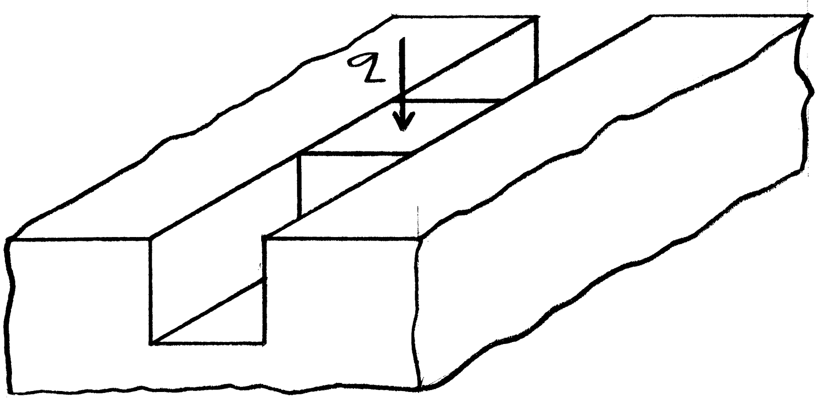
MEHANIKA TRDNIH TELES **IZPIT 1. 9. 2000**

1. V absolutno togi podlagi je narejen žleb kvadratnega prečnega prereza s stranico $a = 20\,\text{mm}$. V žleb je tesno, vendar brez napetosti, vstavljena bakrena kocka dimenzij $20 \times 20 \times 20\,\text{mm}$. Trenje med kocko in podlago je zanemarljivo.
   a. Določi napetosti in deformacije kocke, če zgornjo mejno ploskev enakomerno obtežimo z zvezno obtežbo $q = 80\,\text{MPa}$.
   b. Za koliko moramo spremeniti temperaturo kocke, da se zgornja mejna ploskev kocke ne premakne iz začetne (neobtežene) lege? Določi napetosti v kocki ter njene nove dimenzije v tem primeru!
   c. Za koliko moramo spremeniti temperaturo kocke, da med kocko in bočnima stenama žlebu ne bo napetosti? Določi ustrezne dimenzije kocke v tem primeru!



$q = 80\,\text{MPa}$

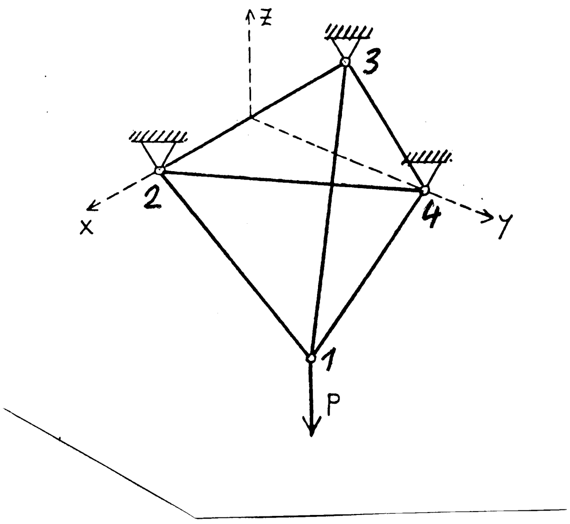
$E = 100\,000\,\text{MPa}$

$\nu = 0.32$

$\alpha_b = 1.7 \cdot 10^{-5}/\text{K}$

(35 točk)

2.



Vse palice prikazanega paličja imajo enake dolžine a in enake prečne prereze A. Določi napetosti v palici $\overline{12}$ in vektor pomika vozlišča 1 glede na koordinatni sistem (x, y, z).

Namig:
Naloga je preprosta. Še laže jo rešiš, če osne sile določiš neposredno iz ravnotežnih pogojev, upoštevaš, da so deformacije majhne, in za določitev navpičnega pomika uporabiš Pitagorov izrek.

$E = 200\,000\,\text{MPa}$

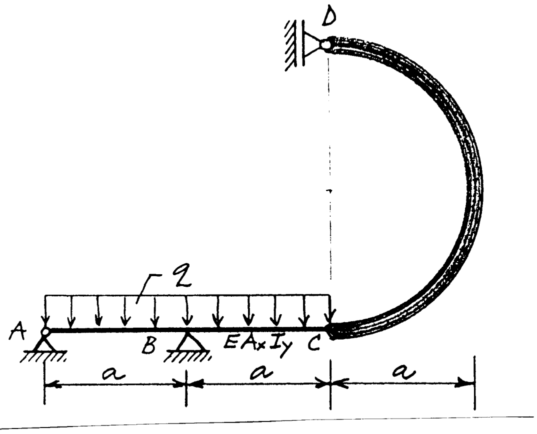
$A = 0.0025\,\text{m}^2$

$a = 6\,\text{m}$

$P = 2.449\,\text{MN}$

(35 točk)

3.



Lahek fasadni element $\overline{CD}$ je zelo tog v primerjavi z nosilcem $\overline{AC}$. V točki C sta oba dela konstrukcije togo povezana. Določi navpični pomik točke D v odvisnosti od velikosti zvezne obtežbe q!

$E = 200\,000\,\text{MPa}$

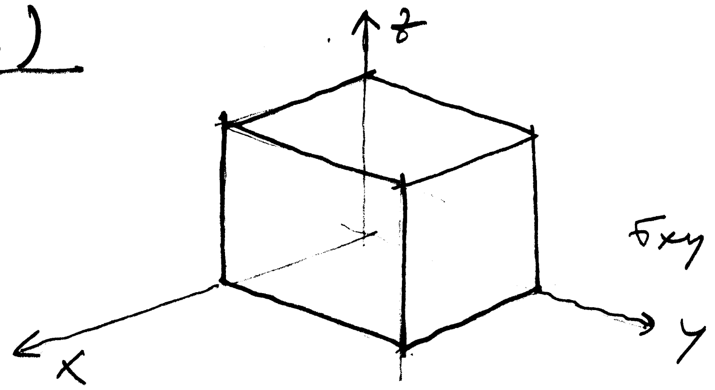
$A_x = 40\,\text{cm}^2$

$I_y = 4000\,\text{cm}^4$

$a = 4\,\text{m}$

(35 točk)

Ad 1.)



$\sigma_{xx} = 0$

$\varepsilon_{yy} = 0$

$\sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0$

$\sigma_{zz} = -q$
$= -80\ MPa$

a) $\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right] = \frac{1}{E}\left[\sigma_{yy} + \nu q\right] = 0$

$$\boxed{\sigma_{yy} = -\nu q} \rightarrow \boxed{\sigma_{yy} = -25{,}6\ MPa}$$

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right] = -\frac{\nu}{E}(-\nu q - q)$$

$$\boxed{\varepsilon_{xx} = \frac{q}{E}\nu(1+\nu)} \rightarrow \boxed{\varepsilon_{xx} = 33{,}792 \cdot 10^{-5}}$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right] = \frac{1}{E}(-q + q\nu^2)$$

$$\boxed{\varepsilon_{zz} = -\frac{q}{E}(1-\nu^2)} \rightarrow \boxed{\varepsilon_{zz} = -71{,}808 \cdot 10^{-5}}$$

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b) $\varepsilon_{yy} = 0$, $\varepsilon_{zz} = 0$, $\sigma_{xx} = 0$, $\sigma_{zz} = -q$

$$\varepsilon_{yy} = \frac{1}{E}(\sigma_{yy} + \nu q) + \alpha_b \Delta T = 0$$
$$\varepsilon_{zz} = \frac{1}{E}(-q - \nu\sigma_{yy}) + \alpha_b \Delta T = 0$$

$$\left.\begin{aligned} \sigma_{yy} + \nu q &= -E\alpha_b \Delta T \\ -q - \nu\sigma_{yy} &= -E\alpha_b \Delta T \end{aligned}\right\} \qquad \boxed{\begin{aligned} \sigma_{yy} &= -q \\ \Delta T &= \frac{q}{E\alpha_b}(1-\nu) \end{aligned}}$$

$$\boxed{\sigma_{yy} = -80\ MPa} \qquad \boxed{\Delta T = 32\ K}$$

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right] + \alpha_B \Delta T$$

$$\boxed{\varepsilon_{xx} = \frac{q}{E}(1+\nu)} \rightarrow \boxed{\varepsilon_{xx} = 105{,}6 \cdot 10^{-5}}$$

Dimenzije kocke: $a'_y = a'_z = 20$ cm

$a'_x = a(1+\varepsilon_{xx}) \rightarrow \boxed{a'_x = 20{,}02112 \text{ cm}}$

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a) $\sigma_{xx} = 0$, $\sigma_{yy} = 0$, $\sigma_{zz} = -q$, $\varepsilon_{yy} = 0$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right] + \alpha_b \Delta T = 0$$

$$\frac{\nu q}{E} + \alpha_b \Delta T = 0 \rightarrow \boxed{\Delta T = -\frac{\nu q}{E \alpha_b}}$$

$$\boxed{\Delta T = -15{,}06 \text{ K}}$$

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right] + \alpha_b \Delta T$$

$$= \frac{\nu q}{E} - \frac{\nu q}{E} \rightarrow \boxed{\varepsilon_{xx} = 0}$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right] + \alpha_b \Delta T$$

$$= -\frac{q}{E} - \frac{\nu q}{E} \rightarrow \boxed{\varepsilon_{zz} = -\frac{q}{E}(1+\nu)}$$

$$\boxed{\varepsilon_{zz} = -105{,}6 \cdot 10^{-5}}$$

$a'_x = a'_y = 20$ cm

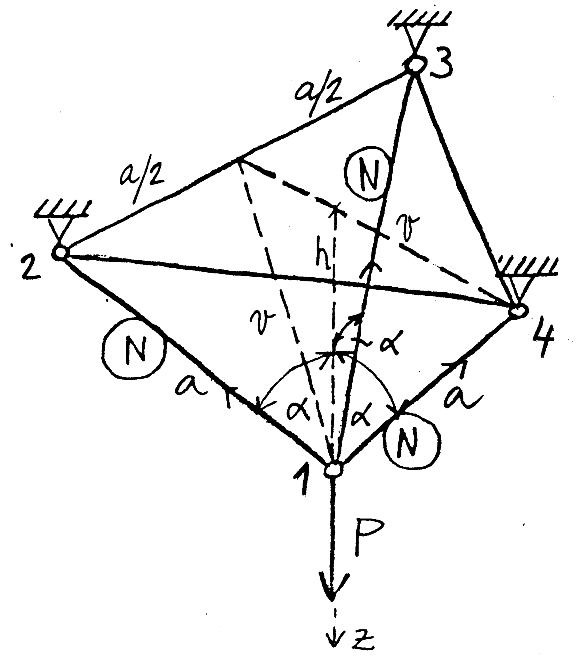
$$\boxed{a'_z = 19{,}9789 \text{ cm}}$$

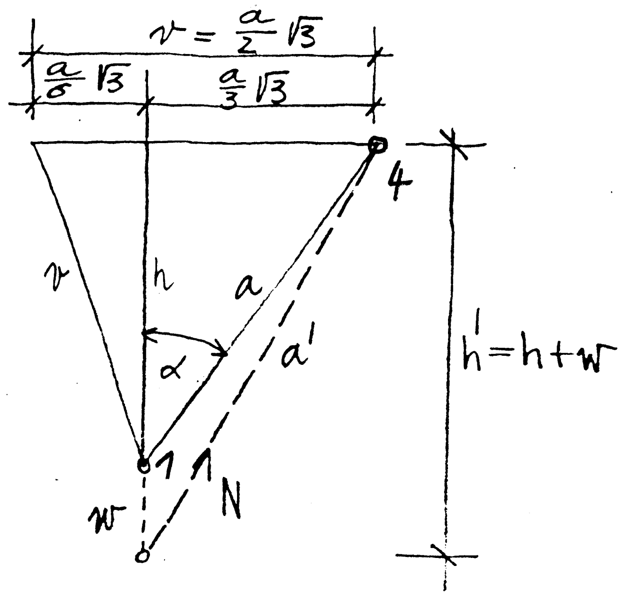
Ad 2.) Koordinate vozlišč 1 in 2:

① $\left(0, \frac{a\sqrt{3}}{6}, \frac{a\sqrt{6}}{3}\right)$ ② $\left(\frac{a}{2}, 0, 0\right)$

Enotski vektor smeri $\overline{12}$:

$$\boxed{\vec{e}_\xi = \frac{1}{2}\vec{e}_x - \frac{\sqrt{3}}{6}\vec{e}_y - \frac{\sqrt{6}}{3}\vec{e}_z}$$





$\sum P_z = 0 \;\ldots\; P - 3N\cos\alpha = 0$; $\quad h = \sqrt{a^2 - \left(\frac{a}{3}\sqrt{3}\right)^2} = \frac{a}{3}\sqrt{6}$

$N = \frac{2,449}{\sqrt{6}} = 1,0\ MN$; $\quad \cos\alpha = \frac{h}{a} = \frac{\sqrt{6}}{3}$

$a' = a + \Delta a = a + a\,D_{\xi\xi} = a\left(1 + \frac{\sigma_{\xi\xi}}{E}\right) = a\left(1 + \frac{N}{EA}\right)$

$a' = \left(1 + \frac{1,0}{2\cdot 10^5 \cdot 0,025}\right) \times 6 = 6,0012\ m$

$w = h' - h = \sqrt{a'^2 - \left(\frac{a}{3}\sqrt{3}\right)^2} - h = \sqrt{6,0012^2 - (2\sqrt{3})^2} - 2\sqrt{6} =$

$w = 4,90045 - 4,89898 = \underline{\underline{0,0015\ m}}$

$\boxed{\vec{u}_0 = 0,0015\ \vec{e}_z}$

$\sigma_{\xi\xi} = \frac{N}{A} = \frac{1}{0,0025} \longrightarrow \underline{\sigma_{\xi\xi} = 400\ MPa}$

$$\sigma_{ij} = \sum_{\alpha}\sum_{\beta} \sigma_{\alpha\beta}\, e_{i\alpha}\, e_{j\beta} = \sigma_{\xi\xi}\, e_{i\xi}\, e_{j\xi}$$

$$\sigma_{xx} = \sigma_{\xi\xi}\, e_{x\xi}^2 = 400 \cdot \left(\tfrac{1}{2}\right)^2 \quad \dots \quad \sigma_{xx} = 100\ \text{MPa}$$

$$\sigma_{yy} = \sigma_{\xi\xi}\, e_{y\xi}^2 = 400 \cdot \frac{3}{36} \quad \dots \quad \sigma_{yy} = 33.3\ \text{MPa}$$

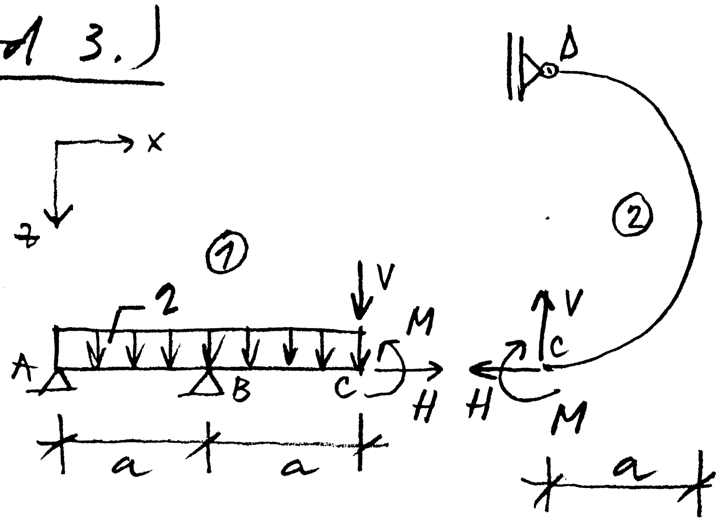
$$\sigma_{zz} = \sigma_{\xi\xi}\, e_{z\xi}^2 = 400 \cdot \frac{6}{9} \quad \dots \quad \sigma_{zz} = 266.7\ \text{MPa}$$

$$\sigma_{xy} = \sigma_{\xi\xi} \cdot e_{x\xi}\, e_{y\xi} = -400 \cdot \frac{\sqrt{3}}{12} \quad \dots \quad \sigma_{xy} = -57{,}7\ \text{MPa}$$

$$\sigma_{yz} = \sigma_{\xi\xi} \cdot e_{y\xi}\, e_{z\xi} = 400 \cdot \frac{\sqrt{18}}{18} \quad \dots \quad \sigma_{yz} = 94{,}3\ \text{MPa}$$

$$\sigma_{zx} = \sigma_{\xi\xi} \cdot e_{z\xi}\, e_{x\xi} = -400 \cdot \frac{\sqrt{6}}{6} \quad \dots \quad \sigma_{zx} = -163{,}3\ \text{MPa}$$

Ad 3.)



(2) : $\boxed{V = 0}$

$$H \cdot 2a + M = 0$$

$$\boxed{H = -M \frac{1}{2a}}$$

$$\vec{r}_D = -2a\,\vec{e}_z$$

$$\vec{u}_C = u_C \vec{e}_x + w_C \vec{e}_z$$

$$\vec{\omega}_C = \omega_C\, \vec{e}_y$$

$$\vec{u}_D = \vec{u}_C + \vec{\omega}_C \times \vec{r}_D = u_C \vec{e}_x + w_C \vec{e}_z - 2a\,\omega_C\, \vec{e}_x$$

$$\vec{u}_D = (u_C - 2a\,\omega_C)\,\vec{e}_x + w_C\, \vec{e}_z \longrightarrow \boxed{w_D = w_C}$$

$$u_x(D) = u_D = 0 \longrightarrow \boxed{u_C = 2a\,\omega_C}$$

(1) $$w_C = \frac{2a^4}{4EI_y} - M\frac{5a^2}{6EI_y}$$

$$\omega_C = -2\,\frac{7a^3}{24EI_y} + M\frac{4a}{3EI_y}$$

$$u_C = H \frac{a}{EA_x} = 2a\left(-q\frac{7a^3}{24EI_y} + M\frac{4a}{3EI_y}\right)$$

$$-\frac{M}{2a}\cdot\frac{a}{A_x} = -q\frac{7a^4}{12EI_y} + M\frac{8a^2}{3I_y}$$

$$\boxed{M = q\,\frac{7a^4A_x}{2(3I_y+16a^2A_x)}} \rightarrow \boxed{M = 34996\ q}$$

$$\boxed{w_C = w_D = 2{,}167\ q}$$