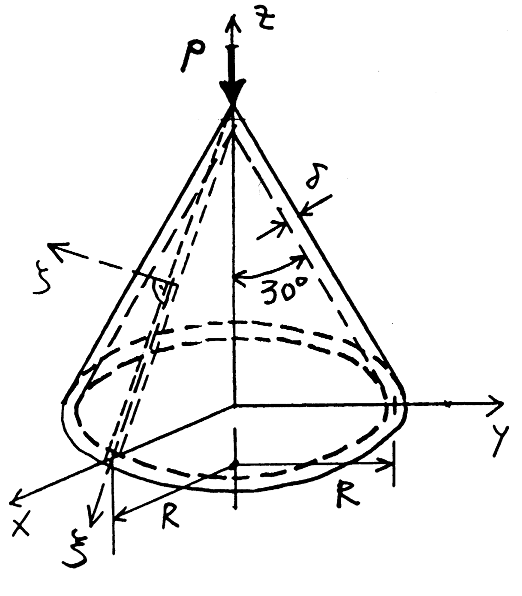
1.



Tanka stožčasta lupina je prilepljena na vodoravno podlago. Srednji polmer osnovne ploskve je R. Vrh stožca je obtežen z navpično silo P. Upoštevajoč, da gre za osnosimetrično napetostno stanje, določi normalno napetost $\sigma_{\xi\xi}$ v steni lupine v odvisnosti od koordinate z ter normalno napetost σ_{zz} in strižno napetost $\sigma_{zx} = \sigma_{zy}$ v onovni ploskvi ($z = 0$)! Ker gre za tanko lupino ($\delta << R$), lahko predpostaviš enakomeren potek napetosti po debelini. Lastne teže lupine ni treba upoštevati.

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2. Kot rešitev mehanskega problema trdnega telesa smo dobili matriko majhnih deformacij ε_{ij}. V točki $T_0(0,0,0)$ je telo nepomično vrtljivo podprto, v točki $T_1(0,3,1)$ pa je preprečen navpični pomik u_z.
   a. Določi vektorsko polje pomikov $\vec{u}$, ki ustreza podanim deformacijam!
   b. Določi specifično spremembo razdalje med točkama T_0 in $T_2(0,\ 0.1,\ 0.1)$! Kolikšno napako narediš, če to spremembo določiš kar z ustrezno vrednostjo tenzorja majhnoh deformacij?
   c. Določi deformirano telesno koordinatno bazo v točki T_0!

$$[\varepsilon_{ij}] = 10^{-3}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 10y & z^2 \\ 0 & z^2 & 4yz \end{bmatrix}$$

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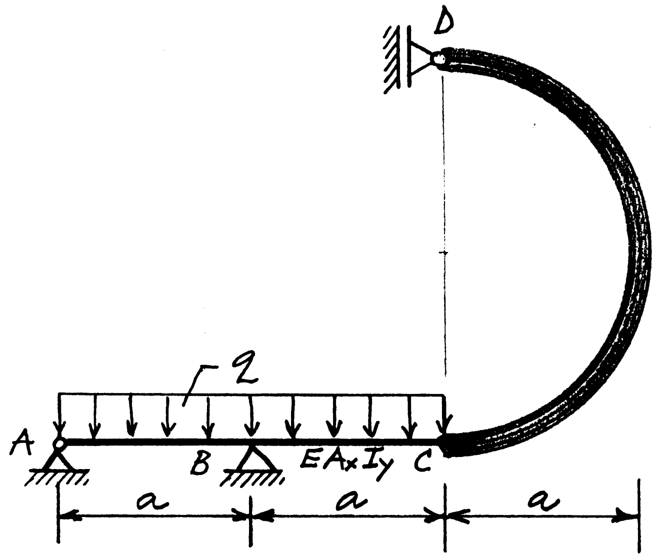
3. Fasadni element $\overline{CD}$ je zelo tog v primerjavi z nosilcem $\overline{AC}$. V točki C sta oba dela konstrukcije togo povezana.
   Določi navpični pomik točke D v odvisnosti od velikosti zvezne obtežbe q!

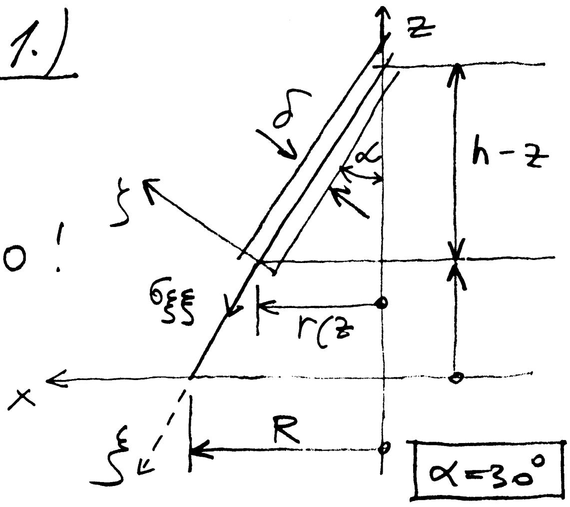


$$E = 200\,000\,\mathrm{MPa}$$
$$A_x = 40\,\mathrm{cm}^2$$
$$I_y = 4000\,\mathrm{cm}^4$$
$$a = 4\,\mathrm{m}$$

MTT IZPIT 7.7.97

Ad 1.)

$\sigma_{\xi\zeta} = 0$!



$r(z) = (h-z)\,\mathrm{tg}\,\alpha$

$$\boxed{r(z) = (h-z)\cdot\frac{1}{\sqrt{3}}}$$

$e_{\xi x} = \frac{1}{2}$

$e_{\xi z} = -\frac{\sqrt{3}}{2}$

$e_{\zeta x} = \frac{\sqrt{3}}{2}$

$e_{\zeta z} = \frac{1}{2}$

$$\sum P_z = 0 \rightarrow -P - 2\pi r(z)\cdot\delta\cdot\sigma_{\xi\xi}\cdot\cos\alpha = 0$$

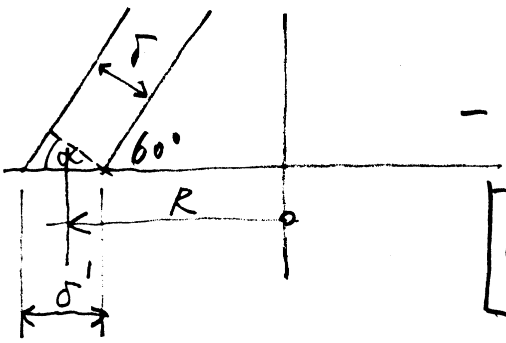
$$\boxed{\sigma_{\xi\xi} = -\frac{P}{\pi\delta(h-z)} = \sigma_{\xi\xi}(z)}$$

$$\sigma_{zz} = \sigma_{\xi\xi}\,e_{\xi z}^2 = \sigma_{\xi\xi}\cdot\frac{3}{4} \rightarrow \boxed{\sigma_{zz} = -\frac{3P}{4\pi\delta(h-z)}}$$

$$\sigma_{zx} = \sigma_{\xi\xi}\,e_{\xi z}\,e_{\xi x} = -\sigma_{\xi\xi}\cdot\frac{\sqrt{3}}{4} \rightarrow \boxed{\sigma_{zx} = \frac{P\sqrt{3}}{4\pi\delta(h-z)}}$$

$$z = 0 \rightarrow \boxed{\sigma_{zz} = -\frac{3P}{4\pi\delta h}} \quad \boxed{\sigma_{zx} = \frac{P\sqrt{3}}{4\pi\delta h}}$$

Kontrola :



$$\delta' = \frac{\delta}{\cos\alpha} = \frac{2\delta}{\sqrt{3}}, \quad R = \frac{h}{\sqrt{3}}$$

$$-2\pi R\delta'\,\sigma_{zz}(z=0) - P = 0$$

$$\boxed{\sigma_{zz}(z=0) = -\frac{3P}{4\pi\delta h}} \quad ✓$$

Ad 2.) (glej izpit 26.1.1990)

a) $$\omega_x = \omega_x^0 + \int_0^y \vec{e}_x (\vec{\nabla} \times \vec{\varepsilon}_y)_{z=0}\, dy + \int_0^z \vec{e}_x (\vec{\nabla} \times \vec{\varepsilon}_z)\, dz$$

$$\vec{e}_x (\vec{\nabla} \times \vec{\varepsilon}_y) = \begin{vmatrix} 1 & 0 & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \varepsilon_{yy} & \varepsilon_{yz} \end{vmatrix} = \frac{\partial \varepsilon_{yz}}{\partial y} - \frac{\partial \varepsilon_{yy}}{\partial z} = 0$$

$$\vec{\varepsilon}_x (\vec{\nabla} \times \vec{\varepsilon}_z) = \begin{vmatrix} 1 & 0 & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0. & \varepsilon_{zy} & \varepsilon_{zz} \end{vmatrix} = \frac{\partial \varepsilon_{zz}}{\partial y} - \frac{\partial \varepsilon_{yz}}{\partial z} = 2z \cdot 10^{-3}$$

$$\omega_x = \omega_x^0 + 10^{-3} \int_0^z 2z\, dz \rightarrow \boxed{\omega_x = \omega_x^0 + 10^{-3} z^2 = \omega_{yz}}$$

$$u_y = u_y^0 + \int_0^y (\varepsilon_{yy})_{z=0}\, dy + \int_0^z (\varepsilon_{zy} + \omega_{zy})\, dz$$

$$u_y = 0 + 10^{-3} \left(\int_0^y 10y\, dy + \int_0^z (z^2 - \omega_x^0 \cdot 10^3 - z^2)\, dz \right)$$

$$\boxed{u_y = -z\,\omega_x^0 + 5y^2 \cdot 10^{-3}}$$

$$u_z = u_z^0 + \int_0^y (\varepsilon_{yz} + \omega_{yz})_{z=0}\, dy + \int_0^z \varepsilon_{zz}\, dz$$

$$u_z = 10^{-3} \left(\int_0^y (z^2 + 10^3 \omega_x^0 + z^2)_{z=0}\, dy + \int_0^z 4yz\, dz \right)$$

$$\boxed{u_z = y\,\omega_x^0 + 2yz^2 \cdot 10^{-3}}$$

$$u_z(T_1) = 3\omega_x^0 + 2 \cdot 3 \cdot 1 \cdot 10^{-3} = 0$$

$$\boxed{\omega_x^0 = -2 \cdot 10^{-3}}$$

$$\boxed{\vec{u} = \left[(5y^2 + 2z)\vec{e}_y + 2y(z^2 - 1)\vec{e}_z \right] \cdot 10^{-3}}$$

b) $\Delta\vec{r} = \vec{r}_2 - \vec{r}_0 = 0{,}1\,\vec{e}_y + 0{,}1\,\vec{e}_z \rightarrow |\Delta\vec{r}| = 0{,}14142$

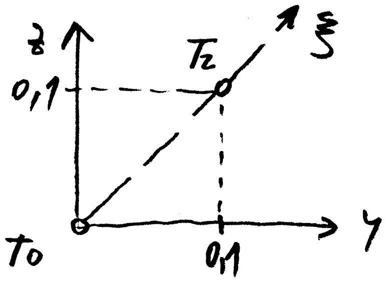
$$\vec{u}_2 = 10^{-3}\left[(5\cdot 0{,}1^2 + 2\cdot 0{,}1)\vec{e}_y + 2\cdot 0{,}1\,(0{,}1^2-1)\vec{e}_z\right]$$

$$\vec{u}_2 = 10^{-3}\left[0{,}250\,\vec{e}_y - 0{,}198\,\vec{e}_z\right]$$

$$\vec{r}_2\,' = \vec{r}_2 + \vec{u}_2 = 0{,}10025\,\vec{e}_y + 0{,}09980\,\vec{e}_z = \Delta\vec{r}\,'$$

$$|\Delta\vec{r}\,'| = 0{,}14146$$

$$D(T_0, T_2) = \frac{|\Delta\vec{r}\,'|}{|\Delta\vec{r}\,|} - 1 \rightarrow \boxed{D = 0{,}2651\cdot 10^{-3}}$$



$$e_{\xi y} = e_{\xi z} = \frac{\sqrt{2}}{2}$$

$$\varepsilon_{\xi\xi} \simeq D = \varepsilon_{yy}\,e_{\xi y} + 2\varepsilon_{yz}\,e_{\xi y}\,e_{\xi z} + \varepsilon_{zz}\,e_{\xi z} = 0$$

$\varepsilon_{ij}(T_0) = 0$ ... Napaka je neskončno velika.

c)

$$\frac{\partial\vec{u}}{\partial y} = 10^{-3}\left[10y\,\vec{e}_y + 2(z^2-1)\,\vec{e}_z\right]$$

$$\frac{\partial\vec{u}}{\partial z} = 10^{-3}\left[2\,\vec{e}_y + 4yz\,\vec{e}_z\right]$$

$$\vec{e}_y\,' = \vec{e}_y + \frac{\partial\vec{u}}{\partial y}$$

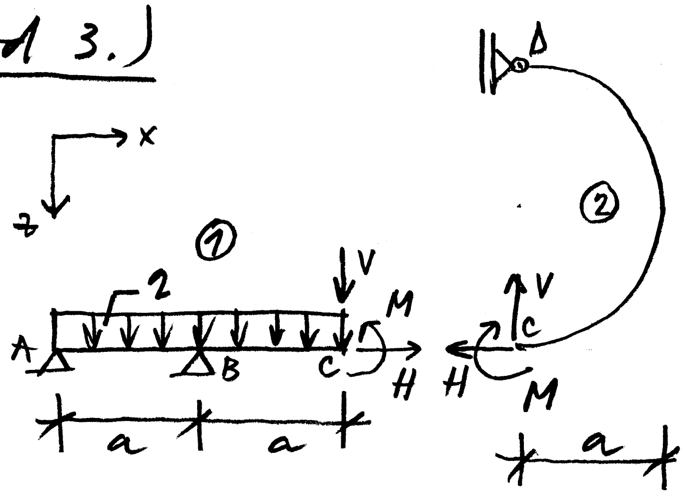
$$\vec{e}_y\,' = (1+0{,}01\,y)\,\vec{e}_y + 0{,}002\,(z^2-1)\,\vec{e}_z$$

$$\vec{e}_z = \vec{e}_z + \frac{\partial\vec{u}}{\partial z}$$

$$\vec{e}_z\,' = 0{,}002\,\vec{e}_y + (1+0{,}004\,yz)\,\vec{e}_z$$

$$T_0:\ \left\{\boxed{\begin{array}{l}\vec{e}_y\,' = \vec{e}_y - 0{,}002\,\vec{e}_z\\ \vec{e}_z\,' = 0{,}002\,\vec{e}_y + \vec{e}_z\end{array}}\right.$$

Ad 3.)



② : $\boxed{V = 0}$

$H \cdot 2a + M = 0$

$$\boxed{H = -M \frac{1}{2a}}$$

$\vec{r}_D = -2a\,\vec{e}_z$

$\vec{u}_c = u_c \vec{e}_x + w_c \vec{e}_z$

$\vec{\omega}_c = \omega_c \vec{e}_y$

$$\vec{u}_D = \vec{u}_c + \vec{\omega}_c \times \vec{r}_D = u_c \vec{e}_x + w_c \vec{e}_z - 2a\,\omega_c \vec{e}_x$$

$$\vec{u}_D = (u_c - 2a\,\omega_c)\vec{e}_x + w_c \vec{e}_z \rightarrow \boxed{w_D = w_c}$$

$$u_x(D) = u_D = 0 \rightarrow \boxed{u_c = 2a\,\omega_c}$$

① $$w_c = \frac{q a^4}{4EI_y} - M\frac{5a^2}{6EI_y}$$

$$\omega_c = -q\frac{7a^3}{24EI_y} + M\frac{4a}{3EI_y}$$

$$u_c = H\frac{a}{EA_x} = 2a\left(-q\frac{7a^3}{24EI_y} + M\frac{4a}{3EI_y}\right)$$

$$-\frac{M}{2a}\cdot\frac{a}{A_x} = -q\frac{7a^4}{12EI_y} + M\frac{8a^2}{3I_y}$$

$$\boxed{M = q\frac{7a^4 A_x}{2(3I_y + 16a^2 A_x)}} \rightarrow \boxed{M = 34996\ q}$$

$$\boxed{w_c = w_D = 2{,}167\ q}$$