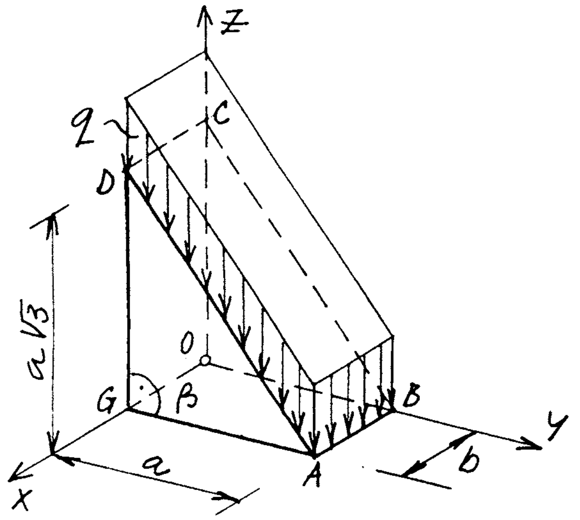
1. Poševna mejna ploskev $ABCD$ majhne homogene trikotne prizme je obtežena z navpično enakomerno površinsko obtežbo q, kot kaže skica. Rezultanta enakomerne površinske obtežbe mejne ploskve $OGAB$ je $\vec{P} = P_y\vec{e}_y + P_z\vec{e}_z$. Komponento P_z poznamo: $P_z = 30\,000\,\mathrm{N}$. Mejni ploskvi OBC in GAD nista obteženi.



a. Pri katerih vrednostih zunanje obtežbe q je normalna napetost v mejni ploskvi $OGDC$ i) natezna, ii) enaka nič, iii) tlačna ? Določi normalno in strižno napetost v mejni ploskvi $OGDC$ pri $q = 170\,\mathrm{MPa}$! Določi komponento P_y in kontroliraj ravnotežje prizme kot celote!

b. Določi dolžine stranic $\overline{OG}$, $\overline{OB}$, in $\overline{OC}$ po deformaciji prizme ter spremembo pravega kota β pri $q = 170\,\mathrm{MPa}$!

$E = 200\,000\,\mathrm{MPa}$ $\quad \nu = 0.3$

$a = 30\,\mathrm{mm}$ $\quad b = 10\,\mathrm{mm}$

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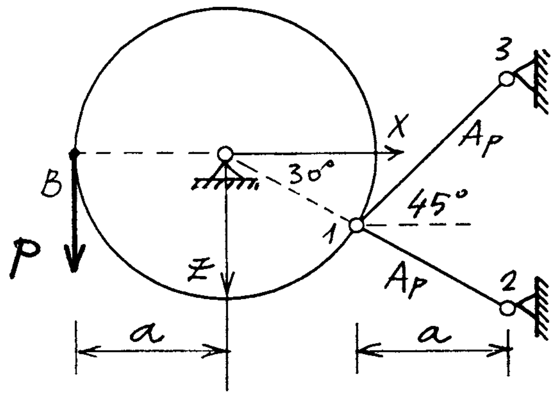
2. Absolutno toga homogena in enakomerno debela krožna plošča je v točki B obtežena z navpično točkovno silo P. Plošča je v središču nepomično vrtljivo podprta, v točki 1 pa je členkasto pritrjena na palici $\overline{12}$ in $\overline{13}$. Določi zasuk plošče okrog središča in osno silo v palici $\overline{13}$! Rezultate izrazi v odvisnosti od sile P!



$E = 20\,000\,\mathrm{kN/cm^2}$

$a = 120\,\mathrm{cm}$

$A_p = 4\,\mathrm{cm^2}$

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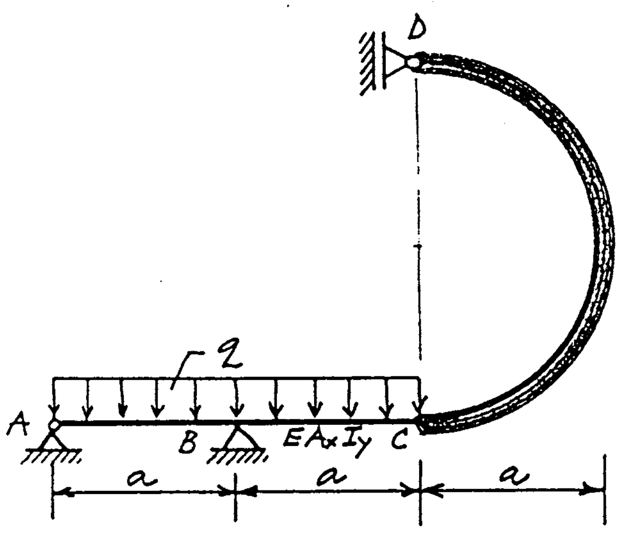
3. Fasadni element $\overline{CD}$ je zelo tog v primerjavi z nosilcem $\overline{AC}$. V točki C sta oba dela konstrukcije togo povezana.

Določi navpični pomik točke D v odvisnosti od velikosti zvezne obtežbe q!



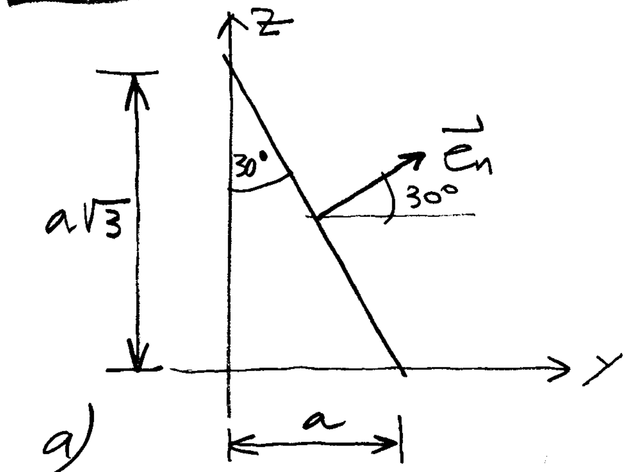
$E = 200\,000\,\mathrm{MPa}$

$A_x = 40\,\mathrm{cm^2}$

$I_y = 4000\,\mathrm{cm^4}$

$a = 4\,\mathrm{m}$

Ad 1.)



$$\vec{e}_n = \frac{\sqrt{3}}{2}\vec{e}_y + \frac{1}{2}\vec{e}_z$$

$$\vec{p}_n = -q\,\vec{e}_z$$

$$\boxed{\sigma_{zz} = -\frac{P}{ab}}$$

$$\sigma_{zz} = -\frac{30000}{30\cdot 10} = \underline{\underline{-100\ \text{MPa}}}$$

a)

$$\sigma_{xx} = 0$$

$$\sigma_{xy} = \sigma_{xz} = 0$$

$$p_{ny} = \sigma_{yy}\, e_{ny} + \sigma_{yz}\, e_{nz} = 0$$

$$p_{nz} = \sigma_{yz}\, e_{ny} + \sigma_{zz}\, e_{nz} = -q$$

$$\frac{\sqrt{3}}{2}\sigma_{yy} + \frac{1}{2}\sigma_{yz} = 0 \longrightarrow \sigma_{yy} = -\frac{\sigma_{yz}}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{2}\sigma_{yz} + \frac{1}{2}\sigma_{zz} = -q$$

$$\sqrt{3}\,\sigma_{yz} - \frac{P}{ab} = -2q \rightarrow \boxed{\sigma_{yz} = \frac{1}{\sqrt{3}}\left(\frac{P}{ab} - 2q\right)}$$

$$\sigma_{yy} = -\frac{\sigma_{yz}}{\sqrt{3}} \rightarrow \boxed{\sigma_{yy} = \frac{1}{3}\left(2q - \frac{P}{ab}\right)}$$

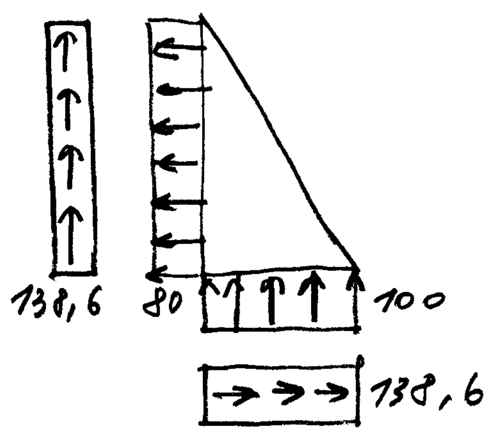
i) $\sigma_{yy} > 0$ für $2q > \frac{P}{ab} \longrightarrow \boxed{q > \frac{P}{2ab}}$

ii) $\sigma_{yy} = 0$ für $\boxed{q = \frac{P}{2ab}}$ $(q = 50\ \text{MPa})$

iii) $\sigma_{yy} < 0$ für $\boxed{q < \frac{P}{2ab}}$

$q = 170$ MPa :

$$\boxed{\begin{aligned} \sigma_{yy} &= 80\ \text{MPa} \\ \sigma_{yz} &= -138{,}6\ \text{MPa} \\ \sigma_{zz} &= -100\ \text{MPa} \end{aligned}}$$



$\Sigma Y = 0$:

$$30\cdot 10\cdot 138{,}6 - 30\cdot\sqrt{3}\cdot 10\cdot 80 = 0 \ \checkmark$$

$\sum Z = 0$! $\quad 30 \cdot \sqrt{3} \cdot 10 \cdot 138{,}6 + 30.000 - 170 \cdot 2 \cdot 30 \cdot 10 = 0$ ✓

b) $I_1^{\sigma} = \sigma_{yy} + \sigma_{zz} = -20$ MPa

$$\frac{1+\nu}{E} = \frac{1{,}3}{200000} = 6{,}5 \cdot 10^{-6}$$

$$\frac{\nu}{E} = \frac{0{,}3}{200000} = 1{,}5 \cdot 10^{-6}$$

$$\varepsilon_{xx} = -\frac{\nu}{E} I_1^{\sigma} = 30 \cdot 10^{-6} \quad , \quad \varepsilon_{xy} = \varepsilon_{xz} = 0$$

$$\varepsilon_{yy} = \frac{1+\nu}{E}\sigma_{yy} - \frac{\nu}{E} I_1^{\sigma} \rightarrow \varepsilon_{yy} = 550 \cdot 10^{-6}$$

$$\varepsilon_{zz} = \frac{1+\nu}{E}\sigma_{zz} - \frac{\nu}{E} I_1^{\sigma} \rightarrow \varepsilon_{zz} = -620 \cdot 10^{-6}$$

$$\varepsilon_{yz} = \frac{1+\nu}{E}\sigma_{yz} \rightarrow \varepsilon_{yz} = -900 \cdot 10^{-6}$$

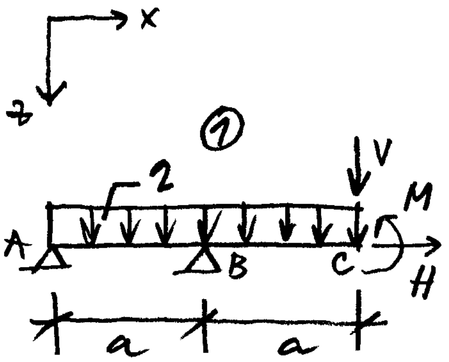
$$\overline{OG}' = \overline{OG}(1+\varepsilon_{xx}) \rightarrow \overline{OG}' = 10{,}0003 \text{ mm}$$

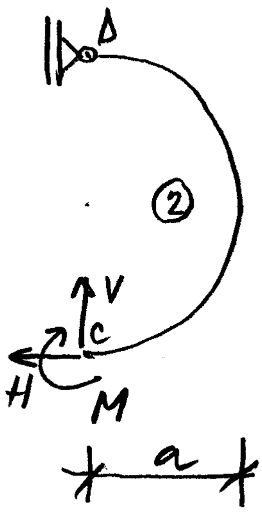
$$\overline{OB}' = \overline{OB}(1+\varepsilon_{yy}) \rightarrow \overline{OB}' = 30{,}0165 \text{ mm}$$

$$\overline{OC}' = \overline{OC}(1+\varepsilon_{zz}) \rightarrow \overline{OC}' = 51{,}9293 \text{ mm}$$

c) $\Delta\varphi = D_{yz} \cong 2\varepsilon_{yz} = -0{,}0018$

Ad 3.)





② : $\boxed{V = 0}$

$$H \cdot 2a + M = 0$$

$$\boxed{H = -M \frac{1}{2a}}$$

$$\vec{r}_D = -2a\,\vec{e}_z$$

$$\vec{u}_C = u_C \vec{e}_x + w_C \vec{e}_z$$

$$\vec{\omega}_C = \omega_C \vec{e}_y$$

$$\vec{u}_D = \vec{u}_C + \vec{\omega}_C \times \vec{r}_D = u_C \vec{e}_x + w_C \vec{e}_z - 2a\,\omega_C \vec{e}_x$$

$$\vec{u}_D = (u_C - 2a\,\omega_C)\vec{e}_x + w_C \vec{e}_z \longrightarrow \boxed{w_D = w_C}$$

$$u_x(D) = u_D = 0 \longrightarrow \boxed{u_C = 2a\,\omega_C}$$

① $$w_C = \frac{q a^4}{4EI_y} - M \frac{5a^2}{6EI_y}$$

$$\omega_C = -q \frac{7a^3}{24EI_y} + M \frac{4a}{3EI_y}$$

$$u_C = H \frac{a}{EA_x} = 2a\left(-q \frac{7a^3}{24EI_y} + M \frac{4a}{3EI_y}\right)$$

$$-\frac{M}{2a} \cdot \frac{a}{A_x} = -q \frac{7a^4}{12EI_y} + M \frac{8a^2}{3I_y}$$

$$\boxed{M = q \frac{7a^4 A_x}{2(3I_y + 16a^2 A_x)}} \rightarrow \boxed{M = 34996\ q}$$

$$\boxed{w_C = w_D = 2{,}167\ q}$$