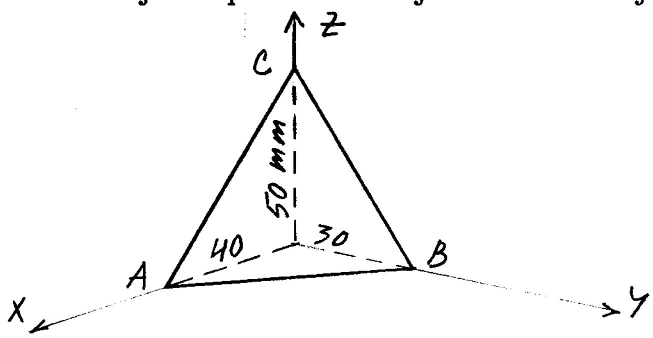
1. Na koordinatno ploskev $y = 0$ prikazane elementarne piramide deluje enakomerna površinska obtežba $\vec{q} = -(80\,\vec{e}_x + 50\,\vec{e}_y + 10\,\vec{e}_z)$. Normalna napetost v koordinatni ravnini $x = 0$ je natezna in znaša 100 MPa. Izmerjeni sta tudi specifična sprememba dolžine stranice $\overline{AC}$, $D_{\overline{AC}} = 272.4 \cdot 10^{-6}$ in celotna sprememba prostornine piramide $\Delta V = 1.90476\,\mathrm{mm}^3$. Določi vse komponente tenzorjev napetosti in majhnih deformacij!

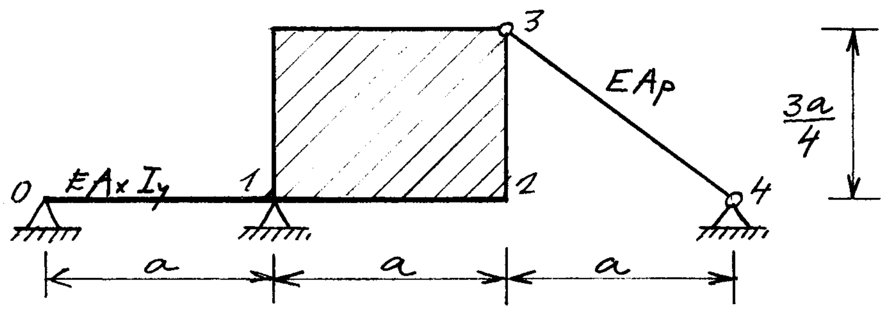


$E = 210\,000\,\mathrm{MPa}$

$\nu = 0.3$

2. Absolutno toga, homogena in enakomerno debela stena skupne teže G je togo pritrjena na nosilec $\overline{02}$, v točki 3 pa je še podprta s palico $\overline{34}$.
   a. določi in skiciraj potek notranjih sil v nosilcu,
   b. določi vektor pomika točke 3!

   Rezultate izrazi v odvisnosti od teže stene G!



$E = 20\,000\,\mathrm{kN/cm^2}$

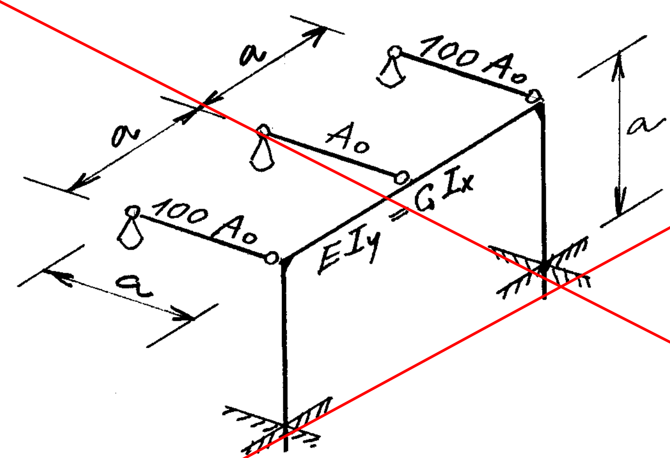
$a = 200\,\mathrm{cm}$

$A_x = 200\,\mathrm{cm}^2$

$I_y = 2800\,\mathrm{cm}^4$

$A_p = 4\,\mathrm{cm}^2$

3. Ravninski okvir je v vodoravni ravnini zgornje prečke podprt s tremi palicami. Določi osne sile, ki nastopijo v palicah, če samo srednjo od njih segrejemo za 40 K!



$E = 20\,000\,\mathrm{kN/cm^2}$

$A_0 = 20\,\mathrm{cm}^2$

$EI_y = GI_x = 4 \cdot 10^7\,\mathrm{cm}^4$

$a = 2\,\mathrm{m}$

$\alpha_T = 1{,}25 \cdot 10^{-5}/K$

Ad 1) plocha $y=0$; $\vec{e_n} = -\vec{e_y}$

$e_{nx} = e_{nz} = 0$, $e_{ny} = -1$

$$\vec{q} = \vec{p_n} = -80\,\vec{e_x} - 50\,\vec{e_y} - 10\,\vec{e_z}$$

$$p_{nx} = \sigma_{xx} e_{nx} + \sigma_{yx} e_{ny} + \sigma_{zx} e_{nz} = -\sigma_{yx} = -80$$

$$\boxed{\sigma_{xy} = 80 \text{ MPa}}$$

$$p_{ny} = \sigma_{xy} e_{nx} + \sigma_{yy} e_{ny} + \sigma_{zy} e_{nz} = -\sigma_{yy} = -50$$

$$\boxed{\sigma_{yy} = 50 \text{ MPa}}$$

$$p_{nz} = \sigma_{xz} e_{nx} + \sigma_{yz} e_{ny} + \sigma_{zz} e_{nz} = -\sigma_{yz} = -10$$

$$\boxed{\sigma_{yz} = 10 \text{ MPa}}$$

$$[\sigma_{ij}] = \begin{bmatrix} 100 & 80 & \cdot \\ 80 & 50 & 10 \\ \cdot & 10 & \cdot \end{bmatrix} (\text{MPa}) \qquad \boxed{\sigma_{xx} = 100 \text{ MPa}}$$

$$\frac{1+\nu}{E} = 6{,}19 \cdot 10^{-6} \quad ; \quad \frac{\nu}{E} = 1{,}43 \cdot 10^{-6}$$

$$I_1^{\varepsilon} = \frac{1-2\nu}{E} I_1^{\sigma} \quad ; \quad V = \frac{1}{2} \cdot 30 \cdot 40 \cdot \frac{50}{3} = 10.000 \text{ mm}^3$$

$$D_V \cong \varepsilon_V = I_1^{\varepsilon} = \frac{\Delta V}{V} = \frac{1.90476}{10000} \rightarrow \boxed{I_1^{\varepsilon} = 190{,}476 \cdot 10^{-6}}$$

$$I_1^{\sigma} = \frac{E}{1-2\nu} I_1^{\varepsilon} \rightarrow \boxed{I_1^{\sigma} = 100 \text{ MPa}}$$

$$\sigma_{zz} = I_1^{\sigma} - \sigma_{xx} - \sigma_{yy} \rightarrow \boxed{\sigma_{zz} = -50 \text{ MPa}}$$

$$\varepsilon_{xx} = \frac{1+\nu}{E}\sigma_{xx} - \frac{\nu}{E} I_1^{\sigma} \rightarrow \varepsilon_{xx} = 476{,}2 \cdot 10^{-6}$$

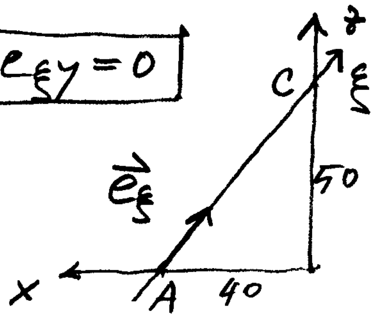
$$\varepsilon_{yy} = \frac{1+\nu}{E}\sigma_{yy} - \frac{\nu}{E} I_1^{\sigma} \rightarrow \varepsilon_{yy} = 166{,}7 \cdot 10^{-6}$$

$$\varepsilon_{zz} = \frac{1+\nu}{E}\sigma_{zz} - \frac{\nu}{E} I_1^{\sigma} \rightarrow \varepsilon_{zz} = -452{,}4 \cdot 10^{-6}$$

$$\varepsilon_{xy} = \frac{1+\nu}{E}\sigma_{xy} \rightarrow \varepsilon_{xy} = 495{,}2 \cdot 10^{-6}$$

$$\varepsilon_{yz} = \frac{1+\nu}{E}\sigma_{yz} \rightarrow \varepsilon_{yz} = 61{,}9 \cdot 10^{-6}$$

$\boxed{e_{\xi y} = 0}$



$\overline{AC} = \sqrt{40^2 + 50^2} = 64{,}03$

$e_{\xi x} = -\frac{40}{64{,}03} \rightarrow e_{\xi x} = -0{,}625$

$e_{\xi z} = \frac{50}{64{,}03} \rightarrow e_{\xi z} = 0{,}781$

$$D_{\overline{AC}} = \varepsilon_{\xi\xi} = \varepsilon_{xx}\, e_{\xi x}^2 + 2\,\varepsilon_{xz}\, e_{\xi x}\, e_{\xi z} + \varepsilon_{zz}\, e_{\xi z}^2$$

$$272{,}3577 \cdot 10^{-6} = 10^{-6}\left(476{,}2 \cdot 0{,}625^2 - 452{,}4 \cdot 0{,}781^2\right) - 2 \cdot 0{,}625 \cdot 0{,}781 \cdot \varepsilon_{xz}$$

$$\boxed{\varepsilon_{xz} = -371{,}4 \cdot 10^{-6}}$$

$\sigma_{xz} = \frac{E}{1+\nu}\,\varepsilon_{xz} \longrightarrow \boxed{\sigma_{xz} = -60 \text{ MPa}}$

$[\sigma_{ij}] =$

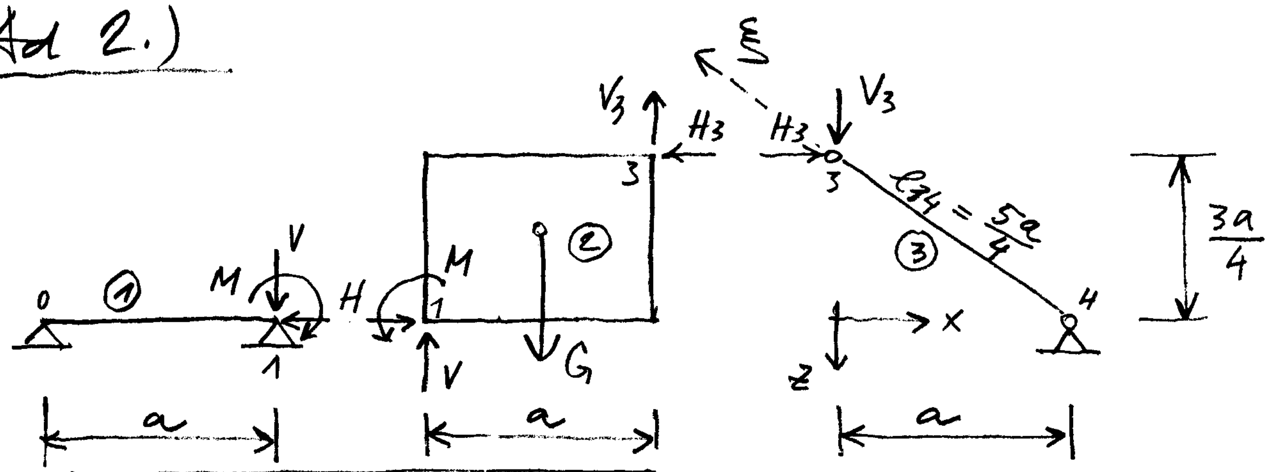
100	80	-60
80	50	10
-60	10	-50

MPa ;

$[\varepsilon_{ij}] = 10^{-6}$

476,2	495,2	-371,4
495,2	166,7	61,9
-371,4	61,9	-452,4

Ad 2.)



① $\boxed{\omega_y^{①}(1) = -M\frac{a}{3EI_y} = \omega_1}$

② $\sum M^1 = M - G\frac{a}{2} + V_3\, a + H_3\, \frac{3a}{4} = 0$

$$\boxed{M = G\,\frac{a}{2} - V_3\, a - H_3\,\frac{3a}{4}}$$

$\vec{u}_1 = \vec{0}$, $\vec{\omega}_1 = \omega_1 \vec{e}_y$, $\vec{r}_3 = a\vec{e}_x - \frac{3a}{4}\vec{e}_z$

$$\vec{u}_3 = \vec{u}_1 + \vec{\omega}_1 \times \vec{r}_3 = \omega_1 \vec{e}_y \times \left(a\vec{e}_x - \frac{3a}{4}\vec{e}_z\right)$$

$$\boxed{\vec{u}_3 = -\frac{3a}{4}\omega_1 \vec{e}_x - a\omega_1 \vec{e}_z}$$

$$u_x^3 = +\frac{3a}{4}\cdot\frac{a}{3EI_y}\left(G\frac{a}{2} - V_3 a - H_3\frac{3a}{4}\right)$$

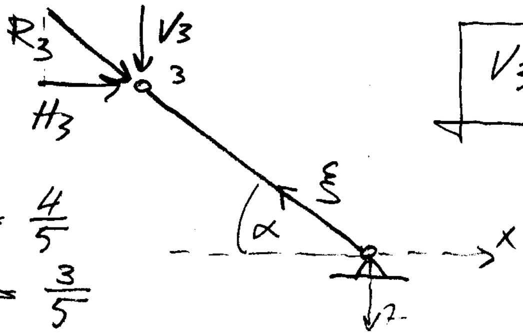
$$u_z^3 = a\frac{a}{3EI_y}\left(G\frac{a}{2} - V_3 a - H_3\frac{3a}{4}\right)$$

$$\boxed{u_x^3 = G\frac{a^3}{8EI_y} - V_3\frac{a^3}{4EI_y} - H_3\frac{3a^3}{16}}$$

$$\boxed{u_z^3 = G\frac{a^3}{6EI_y} - V_3\frac{a^3}{3EI_y} - H_3\frac{a^3}{4EI_y}}$$

(3) : $\sum M^4 = V_3 a - H_3\frac{3a}{4} = 0 \rightarrow \boxed{V_3 = \frac{3}{4}H_3}$

$$\boxed{V_3 = \frac{3}{5}R} \quad \boxed{H_3 = \frac{4}{5}R}$$



$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

$e_{\xi x} = -\frac{4}{5}$

$e_{\xi z} = -\frac{3}{5}$

$$\boxed{\begin{aligned} u_x^3 &= G\frac{a^3}{8EI_y} - R\frac{3a^3}{10EI_y} \\ u_z^3 &= G\frac{a^3}{6EI_y} - R\frac{2a^3}{5EI_y} \end{aligned}}$$

$$u_\xi^{(3)}(3) = u_\xi = -R\frac{5a}{4EA_p}$$

$$u_\xi = u_x e_{\xi x} + u_z e_{\xi z}$$

$$-R\frac{5a}{4EA_p} = \left(G\frac{a^3}{8EI_y} - R\frac{3a^3}{10EI_y}\right)\cdot\left(-\frac{4}{5}\right) +$$

$$+\left(G\frac{a^3}{6EI_y} - R\frac{2a^3}{5EI_y}\right)\cdot\left(-\frac{3}{5}\right)$$

$$R\cdot\left(\frac{5}{4A_p} + \frac{12a^2}{25I_y}\right) = G\cdot\frac{a^2}{5I_y} \rightarrow \boxed{R = \frac{20a^2 A_p\cdot G}{125I_y + 48a^2 A_p}}$$

$$R = \frac{20 \cdot 200^2 \cdot 4}{125 \cdot 2800 + 48 \cdot 200^2 \cdot 4}\, G \rightarrow \boxed{R = 0{,}4\, G}$$