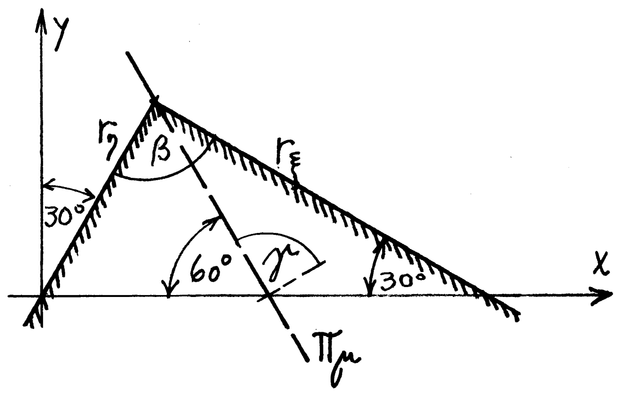
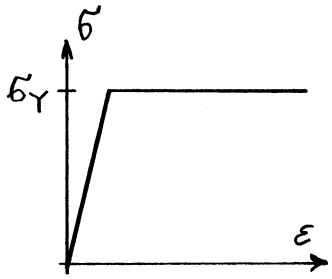
1. Na vogalu enakomerno debele homogene stene, v kateri vlada **homogeno ravninsko napetostno stanje**, izrežemo elementarni del, kakor je prikazano na skici.
Na robu $\mathbf{r}_\xi$ deluje enakomerna zvezna obtežba $\vec{\mathbf{p}}_\xi = -q\,(\vec{\mathbf{e}}_x + \sqrt{3}\,\vec{\mathbf{e}}_y)$, na robu $\mathbf{r}_\eta$ pa enakomerna zvezna obtežba $\vec{\mathbf{p}}_\eta = 3q\,(-\sqrt{3}\,\vec{\mathbf{e}}_x + \vec{\mathbf{e}}_y)$.

   a. Skiciraj obtežbo ter ustrezni zunanji normali robov obravnavane stene!
   b. Določi komponente tenzorja napetosti glede na kartezijski koordinatni sistem $(x,\,y,\,z)$!
   c. Določi rezultirajoči vektor napetosti v ravnini Π_μ z njegovimi komponentami v koordinatnem sistemu $(x,\,y,\,z)$ ter normalno in strižno napetost v tej ravnini!
   d. Določi spremembi pravih kotov β in γ v odvisnosti od obtežbe q!
   e. Določi velikosti in smeri glavnih normalnih napetosti!
   f. Obravnavano telo je narejeno iz bilinearnega elastično–plastičnega materiala. Pri enoosnem napetostnem stanju je prehod iz elastičnega v plastično stanje določen z mejo plastičnega tečenja σ_Y. Upoštevajoč Misesov pogoj tečenja določi obtežbo q_Y, pri kateri pri podanem napetostnem stanju nastopijo prve plastične deformacije!





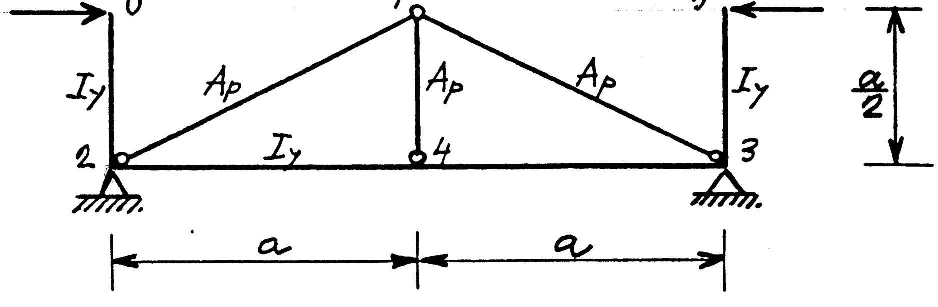
$$E = 200\,000\,\mathrm{MPa}$$
$$\sigma_Y = 360\,\mathrm{MPa}$$
$$\nu = 0.3$$

2. Na jeklen valj premera $D_v = 200\,\mathrm{mm}$ želimo natakniti jekleno cev z notranjim premerom $D_0 = 199.8\,\mathrm{mm}$. Debelina stene cevi je $\delta = 2.2\,\mathrm{mm}$. Trenje med valjem in cevjo je zanemarljivo. V vzdolžni smeri se lahko cev in valj neovirano deformirata.

   a. Za koliko moramo segreti cev, da bi jo lahko nataknili na valj?
   b. Določi napetosti v valju in cevi po ohladitvi sestava na prvotno temperaturo!
   c. Določi premer valja po ohladitvi!

$$E = 200\,000\,\mathrm{MPa}, \qquad \nu = 0.3\,, \qquad \alpha_T = 1.25\cdot 10^{-5}/\mathrm{K}\,.$$

3. Nosilec $\overline{\mathbf{05}}$ je podprt s trikotnim vešalom. Določi vodoravni pomik točke 0!



$$H = 0.3\,\mathrm{MN}$$
$$E = 200\,000\,\mathrm{MPa}$$
$$A_p = 56\,\mathrm{cm}^2$$
$$I_y = 5400\,\mathrm{cm}^4$$
$$a = 4\,\mathrm{m}$$

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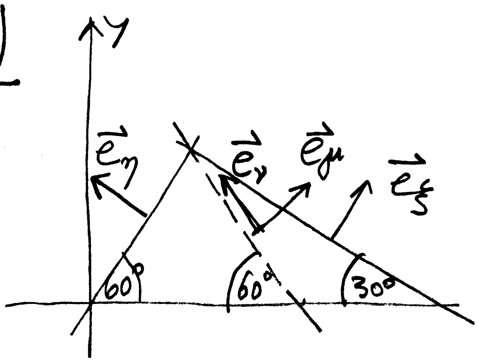
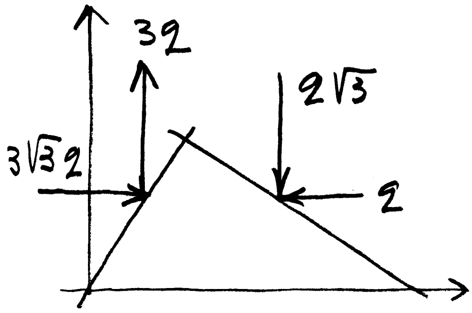
Ad 1.)

a)





b) $r_\xi: \quad \vec{e}_\xi = \frac{1}{2}(\vec{e}_x + \sqrt{3}\,\vec{e}_y) \longrightarrow \vec{p}_\xi = -q(\vec{e}_x + \sqrt{3}\,\vec{e}_y)$

$r_\eta: \quad \vec{e}_\eta = \frac{1}{2}(-\sqrt{3}\,\vec{e}_x + \vec{e}_y) \longrightarrow \vec{p}_\eta = 3q(-\sqrt{3}\,\vec{e}_x + \vec{e}_y)$

$$p_{\xi x} = -q = \frac{1}{2}\sigma_{xx} + \frac{\sqrt{3}}{2}\sigma_{xy} \longrightarrow \sigma_{xx} + \sqrt{3}\,\sigma_{xy} = -2q \quad (A)$$

$$p_{\xi y} = -q\sqrt{3} = \frac{1}{2}\sigma_{xy} + \frac{\sqrt{3}}{2}\sigma_{yy} \longrightarrow \sqrt{3}\,\sigma_{yy} + \sigma_{xy} = -2\sqrt{3}\,q \quad (B)$$

$$p_{\eta x} = -3\sqrt{3}\,q = -\frac{\sqrt{3}}{2}\sigma_{xx} + \frac{1}{2}\sigma_{xy} \longrightarrow \sqrt{3}\,\sigma_{xx} - \sigma_{xy} = 6\sqrt{3}\,q \quad (C)$$

$$p_{\eta y} = 3q = -\frac{\sqrt{3}}{2}\sigma_{xy} + \frac{1}{2}\sigma_{yy} \longrightarrow \sigma_{yy} - \sqrt{3}\,\sigma_{xy} = 6q \quad (D)$$

$$\left.\begin{aligned} (B) &\rightarrow 3\sigma_{yy} + \sqrt{3}\,\sigma_{xy} = -6q \\ (D) &\rightarrow \sigma_{yy} - \sqrt{3}\,\sigma_{xy} = 6q \end{aligned}\right\} + \longrightarrow \boxed{\sigma_{yy} = 0}$$

$(B) \longrightarrow \boxed{\sigma_{xy} = -2\sqrt{3}\,q} \qquad (A) \rightarrow \boxed{\sigma_{xx} = 4q}$

Kontrola: $(C) \rightarrow 4\sqrt{3}\,q + 2\sqrt{3}\,q = 6\sqrt{3}\,q$ ✓

$$[\sigma_{ij}] = \begin{bmatrix} 4q & -2\sqrt{3}\,q & 0 \\ -2\sqrt{3}\,q & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c) $$\begin{Bmatrix} \sigma_{\mu x} \\ \sigma_{\mu y} \\ \sigma_{\mu z} \end{Bmatrix} = [\sigma_{ij}] \begin{Bmatrix} e_{\mu x} \\ e_{\mu y} \\ e_{\mu z} \end{Bmatrix}$$

$\vec{e}_\mu = \frac{1}{2}(\sqrt{3}\,\vec{e}_x + \vec{e}_y)$

$\vec{e}_\nu = \frac{1}{2}(-\vec{e}_x + \sqrt{3}\,\vec{e}_y)$

$$\boxed{\vec{\sigma}_\mu = q\sqrt{3}\,\vec{e}_x - 3q\,\vec{e}_y}$$

$$\sigma_{\mu\mu} = \vec{\sigma}_{\mu}\,\vec{e}_{\mu} \longrightarrow \boxed{\sigma_{\mu\mu} = 0}$$

$$\sigma_{\mu\nu} = \vec{\sigma}_{\mu}\,\vec{e}_{\nu} \longrightarrow \boxed{\sigma_{\mu\nu} = -2q\sqrt{3}}$$

d) $\vec{\sigma}_{\xi} = \vec{p}_{\xi} = -q\,(\vec{e}_x + \sqrt{3}\,\vec{e}_y)$

$$\sigma_{\xi\eta} = \vec{\sigma}_{\xi}\,\vec{e}_{\eta} \longrightarrow \sigma_{\xi\eta} = 0 \longrightarrow \boxed{\Delta\beta = D_{\xi\eta} = 0}$$

$$\vec{\sigma}_{\eta} = \vec{p}_{\eta} = 3q\,(-\sqrt{3}\,\vec{e}_x + \vec{e}_y) \quad (*)$$

e) $\sigma_{\xi\xi} = \sigma_{11} = \vec{\sigma}_{\xi}\,\vec{e}_{\xi} \longrightarrow \boxed{\sigma_{11} = \sigma_{\xi\xi} = -2q}$

$$\sigma_{\eta\eta} = \sigma_{22} = \vec{\sigma}_{\eta}\,\vec{e}_{\eta} \longrightarrow \boxed{\sigma_{22} - \sigma_{\eta\eta} = 6q}$$

$$\boxed{\vec{e}_1 \equiv \vec{e}_{\xi}} \quad \boxed{\vec{e}_2 \equiv \vec{e}_{\eta}}$$

Kontrola :

$$\sigma_{11,22} = 2q \pm \sqrt{(2q)^2 + 12q^2} = \begin{cases} 6q \\ -2q \end{cases} \quad \checkmark$$

$(*)$ $\Delta\gamma = 2\varepsilon_{\mu\nu} = 2\,\dfrac{1+\nu}{E}\,\sigma_{\mu\nu} = -q\,\dfrac{4\sqrt{3}\,(1+\nu)}{E}$

$$\boxed{\Delta\gamma = D_{\mu\nu} = 4{,}5\cdot 10^{-5}\,q} \qquad [q \text{ v MPa}]\,!$$

f) Mises: $(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 = 2\sigma_Y^2$

$\sigma_{11} = -2q\,, \quad \sigma_{22} = 6q\,, \quad \sigma_{33} = 0\,;$

$$(-2q-6q)^2 + (6q)^2 + (2q)^2 = 2\sigma_Y^2$$

$$\boxed{q_Y = \pm\frac{\sigma_Y}{\sqrt{52}}} \longrightarrow \boxed{q_Y = \pm\, 49{,}9 \text{ MPa}}$$

Ad 2.)

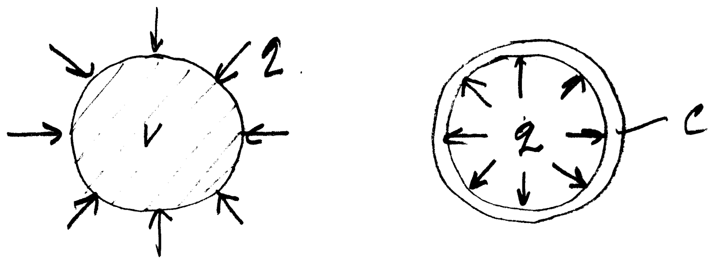
a) $\varepsilon_{xx}^{v} = \varepsilon_{yy}^{v} = \varepsilon_{ss}^{c} = \frac{D_v - D_o}{D_o} = \frac{200}{199{,}8} - 1 = 0{,}001$

$$\varepsilon_{ss}^{c} = \alpha_T \Delta T = \frac{D_v - D_o}{D_o} \rightarrow \Delta T = \frac{D_v - D_o}{\alpha_T D_o} =$$

$$= \frac{200 - 199{,}8}{199{,}8 \cdot 1{,}25 \cdot 10^{-5}}$$

$$\boxed{\Delta T = 80\,^{\circ}C}$$

b)



Valec:

$\sigma_{xx} = \sigma_{yy} = -q$

$\sigma_{zz} = 0$

$$\varepsilon_{xx}^{v} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy}) = -q\,\frac{1-\nu}{E}$$

Cev: $\sigma_{rr} = -q$, $\sigma_{ss} = \frac{q D_v}{2\delta}$, $\sigma_{zz} = 0$

$$\varepsilon_{ss}^{c} = \frac{1}{E}(\sigma_{ss} - \nu\sigma_{rr}) - \alpha_T \Delta T = \frac{1}{E}\left(\frac{q D_v}{2\delta} + \nu q\right) - \alpha_T \Delta T$$

$$\varepsilon_{xx}^{v} = \varepsilon_{ss}^{c} \rightarrow -\frac{1-\nu}{E}q = \frac{q}{E}\left(\frac{D_v}{2\delta} + \nu\right) - \alpha_T \Delta T$$

$$\boxed{q = \frac{2\delta E \alpha_T \Delta T}{D_v + 2\delta}} \longrightarrow \boxed{q = 4{,}305\ MPa}$$

Valec: $\boxed{\sigma_{xx} = \sigma_{yy} = -4{,}305\ MPa}$

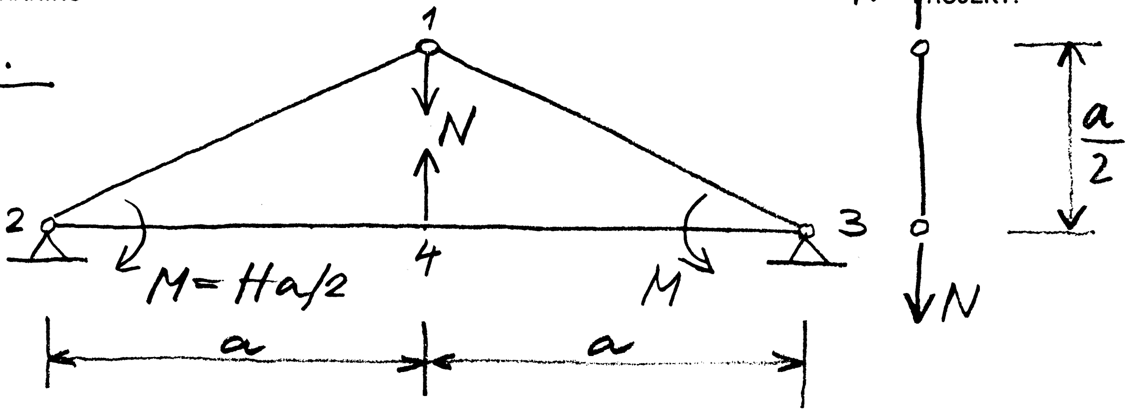
Cev: $\boxed{\sigma_{rr} = -4{,}305\ MPa}$

$$\sigma_{ss} = q\,\frac{D_v}{2\delta} \longrightarrow \boxed{\sigma_{ss} = 195{,}7\ MPa}$$

c) $\varepsilon_{xx}^{v} = -q\,\frac{1-\nu}{E} = \frac{D' - D_v}{D_v} = -4{,}305 \cdot \frac{1-0{,}3}{2 \cdot 10^5} = -1{,}507 \cdot 10^{-5}$

$$D' = D_v(1 + \varepsilon_{xx}^{v}) \longrightarrow \boxed{D' = 199{,}997\ mm}$$

Ad 3.



$$w_4 = w_1 + \Delta l_p = w_1 + N\frac{a}{2EA_p}$$

$$w_1 = N\frac{5a\sqrt{5}}{4EA_p} \longrightarrow \boxed{w_1 = 0{,}00998\ N}$$

$$w_4 = H\frac{a}{2}\frac{(2a)^2}{8EI_y} - N\frac{(2a)^3}{48EI_y} = H\frac{a^3}{4EI_y} - N\frac{a^3}{6EI_y}$$

$$\boxed{w_4 = 0{,}44444 - 0{,}98765\ N}$$

$$\Delta l_p = N\frac{a}{2EA_p} \longrightarrow \boxed{\Delta l_p = 0{,}00179\ N}$$

$$0{,}44444 - 0{,}98765\ N = 0{,}00998\ N + 0{,}00179\ N$$

$$\boxed{N = 0{,}4447\ MN}$$

$$\omega_y(2) = -H\frac{a}{2}\frac{2a}{2EI_y} + N\frac{(2a)^2}{16EI_y} \rightarrow \boxed{\omega_y(2) = -0{,}0575}$$

$$u_0 = -\frac{a}{2}\omega_y(2) + H\frac{\left(\frac{a}{2}\right)^3}{3EI_y} \longrightarrow \boxed{u_0 = 0{,}189\ m}$$