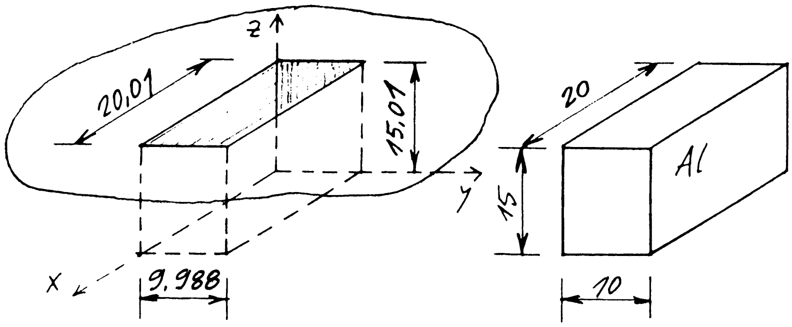
1. V absolutno togi podlagi je narejena pravokotna prizmatična luknja dimenzij $20.01 \times 9.988 \times 15.01\,\text{cm}$. V luknjo želimo vstaviti pravokoten aluminijast kvader dimenzij $20 \times 10 \times 15\,\text{cm}$.
   a. Za koliko moramo spremeniti temperaturo kvadra, da ga lahko vstavimo v odprtino? Kolikšne so tedaj dimenzije kvadra?
   b. Določi napetosti v kvadru in njegove dimenzije, ko se njegova temperatura spet izenači s temperaturo okolice!
   c. Za koliko moramo sedaj spremeniti temperaturo kvadra, da v tlorisnem pogledu popolnoma zapolni odprtino? Kolikšne so tedaj napetosti v kvadru in njegova višina?



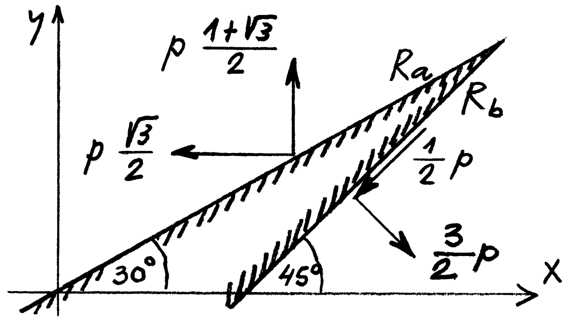
$E = 7200\,\text{kN/cm}^2$

$\nu = 0.34$

$\alpha_T = 2.4 \cdot 10^{-5}/\text{K}$

(35 točk)

2. Na skici so prikazane intenzitete enakomerne zvezne obtežbe na robovih R_a in R_b tanke stene, v kateri vlada homogeno ravninsko napetostno stanje ($\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$).
   a. Določi napetosti v koordinatnem sistemu (x, y)!
   b. Določi specifično spremembo dolžine robu R_a!
   c. Določi velikosti in smeri glavnih linearnih deformacij!



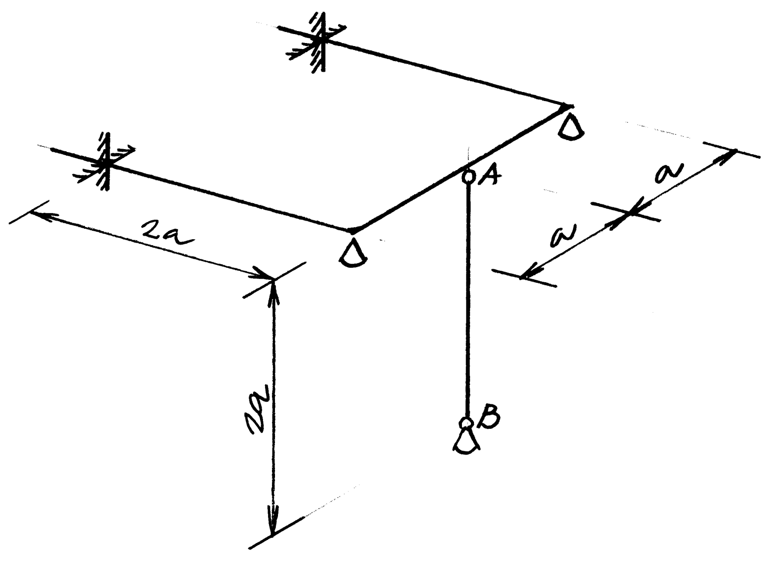
$E = 2.1 \cdot 10^5\,MPa$

$\nu = 0.3$

$p = 100\,MPa$

(40 točk)

3. Določi osno silo, ki nastopi v palici $\overline{AB}$, če celotno konstrukcijo enakomerno segrejemo za ΔT!



$EI_y = GI_x = 8\,\text{MNm}^2$

$I_y = 4000\,\text{cm}^4$

$A_p = 16\,\text{cm}^2$

$a = 2\,\text{m}$

$\alpha_T = 1.2 \cdot 10^{-5}/\text{K}$

$\Delta T = 71.6\,\text{K}$

(35 točk)

Ad 1.)

a) $\Delta l_y = 9{,}988 - 10 = -0{,}012\ cm = \varepsilon_{yy} \cdot l_y = \alpha_T \Delta T_a l_y$

$$\Delta T_a = \frac{\Delta l_y}{\alpha_T l_y} = \frac{-0{,}012}{2{,}4 \cdot 10^{-5} \cdot 10} \rightarrow \boxed{\Delta T_a = -50\ K}$$

$$\varepsilon_{xx} = \varepsilon_{zz} = \alpha_T \Delta T = -50 \cdot 2{,}4 \cdot 10^{-5} = -0{,}0012$$

$$l_x^a = l_x(1+\varepsilon_{xx}) = 20(1-0{,}0012) \rightarrow l_x^a = 19{,}976\ cm$$
$$l_z^a = l_z(1+\varepsilon_{zz}) = 15(1-0{,}0012) \rightarrow l_z^a = 14{,}982\ cm$$

b) $\sigma_{xx} = \sigma_{zz} = 0$, $\Delta T_b = +50\ K$

$$\varepsilon_{yy} = \frac{1}{E}[\sigma_{yy} - \nu(\sigma_{xx}+\sigma_{zz})] + \alpha_T \Delta T_b = 0$$

$$\sigma_{yy} = -E\alpha_T \Delta T = -7200 \cdot 1{,}25 \cdot 10^{-5} \cdot 50$$
$$\boxed{\sigma_{yy}^b = -8{,}64\ kN/cm^2}$$

$$\varepsilon_{xx} = \varepsilon_{zz} = \frac{1}{E}[\sigma_{xx} - \nu(\sigma_{yy}+\sigma_{zz})] + \alpha_T \Delta T$$

$$\varepsilon_{xx} = \varepsilon_{zz} = (1+\nu)\alpha_T \Delta T_b = 1{,}34 \cdot 2{,}4 \cdot 10^{-5} \cdot 50$$
$$\boxed{\varepsilon_{xx} = \varepsilon_{zz} = 0{,}00161}$$

$$l_x^b = 19{,}976 \cdot (1+0{,}00161) \rightarrow l_x^b = 20{,}008\ cm$$
$$l_z^b = 14{,}982 \cdot (1+0{,}00161) \rightarrow l_z^b = 15{,}006\ cm$$

c) $\sigma_{xx} = \sigma_{zz} = 0$

$$\varepsilon_{xx} = \frac{20{,}01 - 19{,}976}{19{,}976} = 0{,}0017$$

$$\varepsilon_{yy} = \frac{\sigma_{yy}^c}{E} + \alpha_T \Delta T_c = 0 \rightarrow \sigma_{yy} = -E\alpha_T \Delta T_c$$

$$\varepsilon_{xx} = \frac{1}{E}[\sigma_{xx} - \nu(\sigma_{yy}+\sigma_{zz})] + \alpha_T \Delta T$$

$$\varepsilon_{xx} = (1+\nu)\alpha_T \Delta T_c = 0{,}0017$$

$$\Delta T_c = \frac{0{,}0017}{1{,}34 \cdot 2{,}4 \cdot 10^{-5}} \rightarrow \Delta T_c = 52{,}9\ K$$

$$\boxed{\Delta\Delta T_c = 52{,}9 - 50 = 2{,}9\ K}$$

$$\sigma_{yy} = -7200 \cdot 2{,}4 \cdot 10^{-5} \cdot 52{,}9$$

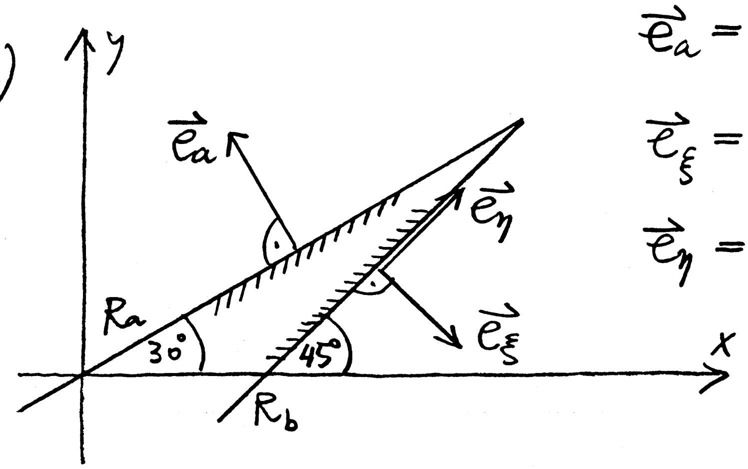
$$\boxed{\sigma_{yy} = -9{,}14\ kN/cm^2}$$

$$\varepsilon_{zz} = \varepsilon_{xx} = 0{,}0017$$

$$l_z^c = 14{,}982 \cdot (1+0{,}0017) \rightarrow \boxed{l_z^c = 15{,}007\ cm}$$

Ad 2.)

a)



$$\vec{e}_a = -\frac{1}{2}\vec{e}_x + \frac{\sqrt{3}}{2}\vec{e}_y$$

$$\vec{e}_\xi = \frac{\sqrt{2}}{2}\vec{e}_x - \frac{\sqrt{2}}{2}\vec{e}_y$$

$$\vec{e}_\eta = \frac{\sqrt{2}}{2}\vec{e}_x + \frac{\sqrt{2}}{2}\vec{e}_y$$

$$\sigma_{\xi\xi} = \frac{3}{2}p$$

$$\sigma_{\xi\eta} = -\frac{1}{2}p$$

$$R_a: \quad \vec{p}_a = -\frac{\sqrt{3}}{2}p\,\vec{e}_x + \frac{1+\sqrt{3}}{2}p\,\vec{e}_y$$

$$R_b: \quad \vec{\sigma}_\xi = \frac{3}{2}p\,\vec{e}_\xi + \frac{1}{2}p\,\vec{e}_\eta$$

R_a:

$$-\frac{\sqrt{3}}{2}p = \sigma_{xx}e_{ax} + \sigma_{xy}e_{ay} = -\frac{1}{2}\sigma_{xx} + \frac{\sqrt{3}}{2}\sigma_{xy}$$

$$\frac{1+\sqrt{3}}{2}p = \sigma_{xy}e_{ax} + \sigma_{yy}e_{ay} = -\frac{1}{2}\sigma_{xy} + \frac{\sqrt{3}}{2}\sigma_{yy}$$

R_b:
$$\sigma_{\xi\xi} = \sigma_{xx}\, e_{\xi x}^2 + 2\sigma_{xy}\, e_{\xi x} e_{\xi y} + \sigma_{yy}\, e_{\xi y}^2$$

$$\sigma_{\xi\eta} = \sigma_{xx}\, e_{\xi x} e_{\eta x} + \sigma_{xy}(e_{\xi x} e_{\eta y} + e_{\xi y} e_{\eta x}) + \\ + \sigma_{yy}\, e_{\xi y} e_{\eta y}$$

R_a:
$$\sigma_{xx} - \sqrt{3}\,\sigma_{xy} = \sqrt{3}\,p \rightarrow \sigma_{xx} = \sqrt{3}\,p + \sqrt{3}\,\sigma_{xy}$$
$$\sqrt{3}\,\sigma_{yy} - \sigma_{xy} = (1+\sqrt{3})\,p \rightarrow \sigma_{yy} = \frac{1}{\sqrt{3}}(1+\sqrt{3})\,p + \frac{1}{\sqrt{3}}\sigma_{xy}$$

R_b:
$$\frac{3}{2}p = \sigma_{xx}\cdot\frac{1}{2} - 2\sigma_{xy}\cdot\frac{1}{2} + \sigma_{yy}\cdot\frac{1}{2} \quad (= \sigma_{\xi\xi})$$

$$\sigma_{xx} - 2\sigma_{xy} + \sigma_{yy} = 3p$$

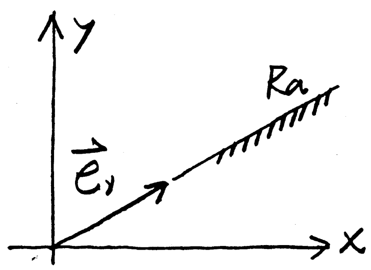
$$\sqrt{3}\,p + \sqrt{3}\,\sigma_{xy} - 2\sigma_{xy} + \frac{1}{\sqrt{3}}(1+\sqrt{3})\,p + \frac{1}{\sqrt{3}}\sigma_{xy} = 3p \,\Big|\cdot\sqrt{3}$$

$$\boxed{\sigma_{xy} = -p} \longrightarrow \boxed{\sigma_{xx} = 0}\ ; \quad \boxed{\sigma_{yy} = p}$$

Kontrola: (R_b)

$$\sigma_{\xi\eta} = \sigma_{xx}\cdot\frac{1}{2} + \sigma_{xy}\left(\frac{1}{2} - \frac{1}{2}\right) + \sigma_{yy}\cdot\left(-\frac{1}{2}\right) = -\frac{p}{2} \ \checkmark$$

b)



$$\vec{e}_\nu = \frac{\sqrt{3}}{2}\vec{e}_x + \frac{1}{2}\vec{e}_y$$

$$I_1^\sigma = \sigma_{xx} + \sigma_{yy} \rightarrow \boxed{I_1^\sigma = p}$$

$$D_{\nu\nu} = \varepsilon_{\nu\nu} = \frac{1+\nu}{E}\sigma_{\nu\nu} - \frac{\nu}{E}I_1^\sigma$$

$$\sigma_{\nu\nu} = \sigma_{xx}\, e_{\nu x}^2 + 2\sigma_{xy}\, e_{\nu x} e_{\nu y} + \sigma_{yy}\, e_{\nu y}^2$$

$$\sigma_{\nu\nu} = -2p\frac{\sqrt{3}}{4} + \frac{p}{4} \longrightarrow \sigma_{\nu\nu} = \frac{p}{4}(1-2\sqrt{3})$$

$$\boxed{\sigma_{\nu\nu} = -61{,}6 \ \text{MPa}}$$

$$D_{\nu\nu} = \frac{1}{2{,}1\cdot 10^5}\left[1{,}3\cdot(-61{,}6) - 0{,}3\cdot 100\right]$$

$$\boxed{D_{\nu\nu} = -52{,}42\cdot 10^{-5}}$$

c)
$$\sigma_{11,22} = \frac{p}{2} \pm \sqrt{\frac{p^2}{4} + p^2} \qquad \boxed{\begin{aligned}\sigma_{11} &= \frac{p}{2}(1+\sqrt{5})\\ \sigma_{22} &= \frac{p}{2}(1-\sqrt{5})\end{aligned}}$$

$$\boxed{\begin{aligned}\sigma_{11} &= 161{,}80 \text{ MPa}\\ \sigma_{22} &= -61{,}80 \text{ MPa}\end{aligned}}$$

$$\operatorname{tg} 2\alpha_\sigma = \frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}} = 2 \longrightarrow \boxed{\alpha_\sigma = 31{,}72^\circ}$$

$$\varepsilon_{11} = \frac{1+\nu}{E}\sigma_{11} - \frac{\nu}{E}I_1^\sigma = \frac{1}{2{,}1\cdot 10^5}\left[1{,}3\cdot 161{,}80 - 0{,}3\cdot 100\right]$$

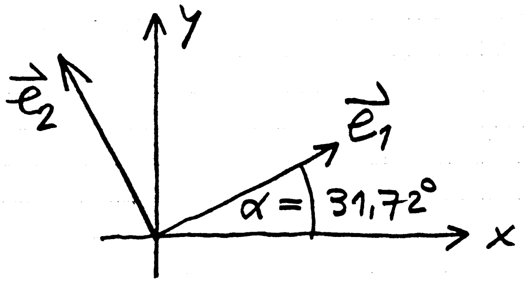
$$\boxed{\varepsilon_{11} = 85{,}88\cdot 10^{-5}}$$

$$\varepsilon_{22} = \frac{1+\nu}{E}\sigma_{22} - \frac{\nu}{E}I_1^\sigma = \frac{1}{2{,}1\cdot 10^5}\left[-1{,}3\cdot 61{,}80 - 0{,}3\cdot 100\right]$$

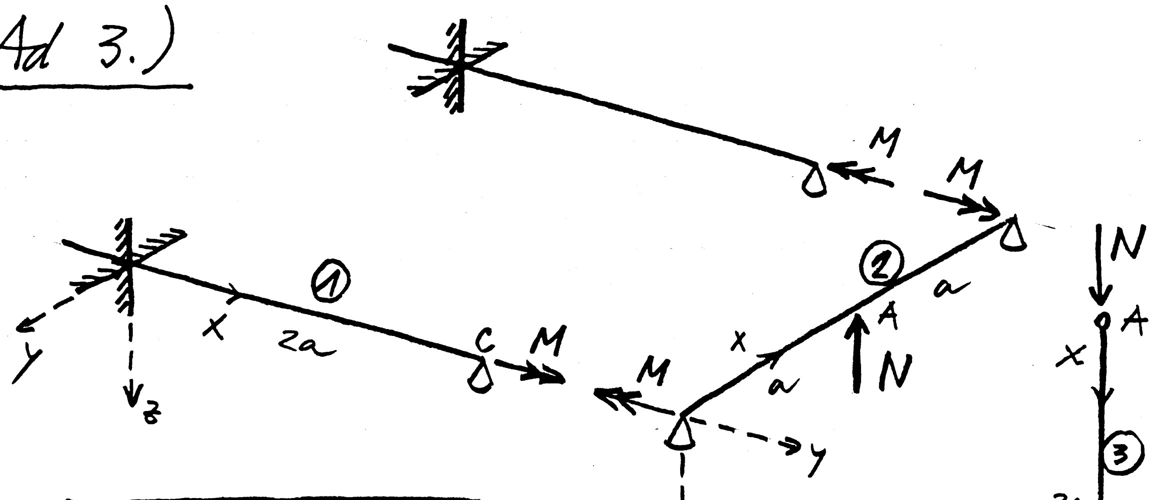
$$\boxed{\varepsilon_{22} = -52{,}54\cdot 10^{-5}}$$

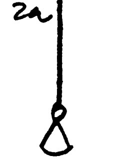
$$\varepsilon_{33} = -\frac{\nu}{E}I_1^\sigma = -\frac{0{,}3}{2{,}1\cdot 10^5}\cdot 100$$

$$\boxed{\varepsilon_{33} = \varepsilon_{zz} = -14{,}29\cdot 10^{-5}} \qquad \boxed{\alpha_\varepsilon = \alpha_\sigma = 31{,}72^\circ}$$



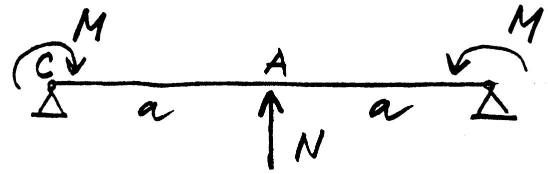
Ad 3.)





① $\boxed{\omega_x^{①}(c) = M \cdot \dfrac{2a}{GI_x}}$

②



$$w_A^{②} = M\frac{(2a)^2}{8EI_y} - N\frac{(2a)^3}{48EI_y} \longrightarrow \boxed{w_A^{②} = M\frac{a^2}{2EI_y} - N\frac{a^3}{6EI_y}}$$

$$\omega_y^{②}(c) = -M\frac{2a}{2EI_y} + N\frac{(2a)^2}{16EI_y} \longrightarrow$$

$$\boxed{\omega_y^{②}(c) = -M\frac{a}{EI_y} + N\frac{a^2}{4EI_y}}$$

③ $\boxed{u_A^{③} = N\dfrac{2a}{EA_p} - 2a\,\alpha_T\,\Delta T}$

$$\omega_x^{①}(c) = \omega_y^{②}(c) \longrightarrow M\cdot\frac{2a}{GI_x} = -M\frac{a}{EI_y} + N\frac{a^2}{4EI_y}$$

$$\boxed{M = N\frac{a}{12}}$$

$$w_A^{②} = N\frac{a}{12}\cdot\frac{a^2}{2EI_y} - N\frac{a^3}{6EI_y} \longrightarrow \boxed{w_A^{②} = -N\frac{a^3}{8EI_y}}$$

$$w_A^{②} = u_A^{③} \longrightarrow -N\frac{a^3}{8EI_y} = N\frac{2a}{EA_p} - 2a\,\alpha_T\,\Delta T$$

$$\boxed{N = \frac{16\,\alpha_T\,EA_p I_y}{a^2 A_p + 16 I_y}\,\Delta T = \underline{\underline{0{,}025\ MN}}}$$