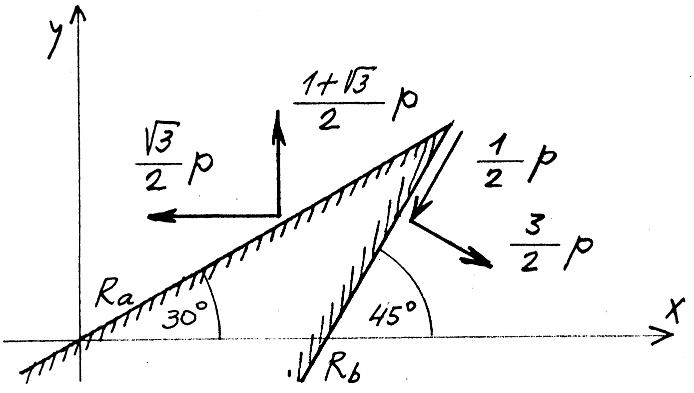
1. Na skici so prikazane intenzitete enakomerne zvezne obtežbe na robovih R_a in R_b tanke stene, v kateri vlada homogeno ravninsko napetostno stanje ($\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$).
   a. Določi napetosti v koordinatnem sistemu (x, y)!
   b. Določi specifično spremembo dolžine robu R_a!
   c. Določi velikosti in smeri glavnih linearnih deformacij !



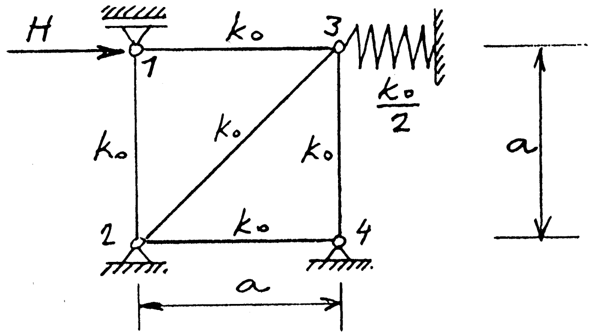
$E = 2.1 \cdot 10^5\, MPa$

$\nu = 0.3$

$p = 100\, MPa$

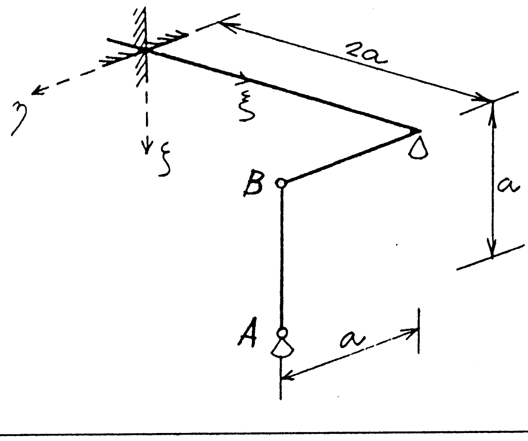
(40 točk)

2. Vse palice prikazane konstrukcije imajo enako osno togost k_0. Vozlišče 3 je v vodoravni smeri elastično podprto z vzmetjo z vzmetno konstanto $k_x^3 = 0.5\, k_0$. Določi osno silo v palici $\overline{23}$! Rezultat izrazi v odvisnosti od obtežbe H in togostnega koeficienta k_0!
(Nasvet: zapiši ravnotežne enačbe vozlišč v razviti obliki!)



(40 točk)

3. Določi pomike točke B in osno silo v palici $\overline{AB}$, če konstrukcijo segrejemo za $\Delta T = 60\,$K! Skiciraj tudi potek in značilne vrednosti notranjih sil!



$E = 20\,000\, kN/cm^2$

$\nu = 0.25$

$\alpha_T = 1.2 \cdot 10^{-5}/K$

$A_x = 50\, cm^2$

$I_x = 2040\, cm^4$

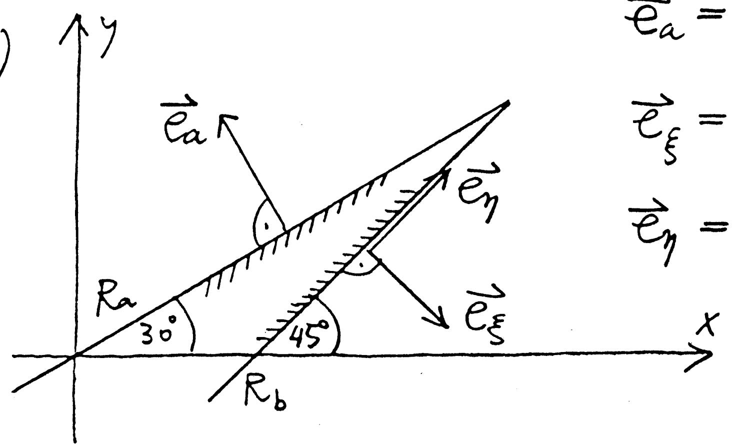
$I_y = 1020\, cm^4$

$a = 2.5\, m$

(30 točk)

Ad 1.)

a)



$$\vec{e}_a = -\frac{1}{2}\vec{e}_x + \frac{\sqrt{3}}{2}\vec{e}_y$$

$$\vec{e}_\xi = \frac{\sqrt{2}}{2}\vec{e}_x - \frac{\sqrt{2}}{2}\vec{e}_y$$

$$\vec{e}_\eta = \frac{\sqrt{2}}{2}\vec{e}_x + \frac{\sqrt{2}}{2}\vec{e}_y$$

$$\sigma_{\xi\xi} = \frac{3}{2}p$$

$$\sigma_{\xi\eta} = -\frac{1}{2}p$$

$$Ra: \quad \vec{p}_a = -\frac{\sqrt{3}}{2}p\,\vec{e}_x + \frac{1+\sqrt{3}}{2}p\,\vec{e}_y$$

$$Rb: \quad \vec{\sigma}_\xi = \frac{3}{2}p\,\vec{e}_\xi + \frac{1}{2}p\,\vec{e}_\eta$$

Ra:
$$-\frac{\sqrt{3}}{2}p = \sigma_{xx}e_{ax} + \sigma_{xy}e_{ay} = -\frac{1}{2}\sigma_{xx} + \frac{\sqrt{3}}{2}\sigma_{xy}$$
$$\frac{1+\sqrt{3}}{2}p = \sigma_{xy}e_{ax} + \sigma_{yy}e_{ay} = -\frac{1}{2}\sigma_{xy} + \frac{\sqrt{3}}{2}\sigma_{yy}$$

Rb:
$$\sigma_{\xi\xi} = \sigma_{xx}e_{\xi x}^2 + 2\sigma_{xy}e_{\xi x}e_{\xi y} + \sigma_{yy}e_{\xi y}^2$$
$$\sigma_{\xi\eta} = \sigma_{xx}e_{\xi x}e_{\eta x} + \sigma_{xy}(e_{\xi x}e_{\eta y} + e_{\xi y}e_{\eta x}) + {}$$
$$+ \sigma_{yy}e_{\xi y}e_{\eta y}$$

Ra:
$$\sigma_{xx} - \sqrt{3}\sigma_{xy} = \sqrt{3}p \quad \rightarrow \quad \sigma_{xx} = \sqrt{3}p + \sqrt{3}\sigma_{xy}$$
$$\sqrt{3}\sigma_{yy} - \sigma_{xy} = (1+\sqrt{3})p \rightarrow \sigma_{yy} = \frac{1}{\sqrt{3}}(1+\sqrt{3})p + \frac{1}{\sqrt{3}}\sigma_{xy}$$

Rb:
$$\frac{3}{2}p = \sigma_{xx}\cdot\frac{1}{2} - 2\sigma_{xy}\cdot\frac{1}{2} + \sigma_{yy}\cdot\frac{1}{2} \quad (= \sigma_{\xi\xi})$$

$$\sigma_{xx} - 2\sigma_{xy} + \sigma_{yy} = 3p$$

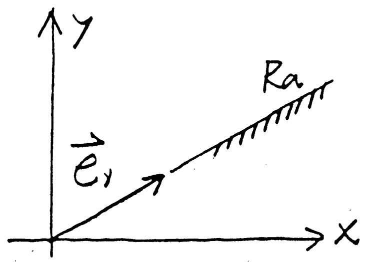
$$\sqrt{3}p + \sqrt{3}\sigma_{xy} - 2\sigma_{xy} + \frac{1}{\sqrt{3}}(1+\sqrt{3})p + \frac{1}{\sqrt{3}}\sigma_{xy} = 3p \;\Big|\cdot\sqrt{3}$$

$$\boxed{\sigma_{xy} = -p} \longrightarrow \boxed{\sigma_{xx} = 0} \; ; \; \boxed{\sigma_{yy} = p}$$

Kontrola: (R_b)

$$\sigma_{\xi\eta} = \sigma_{xx}\cdot\frac{1}{2} + \sigma_{xy}\left(\frac{1}{2}-\frac{1}{2}\right) + \sigma_{yy}\cdot\left(-\frac{1}{2}\right) = -\frac{p}{2}$$

b)



$$\vec{e}_\nu = \frac{\sqrt{3}}{2}\vec{e}_x + \frac{1}{2}\vec{e}_y$$

$$I_1^\sigma = \sigma_{xx} + \sigma_{yy} \rightarrow \boxed{I_1^\sigma = p}$$

$$D_{\nu\nu} = \varepsilon_{\nu\nu} = \frac{1+\nu}{E}\sigma_{\nu\nu} - \frac{\nu}{E} I_1^\sigma$$

$$\sigma_{\nu\nu} = \sigma_{xx} e_{\nu x}^2 + 2\sigma_{xy} e_{\nu x} e_{\nu y} + \sigma_{yy} e_{\nu y}^2$$

$$\sigma_{\nu\nu} = -2p\frac{\sqrt{3}}{4} + \frac{p}{4} \longrightarrow \sigma_{\nu\nu} = \frac{p}{4}(1-2\sqrt{3})$$

$$\boxed{\sigma_{\nu\nu} = -61{,}6 \text{ MPa}}$$

$$D_{\nu\nu} = \frac{1}{2{,}1\cdot 10^5}\left[1{,}3\cdot(-61{,}6) - 0{,}3\cdot 100\right]$$

$$\boxed{D_{\nu\nu} = -52{,}42\cdot 10^{-5}}$$

c)

$$\sigma_{11,22} = \frac{p}{2} \pm \sqrt{\frac{p^2}{4} + p^2} \quad \boxed{\begin{aligned}\sigma_{11} &= \frac{p}{2}(1+\sqrt{5})\\ \sigma_{22} &= \frac{p}{2}(1-\sqrt{5})\end{aligned}}$$

$$\boxed{\begin{aligned}\sigma_{11} &= 161{,}80 \text{ MPa}\\ \sigma_{22} &= -61{,}80 \text{ MPa}\end{aligned}}$$

$$\operatorname{tg} 2\alpha_\sigma = \frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}} = 2 \longrightarrow \boxed{\alpha_\sigma = 31{,}72^\circ}$$

$$\varepsilon_{11} = \frac{1+\nu}{E}\sigma_{11} - \frac{\nu}{E} I_1^\sigma = \frac{1}{2{,}1\cdot 10^5}\left[1{,}3\cdot 161{,}80 - 0{,}3\cdot 100\right]$$

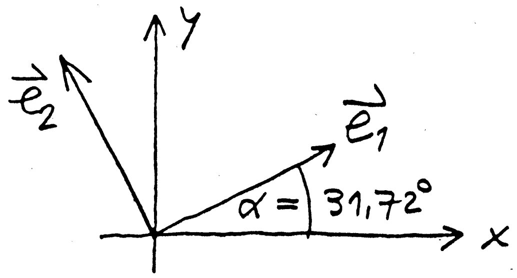
$$\boxed{\varepsilon_{11} = 85{,}88 \cdot 10^{-5}}$$

$$\varepsilon_{22} = \frac{1+\nu}{E}\,\sigma_{22} - \frac{\nu}{E} I_1^{\sigma} = \frac{1}{2{,}1 \cdot 10^5}\left[-1{,}3 \cdot 61{,}80 - 0{,}3 \cdot 100\right]$$

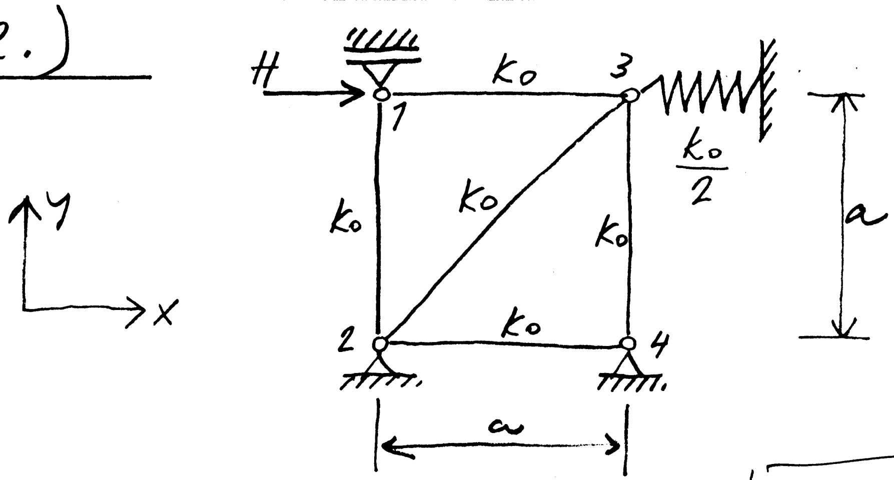
$$\boxed{\varepsilon_{22} = -52{,}54 \cdot 10^{-5}}$$

$$\varepsilon_{33} = -\frac{\nu}{E} I_1^{\sigma} = -\frac{0{,}3}{2{,}1 \cdot 10^5} \cdot 100$$

$$\boxed{\varepsilon_{33} = \varepsilon_{zz} = -14{,}29 \cdot 10^{-5}} \qquad \boxed{\alpha_{\varepsilon} = \alpha_{\sigma} = 31{,}72^{\circ}}$$



Ad 2.)



$$\boxed{H_3 = R_3^{x} = -\frac{3H}{5}}$$

$$[K_{12}] = [K_{34}] = \begin{bmatrix} 0 & 0 \\ 0 & k_0 \end{bmatrix}$$

$$[K_{13}] = \begin{bmatrix} k_0 & 0 \\ 0 & 0 \end{bmatrix} \qquad [K_{23}] = \frac{1}{2}\begin{bmatrix} k_0 & k_0 \\ k_0 & k_0 \end{bmatrix}$$

$$[K_{11}] = -[K_{12}] - [K_{13}] \rightarrow [K_{11}] = -\begin{bmatrix} k_o & 0 \\ 0 & k_o \end{bmatrix}$$

$$[K_{33}] = -[K_{31}] - [K_{32}] - [K_{34}] = -\frac{1}{2}\begin{bmatrix} 3k_o & k_o \\ k_o & 3k_o \end{bmatrix}$$

Vozlišče 1:

$$[K_{11}]\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} + [K_{13}]\begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_1^y \end{Bmatrix} + \begin{Bmatrix} H \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} -k_o & 0 \\ 0 & -k_o \end{bmatrix}\begin{Bmatrix} u_1 \\ 0 \end{Bmatrix} + \begin{bmatrix} k_o & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_1^y \end{Bmatrix} = \begin{Bmatrix} -H \\ 0 \end{Bmatrix}$$

$$\boxed{-k_o u_1 + k_o u_3 = -H \quad (a)} \qquad \boxed{R_1^y = 0}$$

Vozlišče 3:

$$[K_{31}]\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} + [K_{33}]\begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix} + \begin{Bmatrix} R_3^x \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} k_o & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} u_1 \\ 0 \end{Bmatrix} - \frac{1}{2}\begin{bmatrix} 3k_o & k_o \\ k_o & 3k_o \end{bmatrix}\begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix} + \begin{Bmatrix} -k_x^3 u_3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$k_o u_1 - 2k_o u_3 - \frac{1}{2}k_o v_3 = 0 \rightarrow \boxed{2u_1 - 4u_3 - v_3 = 0}$$

$$-\frac{1}{2}k_o u_3 - \frac{3}{2}k_o v_3 = 0 \rightarrow \boxed{u_3 + 3v_3 = 0}$$

$$\boxed{v_3 = -\frac{1}{3}u_3} \qquad 2u_1 - 4u_3 - \frac{1}{3}u_3 = 0$$

$$\boxed{u_1 = \frac{11}{6}u_3} \qquad (a):\ -u_1 + u_3 = -\frac{H}{k_o}$$

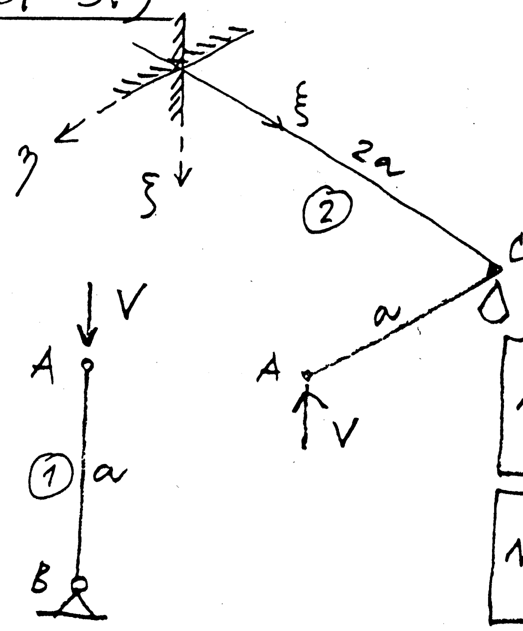
$$\boxed{u_3 = H\frac{6}{5k_o}} \qquad \boxed{u_1 = H\frac{11}{5k_o}} \qquad \boxed{v_3 = -H\frac{2}{5k_o}}$$

$$N_{23} = k_{23}\left[(u_3-u_2)\cos\alpha_{23} + (v_3-v_2)\cos\beta_{23}\right]$$

$$N_{23} = k_0 \cdot \frac{H}{5k_0}\left[(6-0)\frac{\sqrt{2}}{2} + (-2-0)\frac{\sqrt{2}}{2}\right]$$

$$\boxed{N_{23} = H\,\frac{2\sqrt{2}}{5}}$$

Ad 3.)



$$\omega_\xi^{(2)}(C) = -Va\cdot\frac{2a}{GI_x}$$

$$u_\xi^{(2)}(A) = w_A^{(2)} =$$

$$= -V\frac{a^3}{3EI_y} - \omega_\xi^{(2)}(C)\cdot a$$

$$\boxed{w_A^{(2)} = -\frac{Va^3}{3EI_y} - \frac{2Va^3}{GI_x}}$$

$$\boxed{w_A^{(1)} = V\frac{a}{EA_x} - a\alpha_T\Delta T}$$

$$-V\frac{a^3}{3EI_y} - V\frac{2a^3}{GI_x} = V\frac{a}{EA_x} - a\alpha_T\Delta T$$

$$V\left(\frac{a^3}{3EI_y} + \frac{2a^3}{GI_x} + \frac{a}{EA_x}\right) = a\alpha_T\Delta T$$

$$2{,}1704\;V = 0{,}18 \longrightarrow \boxed{V = 0{,}083\text{ kN}}$$

$$w_A = 0{,}083\cdot\frac{250}{20000\cdot 50} - 0{,}18$$

$$\boxed{w_A = -0{,}18\text{ cm}}$$