega_x(B) = \frac{qa^2}{2} \cdot \frac{a}{GI_x} = q\frac{a^3}{2GI_x}$$

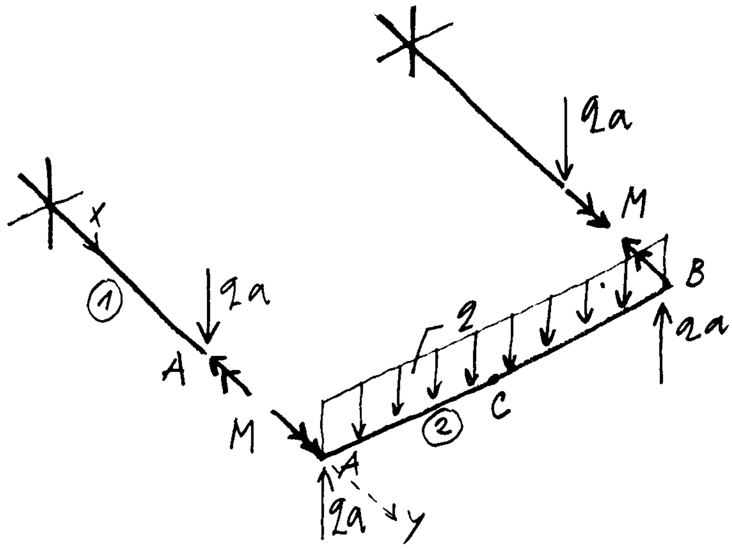
$$w_B = qa\frac{a^3}{3EI_y} = q\frac{a^4}{3EI_y}$$

$$w_C = w_B + \omega_x(B)\cdot a + \frac{qa^4}{8EI_y}$$

$$EI_y = GI_x \longrightarrow \quad w_C = \frac{qa^4}{EI_y}\left(\frac{1}{3} + \frac{1}{2} + \frac{1}{8}\right)$$

$$\boxed{w_C = q\frac{23a^4}{24EI_y}} \longrightarrow \boxed{w_C = 0{,}0776\ m}$$

b) brez členka ;



①: $w_A^{①} = q\frac{a^4}{3EI_y}$

$$\omega_x^{①}(A) = \omega_y^{②}(A) = -M\frac{a}{GI_x}$$

②:

$$\omega_y^{②}(A) = -q\frac{(2a)^3}{24EI_y} + M\frac{2a}{2EI_y}$$

$$\omega_y^{②}(A) = -q\frac{a^3}{3EI_y} + M\frac{a}{EI_y}$$

$$\omega_x^{①}(A) = \omega_y^{②}(A) \ \ldots \ -M\frac{a}{GI_x} = -q\frac{a^3}{3EI_y} + M\frac{a}{EI_y}$$

$$EI_y = GI_x \ \ldots \quad \boxed{M = q\frac{a^2}{6}}$$

$$w_C = w_A + q\,\frac{5(2a)^4}{384EI_y} - M\,\frac{(2a)^2}{8EI_y}$$

$$\boxed{w_C = q\,\frac{11a^4}{24EI_y}} \longrightarrow \boxed{w_C = 0{,}0371\ m}$$