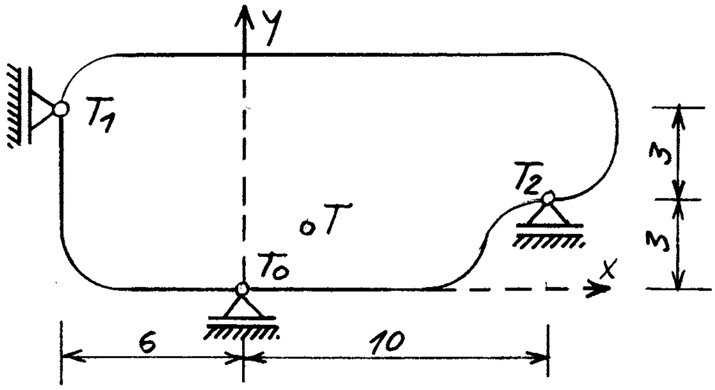
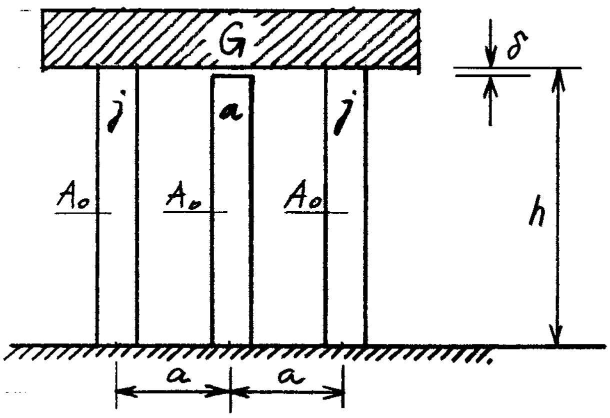
1. V prerezu telesa z ravnino $z = 0$ vlada ravninsko deformacijsko stanje $\varepsilon_{zx} = \varepsilon_{zy} = \varepsilon_{zz} = 0$; $u_z = 0$. Nenične komponente tenzorja majhnih deformacij ε_{ij} so podane v odvisnosti od obtežnega faktorja Ψ. Način podpiranja je simbolično prikazan na skici. Meja plastičnega tečenja uporabljenega materiala je σ_Y. V točki $T(2,2,0)$ določi:
   a. pomike in zasuke,
   b. velikosti glavnih normalnih napetosti,
   c. kritično vrednost obtežnega faktorja Ψ_Y, pri katerem se v točki T pojavijo plastične deformacije (uporabi Misesov kriterij za začetek plstičnega tečenja).



$$[\varepsilon_{ij}] = 10^{-4}\Psi \begin{bmatrix} 10x & y^2 & 0 \\ y^2 & 4xy & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

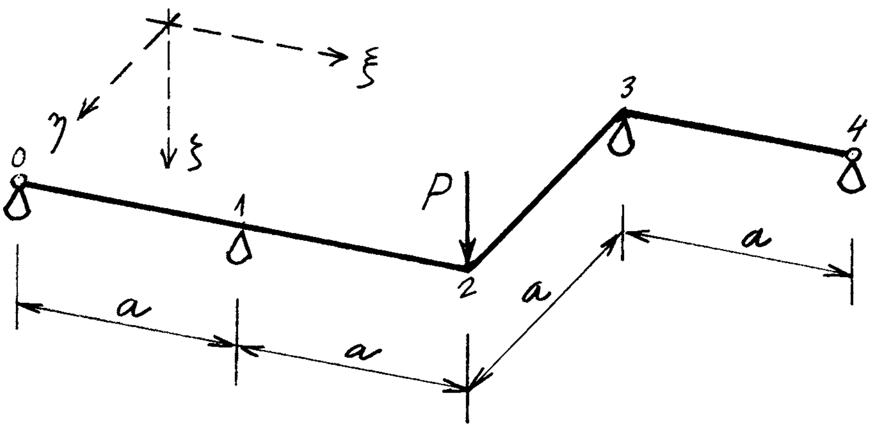
$$2\mu = 16\,000\ \mathrm{kN/cm^2}$$
$$\lambda = 8\,000\,\mathrm{kN/cm^2}$$
$$\sigma_Y = 24\ \mathrm{kN/cm^2}$$

2. Absolutno togo gredo teže G centrično položimo na tri stebre. Krajna stebra sta jeklena, srednji pa je iz aluminija in je pred obtežitvijo za $1\,mm$ krajši od krajnih. Določi napetosti v stebrih in njihovo novo dolžino po obtežitvi! Za koliko moramo spremeniti temperaturo stebrov, da bodo napetosti v vseh stebrih enake?



$$E_j = 21\,000\,\mathrm{kN/cm^2}$$
$$E_a = 7\,000\,\mathrm{kN/cm^2}$$
$$\alpha_j = 1.25 \cdot 10^{-5}/K$$
$$\alpha_a = 2.00 \cdot 10^{-5}/K$$
$$A_o = 20\,\mathrm{cm^2}$$
$$a = 1\,\mathrm{m}$$
$$h = 3\,\mathrm{m} \qquad \delta = 1\,\mathrm{mm}$$
$$G = 700\,\mathrm{kN}$$

3. Določi pomik točke 2! Zaradi lažjega računanja vzamemo $GI_x = EI_y$.



MTT IZPIT 4. 2. 1998

Ad 1.) $\boxed{\psi=1}$ $\quad \vec{\varepsilon}_z = \vec{0}$; $\quad T_0(0,0) \rightarrow u_y(T_0)=0$

a) $$\omega_z = \omega_z(T_0) + \int_0^x \vec{e}_z(\vec{\nabla}\times\vec{\varepsilon}_x)_{y=0}\, dx + \int_0^y \vec{e}_z(\vec{\nabla}\times\vec{\varepsilon}_y)\, dy$$

$$\vec{e}_z(\vec{\nabla}\times\vec{\varepsilon}_x) = \begin{vmatrix} 0 & 0 & 1 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \varepsilon_{xx} & \varepsilon_{xy} & 0 \end{vmatrix} = \frac{\partial \varepsilon_{xy}}{\partial x} - \frac{\partial \varepsilon_{xx}}{\partial y} = 0$$

$$\vec{e}_z(\vec{\nabla}\times\vec{\varepsilon}_y) = \begin{vmatrix} 0 & 0 & 1 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \end{vmatrix} = \frac{\partial \varepsilon_{yy}}{\partial x} - \frac{\partial \varepsilon_{xy}}{\partial y} = 2y\cdot 10^{-4}$$

$$\omega_z = \omega_z(T_0) + 10^{-4}\int_0^y 2y\, dy \rightarrow \boxed{\omega_z = \omega_z(T_0) + 10^{-4} y^2}$$

$$u_x = u_x(T_0) + \int_0^x \varepsilon_{xx}\Big|_{y=0} dx + \int_0^y (\varepsilon_{yx} - \omega_z)\, dy$$

$$u_x = u_x(T_0) + 10^{-4}\left[\int_0^x 10x\, dx + \int_0^y \left(y^2 - 10^4\omega_z(T_0) - y^2\right) dy\right]$$

$$\boxed{u_x = u_x(T_0) - y\,\omega_z(T_0) + 5x^2\cdot 10^{-4}}$$

$$u_y = u_y(T_0) + \int_0^x (\varepsilon_{xy} + \omega_z)_{y=0}\, dy + \int_0^y \varepsilon_{yy}\, dy$$

$$u_y = u_y(T_0) + 10^{-4}\left[\int_0^x \left(y^2 + 10^4\omega_z(T_0) + y^2\right)_{y=0} dx + \int_0^y 4xy\, dy\right]$$

$$u_y(T_0) = 0 \rightarrow \boxed{u_y = x\,\omega_z(T_0) + 2xy^2\cdot 10^{-4}}$$

$T_2(10,3,0) \rightarrow u_y(T_2)=0$

$u_y(T_2) = 10\,\omega_z(T_0) + 2\cdot 10\cdot 3^2\cdot 10^{-4} = 0$

$$\boxed{\omega_z(T_0) = -18\cdot 10^{-4}}$$

$u_x = u_x(T_0) + (18y + 5x^2)\cdot 10^{-4}$

$T_1(-6,6,0) \rightarrow u_x = 0 \rightarrow u_x(T_0) + (18\cdot 6 + 5\cdot 36)\cdot 10^{-4} = 0$

$$\boxed{u_x(T_0) = -288\cdot 10^{-4}}$$

$\psi \neq 1$:

$$\boxed{\begin{aligned} \omega_z &= (y^2-18)\cdot \psi\cdot 10^{-4} \\ u_x &= (5x^2+18y-288)\cdot\psi\cdot 10^{-4} \\ u_y &= (2xy^2-18x)\cdot\psi\cdot 10^{-4} \end{aligned}}$$

$T(2,2,0)$:

$$\boxed{\begin{aligned} \omega_z(T) &= -14\cdot\psi\cdot 10^{-4} \\ u_x(T) &= -232\cdot\psi\cdot 10^{-4} \\ u_y(T) &= -20\cdot\psi\cdot 10^{-4} \end{aligned}} \quad (cm)$$

b) $[\varepsilon_{ij}]_T = \psi\cdot 10^{-4}\begin{bmatrix} 20 & 4 & 0 \\ 4 & 16 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\qquad \varepsilon_{ij} = \psi\cdot 10^{-4}\,\bar{\varepsilon}_{ij}$

$$\bar{\varepsilon}_{11,22} = \frac{20+16}{2} \pm \sqrt{\left(\frac{20-16}{2}\right)^2 + 4^2}$$

$$\boxed{\begin{aligned} \varepsilon_{11} &= 22{,}47\ \psi\cdot 10^{-4} \\ \varepsilon_{22} &= 13{,}53\ \psi\cdot 10^{-4} \end{aligned}}$$

$$\boxed{\varepsilon_{33} = 0}$$

$$\boxed{I_1^{\varepsilon} = 36\ \psi\cdot 10^{-4}}$$

$\sigma_{11} = 2\mu\varepsilon_{11} + \lambda I_1^{\varepsilon} \rightarrow$ $\boxed{\sigma_{11} = 64{,}8\ \psi\ kN/cm^2}$

$\sigma_{22} = 2\mu\varepsilon_{22} + \lambda I_1^{\varepsilon} \rightarrow$ $\boxed{\sigma_{22} = 50{,}4\ \psi\ kN/cm^2}$

$\sigma_{33} = \lambda I_1^{\varepsilon} \rightarrow$ $\boxed{\sigma_{33} = 28{,}8\ \psi\ kN/cm^2}$

c) Mises: $(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2 = 2\sigma_Y^2$

$$\gamma_Y^2\left[(64,8-50,4)^2+(50,4-28,8)^2+(28,8-64,8)^2\right] = 2\sigma_Y^2$$

$$1966,1\;\gamma_Y^2 = 2\sigma_Y^2 \longrightarrow \gamma_Y = \sigma_Y\sqrt{\frac{2}{1966,1}}$$

$$\gamma_Y = 0,032\;\sigma_Y \longrightarrow \boxed{\gamma_Y = 0,765}$$

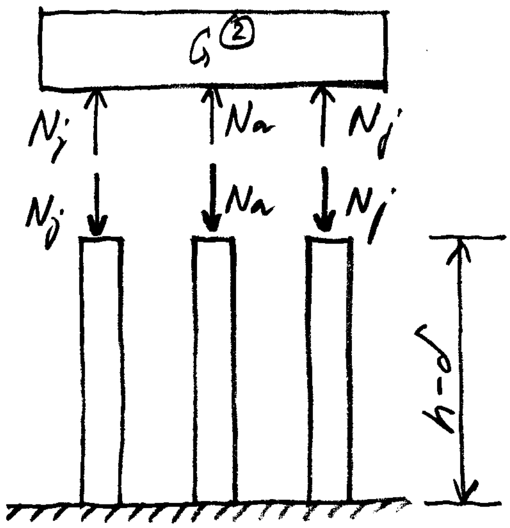
Ad 2.)

$$(1):\ \Delta l_j^{(1)} = \delta = \frac{G^{(1)}}{2}\,\frac{h}{E_j A_0} \longrightarrow G^{(1)} = \frac{2\delta E_j A_0}{h}$$

$$G^{(1)} = \frac{2\cdot 0,1\cdot 21000\cdot 20}{300} \longrightarrow \boxed{G^{(1)} = 280\ \mathrm{kN}}$$

$$G^{(2)} = G - G^{(1)} \longrightarrow \boxed{G^{(2)} = 420\ \mathrm{kN}} \qquad \boxed{\begin{aligned} N_j^{(1)} &= 140\ \mathrm{kN} \\ N_a^{(1)} &= 0 \end{aligned}}$$

(2)



$$\boxed{G^{(2)} = 2N_j^{(2)} + N_a^{(2)}}$$

$$\Delta l_j^{(2)} = \Delta l_a^{(2)}$$

$$N_j^{(2)}\,\frac{h-\delta}{E_j A_0} = N_a^{(2)}\,\frac{h-\delta}{E_a A_0}$$

$$\boxed{N_a^{(2)} = N_j^{(2)}\,\frac{E_a}{E_j}}$$

$$N_j^{(2)}\left(2+\frac{E_a}{E_j}\right) = G^{(2)} \longrightarrow \boxed{N_j^{(2)} = G^{(2)}\,\frac{E_j}{2E_j+E_a}}$$

$$N_j^{(2)} = 420\,\frac{21000}{2\cdot 21000+7000} \longrightarrow \boxed{N_j^{(2)} = 180\ \mathrm{kN}}$$

$$N_a^{(2)} = G^{(2)} - 2N_j^{(2)} \longrightarrow \boxed{N_a^{(2)} = 60\ \mathrm{kN}}$$

$$N_j = N_j^{①} + N_j^{②} \quad \rightarrow \quad \boxed{N_j = 320 \text{ kN}}$$
$$N_a = N_a^{②} \quad \rightarrow \quad \boxed{N_a = 60 \text{ kN}}$$

$$\sigma_j = -\frac{N_j}{A_0} \quad \rightarrow \quad \boxed{\sigma_j = -16 \text{ kN/cm}^2}$$
$$\sigma_a = -\frac{N_a}{A_0} \quad \rightarrow \quad \boxed{\sigma_a = -3 \text{ kN/cm}^2}$$

$$\Delta l_j^{②} = \Delta l_a^{②} = N_j^{②} \cdot \frac{h-\delta}{E_j A_0} = 180 \; \frac{300-0{,}1}{21000 \cdot 20} = 0{,}1285 \text{ cm}$$

$$h' = h - \delta - \Delta l^{②} = 300 - 0{,}1 - 0{,}1285$$
$$\boxed{h' = 299{,}77 \text{ cm}}$$

$$\sigma_j = \sigma_a \quad \rightarrow \quad N_j = N_a = \frac{G}{3}$$

$$\Delta l_j = \Delta l_a + \delta$$

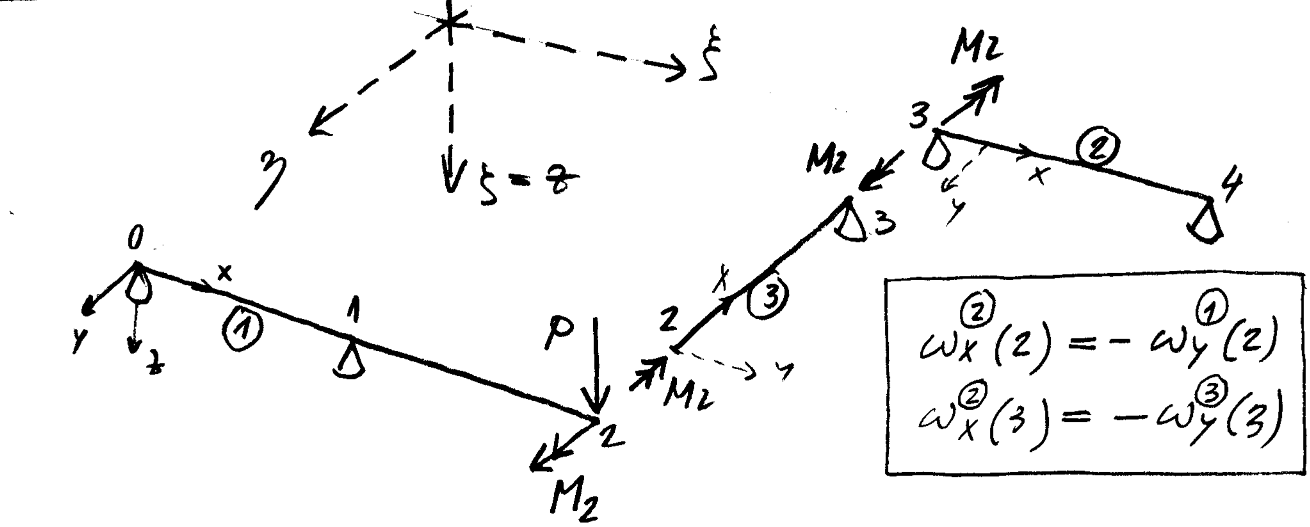
$$\left(-\frac{G}{3E_j A_0} + \alpha_j \Delta T\right) h = \left(-\frac{G}{3E_a A_0} + \alpha_a \Delta T\right)(h-\delta) + \delta$$

$$\Delta T \left[\alpha_a (h-\delta) - \alpha_j h\right] = \frac{G}{3A_0}\left[\frac{h-\delta}{E_a} - \frac{h}{E_j}\right] - \delta$$

$$\boxed{\Delta T = 103{,}7 \text{ K}}$$

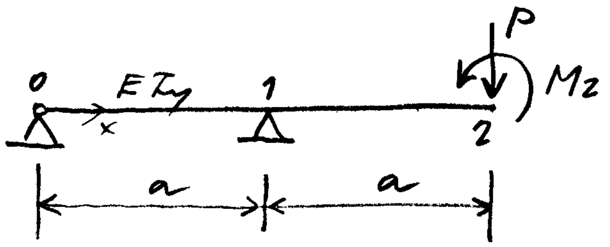
Kontrola:
$$\Delta l_j = \left(-\frac{G}{3E_j A_0} + \alpha_j \Delta T\right) \cdot h = 0{,}2223 \text{ cm}$$
$$\Delta l_a = \left(-\frac{G}{3E_a A_0} + \alpha_a \Delta T\right)(h-\delta) = 0{,}1223 \text{ cm}$$
$$\Delta l_j = \Delta l_a + \delta \quad ✓$$

Ad 3.)



$$\boxed{\begin{aligned}\omega_x^{②}(2) &= -\omega_y^{①}(2)\\ \omega_x^{②}(3) &= -\omega_y^{③}(3)\end{aligned}}$$

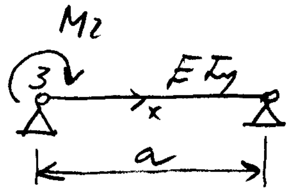
① :



$$\omega_y^{①}(2) = -P\frac{5a^2}{6EI_y} + M_z\frac{4a}{3EI_y}$$

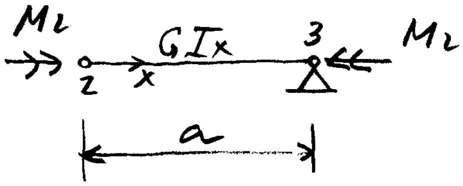
$$w(2) = P\frac{2a^3}{3EI_y} - M_z\frac{5a^2}{6EI_y}$$

③ :



$$\omega^{③}(3) = -M_z\frac{a}{3EI_y}$$

② :



$$M_x = -M_z = GI_x\frac{d\omega_x}{dx}$$

$$\omega_x = -\frac{M_z}{GI_x}x + C_1$$

$$x=0 \rightarrow \omega_x^{②}(2) = -\omega_y^{①}(2) \rightarrow \boxed{C_1 = P\frac{5a^2}{6EI_y} - M_z\frac{4a}{3EI_y}}$$

$$\omega_x^{②} = -M_z\left(\frac{x}{GI_x} + \frac{4a}{3EI_y}\right) + P\frac{5a^2}{6EI_y}$$

$$x=a \rightarrow \omega_x^{②}(3) = -\omega_y^{③}(3) = M_z\frac{a}{3EI_y}$$

$$-M_2\left(\frac{a}{GI_x}+\frac{4a}{3EI_y}\right)+P\,\frac{5a^2}{6EI_y}=M_2\,\frac{a}{3EI_y}$$

$$GI_x = EI_y \longrightarrow \boxed{M_2 = P\,\frac{5a}{16}}$$

$$w_2 = P\,\frac{2a^3}{3EI_y} - P\,\frac{5a}{16}\cdot\frac{5a^2}{6EI_y} \rightarrow \boxed{w_2 = P\,\frac{13a^3}{32EI_y}}$$