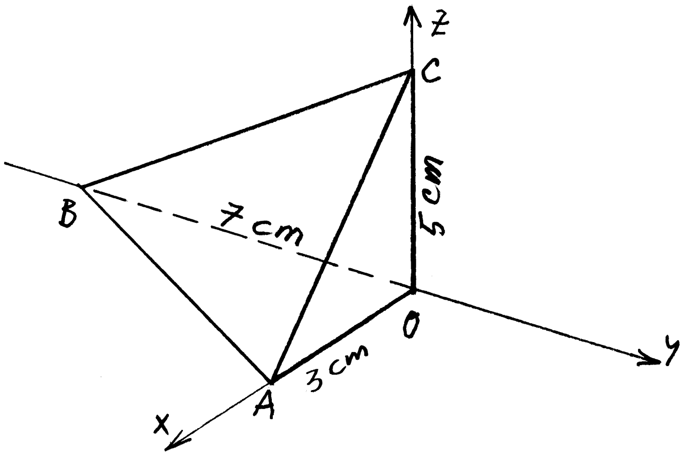
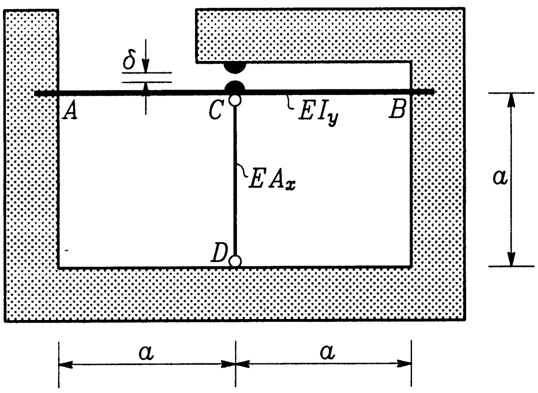
1. Na poševno mejno ploskev ABC prikazane elementarne piramide deluje enakomerna površinska obtežba $\vec{q} = 13.982\,\vec{e}_x + 6.439\,\vec{e}_y + 1.610\,\vec{e}_z$ [kN/cm^2]. Normalna napetost v mejni ploskvi $z = 0$ je tlačna in znaša $10\,\mathrm{kN/cm^2}$. V mejni ploskvi $y = 0$ nastopa samo strižna napetost v smeri x.
   a. Določi vse komponente tenzorja napetosti v koordinatnem sistemu x, y, z!
   b. Razstavi tenzor napetosti na hidrostatični in deviatorični del ter ob upoštevanju Misesovega kriterija plastičnega tečenja preveri, ali je elementarna piramida v elastičnem območju!
   c. Določi dolžini stranic $\overline{AB}$ in $\overline{AO}$ po deformaciji piramide!



$$E = 21\,000\,\mathrm{kN/cm^2}$$
$$\sigma_Y = 36\,\mathrm{kN/cm^2}$$
$$\nu = 0.3$$

(40 točk)

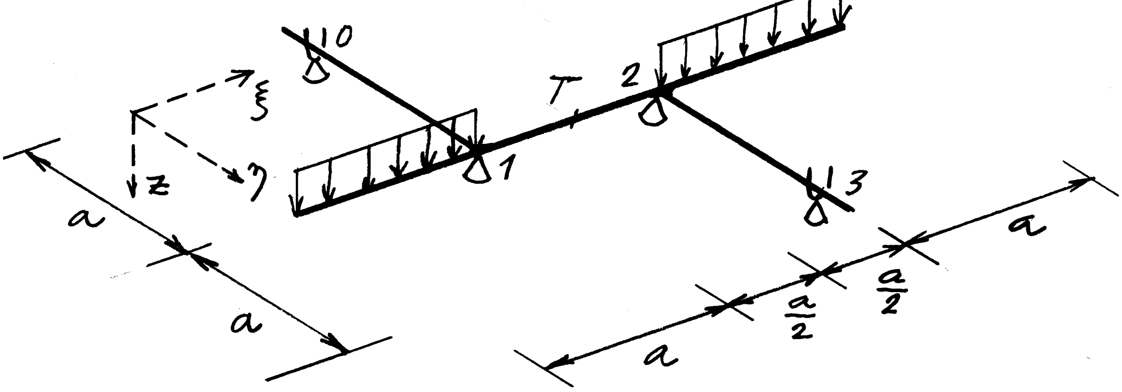
2. Temperaturno stikalo je narejeno iz togega okvirja, vpetega upogibnega elementa AB in paličnega termočlena CD. Določi spremembo temperature ΔT termočlena CD, pri kateri se vzpostavi kontakt !



$$E = 100\,000\,\mathrm{MPa}$$
$$\alpha_T = 3.6 \cdot 10^{-5}/\mathrm{K}$$
$$a = 80\,\mathrm{mm}$$
$$\delta = 0.6\,\mathrm{mm}$$
$$A_x = 10\,\mathrm{mm}^2$$
$$I_y = 2\,\mathrm{mm}^4$$

(40 točk)

2. Določi navpični pomik točke T v odvisnosti od obtežbe q! V podporah 0 in 3 je razen prečnih pomikov preprečen tudi torzijski zasuk nosilca okoli vzdolžne lokalne osi. Zaradi enostavnosti tudi vzamemo : $EI_y = GI_x$.



(točk)

MTT

Ad 1.) Plochu ABC :

$$\frac{x}{3} - \frac{y}{7} + \frac{z}{5} = 1 \longrightarrow \phi = 35x - 15y + 21z - 105 =$$

$$\vec{n} = 35\vec{e}_x - 15\vec{e}_y + 21z$$

$$|\vec{n}| = \sqrt{35^2 + 15^2 + 21^2} = 43{,}486$$

$$\vec{e}_n = \frac{\vec{n}}{n} \longrightarrow \boxed{\vec{e}_n = 0{,}805\,\vec{e}_x - 0{,}345\,\vec{e}_y + 0{,}483\,\vec{e}_z}$$

$$\boxed{\sigma_{zz} = -10 \;,\; \sigma_{yy} = \sigma_{yz} = 0}$$

a)

$$\begin{Bmatrix} p_{nx} \\ p_{ny} \\ p_{nz} \end{Bmatrix} = \begin{Bmatrix} 13{,}982 \\ 6{,}439 \\ 1{,}610 \end{Bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & 0 & 0 \\ \sigma_{zx} & 0 & -10 \end{bmatrix} \begin{Bmatrix} 0{,}805 \\ -0{,}345 \\ 0{,}483 \end{Bmatrix}$$

$$0{,}805\,\sigma_{xx} - 0{,}345\,\sigma_{xy} + 0{,}483\,\sigma_{xz} = 13{,}982$$

$$0{,}805\,\sigma_{xy} = 6{,}439 \longrightarrow \sigma_{xy} = 8\ kN/cm^2$$

$$0{,}805\,\sigma_{xz} - 0{,}483 \cdot 10 = 1{,}610 \longrightarrow \sigma_{xz} = 8\ kN/cm^2$$

$$\rightarrow 0{,}805\,\sigma_{xx} = 13{,}982 + 0{,}345 \cdot 8 - 0{,}483 \cdot 8$$

$$\sigma_{xx} = 16\ kN/cm^2$$

$$[\sigma_{ij}] = \begin{bmatrix} 16 & 8 & 8 \\ 8 & 0 & 0 \\ 8 & 0 & -10 \end{bmatrix} \qquad I_1^\sigma = 16 - 10 = 6$$

$$\sigma^H = \frac{1}{3} I_1^\sigma \rightarrow \sigma^H = 2$$

b.)

$$[\sigma_{ij}^H] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad [s_{ij}^H] = \begin{bmatrix} 14 & 8 & 8 \\ 8 & -2 & 0 \\ 8 & 0 & -12 \end{bmatrix}$$

$$I_2^s = \begin{vmatrix} -2 & 0 \\ 0 & -12 \end{vmatrix} + \begin{vmatrix} 14 & 8 \\ 8 & -12 \end{vmatrix} + \begin{vmatrix} 14 & 8 \\ 8 & -2 \end{vmatrix} \rightarrow \boxed{I_2^s = -300}$$

$$f(s) = |I_2^s| - k_M^2 \quad \dots \quad k_M = \frac{\sigma_Y}{\sqrt{3}}$$

$$f(s) = 300 - \frac{36^2}{3} = -132 \longrightarrow \boxed{EL.\ ST.\ !}$$

c) $$\frac{1+\nu}{E} = 6{,}19 \cdot 10^{-5} \ ; \quad \frac{\nu}{E} = 1{,}43 \cdot 10^{-5}$$

$$\varepsilon_{ij} = \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} I_1^\sigma \delta_{ij}$$

$$\varepsilon_{xx} = 10^{-5} (6{,}19 \cdot 16 - 1{,}43 \cdot 6) \rightarrow \varepsilon_{xx} = 90{,}476 \cdot 10^{-5}$$

$$\varepsilon_{yy} = 10^{-5} (6{,}19 \cdot 0 - 1{,}43 \cdot 6) \rightarrow \varepsilon_{yy} = -8{,}571 \cdot 10^{-5}$$

$$\varepsilon_{zz} = 10^{-5} (6{,}19 \cdot -10 - 1{,}43 \cdot 6) \rightarrow \varepsilon_{zz} = -70{,}476 \cdot 10^{-5}$$

$$\varepsilon_{xy} = \varepsilon_{xz} = 10^{-5} \cdot 6{,}19 \cdot 8 \rightarrow \varepsilon_{xy} = \varepsilon_{xz} = 49{,}523 \cdot 10^{-5}$$

$$\varepsilon_{yz} = 0$$

$$[\varepsilon_{ij}] = 10^{-5} \begin{bmatrix} 90{,}476 & 49{,}523 & 49{,}523 \\ 49{,}523 & -8{,}571 & 0 \\ 49{,}523 & 0 & -70{,}476 \end{bmatrix}$$

$\overline{AO}$: $$\overline{AO}' = (1 + \varepsilon_{xx}) \cdot \overline{AO} \longrightarrow \boxed{\overline{AO}' = 3{,}0027 \text{ cm}}$$

$\overline{AB}$: $$\vec{r} = -3\vec{e}_x - 7\vec{e}_y \rightarrow |\vec{r}| = \sqrt{3^2 + 7^2} = \sqrt{58}$$

$$\vec{e}_s = \frac{\vec{r}}{r} = -0{,}394\,\vec{e}_x - 0{,}919\,\vec{e}_y \ ; \quad e_{sz} = 0 \ !$$

$$\varepsilon_{ss} = \varepsilon_{xx} e_{sx}^2 + 2\varepsilon_{xy} e_{sx} e_{sy} + \varepsilon_{yy} e_{sy}^2$$

$$= 10^{-5} (90{,}476 \cdot 0{,}394^2 + 2 \cdot 49{,}523 \cdot 0{,}394 \cdot 0{,}919 - 8{,}571 \cdot 0{,}919^2) \longrightarrow \boxed{\varepsilon_{ss} = 42{,}670 \cdot 10^{-5}}$$

$$\overline{AB}' = (1 + \varepsilon_{ss}) \cdot \overline{AB} \longrightarrow \boxed{\overline{AB}' = 7{,}6190 \text{ cm}}$$

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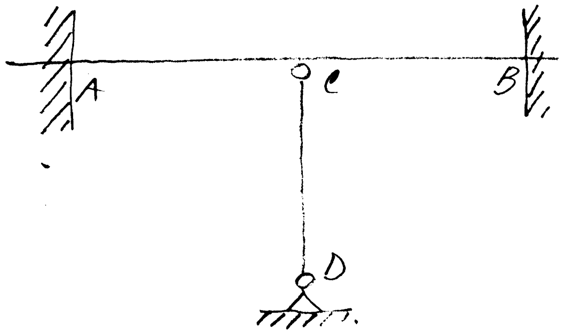
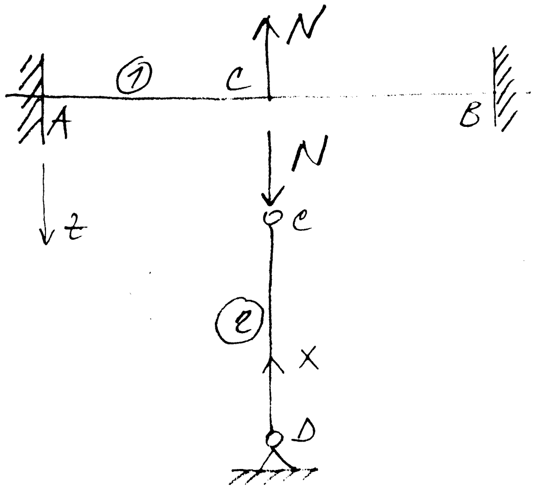
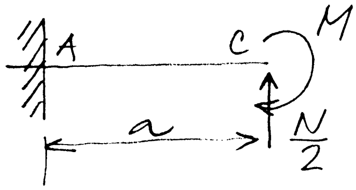
Ad. 2.)





①:



$$\varphi_C = -M\frac{a}{EI_y} + \frac{N}{2}\frac{a^2}{2EI_y} = 0 \rightarrow \boxed{M = N\frac{a}{4}}$$

$$w_C = M\frac{a^2}{2EI_y} - \frac{N}{2}\frac{a^3}{3EI_y} = N\frac{a^3}{EI_y}\left(\frac{1}{8} - \frac{1}{6}\right)$$

$$\boxed{w_C^{(1)} = -N\frac{a^3}{24EI_y}}$$

②: $\boxed{u_C^{(2)} = -\frac{Na}{EA_x} + \alpha_T \Delta T a}$ $\quad\boxed{w_C^{(1)} = -u_C^{(2)}}$

$$-N\frac{a^3}{24EI_y} = N\frac{a}{EA_x} - \alpha_T \Delta T a$$

$$\boxed{N = \frac{24EA_xI_y}{24I_y + a^2A_x}\alpha_T \Delta T} \rightarrow \boxed{w_C = -\frac{a^3A_x\alpha_T}{24I_y + a^2A_x}\Delta T}$$

STIK: $\boxed{w_C = -\delta} \rightarrow \boxed{\Delta T = \frac{24I_y + a^2A_x}{a^3A_x\alpha_T}\delta}$

$$\Delta T = \frac{24\cdot 2 + 80^2\cdot 10}{80^3\cdot 10\cdot 3{,}6\cdot 10^{-5}}\cdot 0{,}6$$

$$\boxed{\Delta T = 208{,}5\ K}$$

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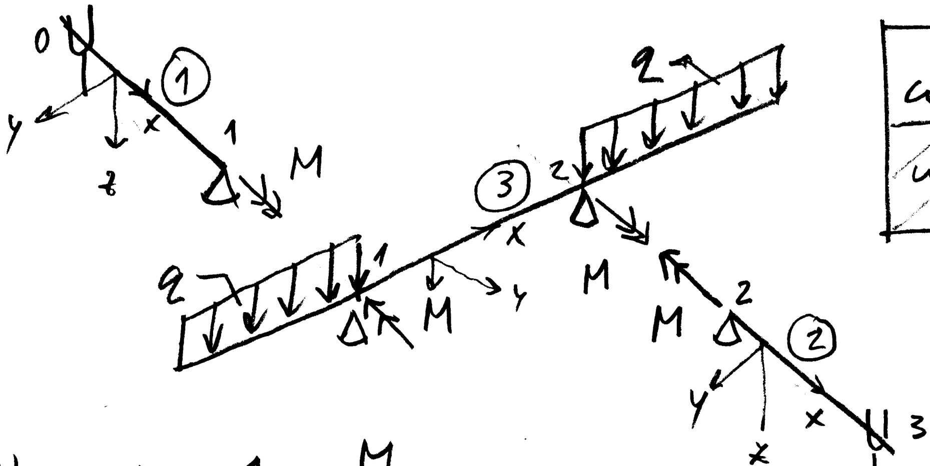
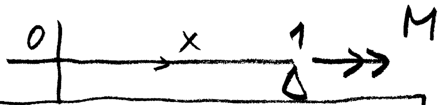
Ad. 3.)



$$\omega_x^{(1)}(1) = \omega_y^{(3)}(1)$$

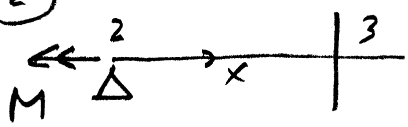
$$\omega_x^{(2)}(2) = \omega_y^{(3)}(2)$$

(1)



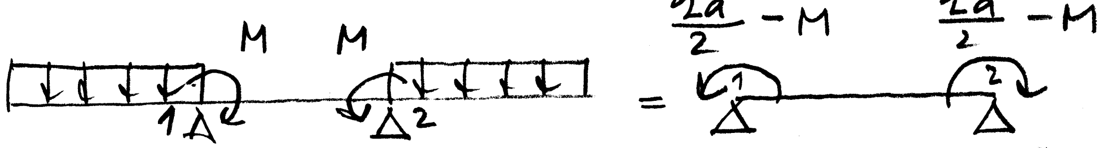
$$\boxed{\omega_x^{(1)}(1) = M\frac{a}{GI_x}}$$

(2)



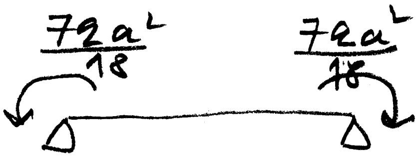
$$\boxed{\omega_x^{(2)}(2) = -M\frac{a}{GI_x}}$$

(3)



$$\boxed{\omega_y^{(3)}(1) = \left(\frac{qa^2}{2} - M\right)\cdot\frac{a}{2EI_y} = -\omega_y^{(3)}(2)}$$

$$\omega_x^{(1)}(1) = \omega_y^{(3)}(1) \quad !$$



$$M\frac{a}{GI_x} = q\frac{a^3}{4EI_y} - M\frac{a}{2EI_y}$$

$$M\left(1+\frac{1}{2}\right) = q\frac{a^2}{4} \longrightarrow \boxed{M = q\frac{a^2}{6}}$$

$$w_S = -\left(\frac{qa^2}{2} - M\right)\cdot\frac{a^2}{8EI_y} \longrightarrow \boxed{w_S = -q\frac{a^4}{24EI_y}}$$