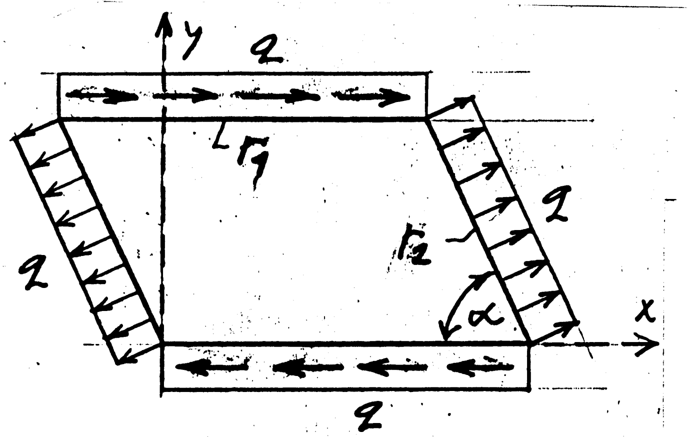
1. Na element enakomerno debele stene deluje zunanja obtežba $q$, kot kaže skica. Ob predpostavki, da vlada v elementu homogeno ravninsko napetostno stanje, določi kot $\alpha$, pri katerem je element v ravnotežju! Določi velikosti in smeri glavnih normalnih deformacij v tem primeru ter jih označi na skici! (NASVET: Zapiši ravnotežna pogoja na robovih $r_1$ in $r_2$, določi $\alpha$, nato pa še preostale napetosti!)

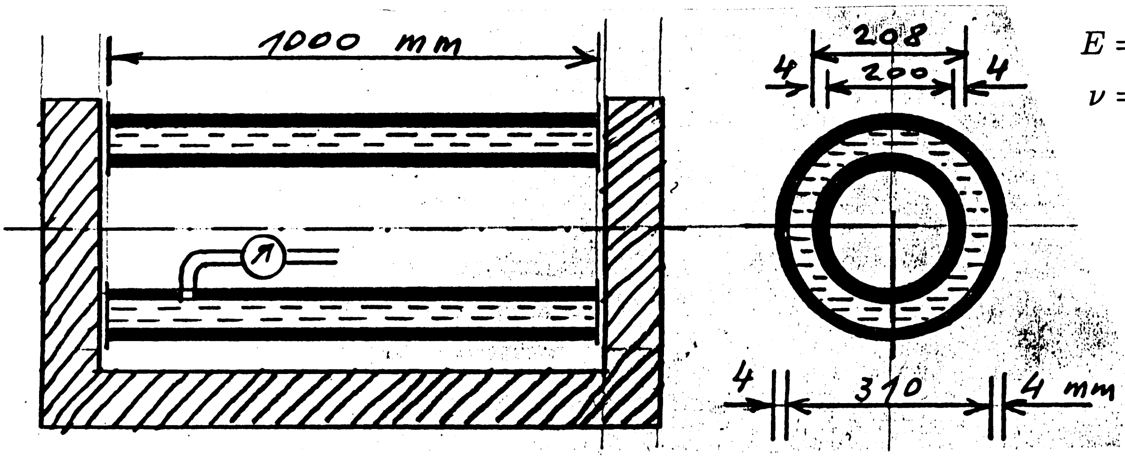



$E = 200\,000\,\mathrm{MPa}$

$\nu = 0.25$

(35 točk)

2. V vmesni prostor med dvema bakrenima cevema z debelino stene $\delta = 4\,\mathrm{mm}$ načrpamo nestisljivo hladilno tekočino. Koliko tekočine porabimo, da znaša hidrostatični tlak $p = 6\,\mathrm{MPa}$? Kolikšne so tedaj normalne napetosti v tangencialni smeri v obeh ceveh? (Tesnila ob nepodajnih priključnih ploščah omogočajo neovirano deformiranje cevi. Vzdolžne normalne napetosti so zanemarljive.)

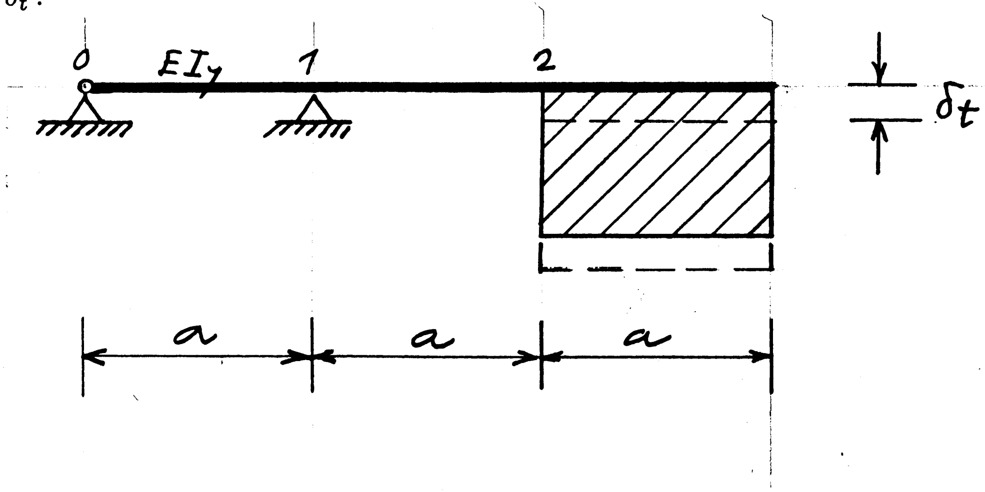



$E = 100\,000\,\mathrm{MPa}$

$\nu = 0.3$

(35 točk)

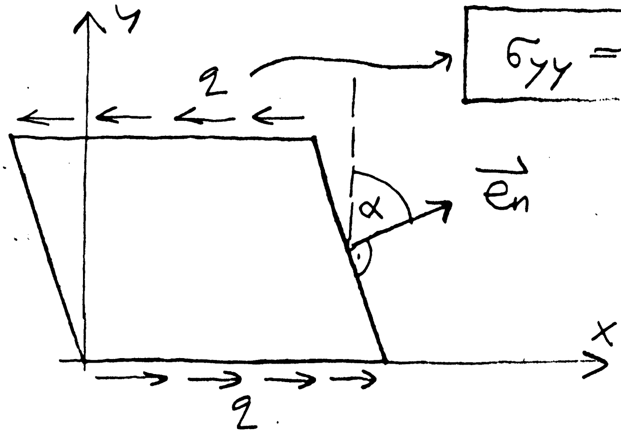
3. Elastičen nosilec je v točki 2 togo vpet v masiven temelj. Določi reakcije, ki nastopijo v podporah 0 in 1, če se masivni temelj enakomerno posede za $\delta_t$! Rezultate izrazi v odvisnosti od posedka $\delta_t$!




(40 točk)

MTT IZPIT

## Ad 1.)




$$\boxed{\sigma_{yy} = 0, \quad \sigma_{yx} = \sigma_{xy} = q}$$

$e_{nx} = \sin\alpha$

$e_{ny} = \cos\alpha$

$e_{nz} = 0$

$\vec{e}_n = \sin\alpha\, \vec{e}_x + \cos\alpha\, \vec{e}_y \qquad \vec{p}_n = q\, \vec{e}_n$

$\vec{p}_n = q \sin\alpha\, \vec{e}_x + q\cos\alpha\, \vec{e}_y = \vec{\sigma}_x e_{nx} + \vec{\sigma}_y e_{ny}$

$p_{nx} = q\sin\alpha = \sigma_{xx} e_{nx} + \sigma_{yx} e_{ny} = \sigma_{xx}\sin\alpha + q\cos\alpha$

$p_{ny} = q\cos\alpha = \sigma_{xy} e_{nx} + \sigma_{yy} e_{ny} = q\sin\alpha$

$\to \cos\alpha = \sin\alpha \longrightarrow \operatorname{tg}\alpha = 1 \longrightarrow \boxed{\alpha = 45^\circ}$

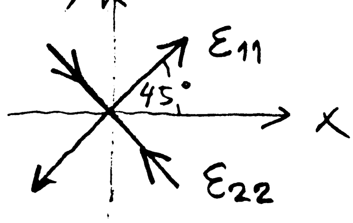
$\sin\alpha = \cos\alpha = \frac{\sqrt{2}}{2} \to q\frac{\sqrt{2}}{2} = \sigma_{xx}\frac{\sqrt{2}}{2} + q\frac{\sqrt{2}}{2}$

$$\boxed{\sigma_{xx} = 0}$$

$$[\sigma_{ij}] = \begin{bmatrix} 0 & q & 0 \\ q & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \sigma_{11,22} = \pm\sqrt{\sigma_{xy}^2} = \pm q$$

$$\boxed{\sigma_{11} = q \quad , \quad \sigma_{22} = -q}$$

$\operatorname{tg} 2\alpha_\sigma = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} = \infty \longrightarrow 2\alpha_\sigma = 90^\circ \to \boxed{\alpha_\sigma = 45^\circ}$




$$\boxed{\begin{aligned} \varepsilon_{11} &= \frac{1+\nu}{E} q \\ \varepsilon_{22} &= -\frac{1+\nu}{E} q \\ \varepsilon_{33} &= 0 \end{aligned}}$$

$$\boxed{I_1^\sigma = 0}$$

Ad 2) $D_1 = 208$ mm , $D_2 = 310$ mm

$$V = \frac{\pi l}{4}(D_2^2 - D_1^2) \quad \ldots \quad V' = \frac{\pi l}{4}(D_2'^2 - D_1'^2)$$

Notranja cev: $\sigma_{ss}^1 = -\frac{p D_1}{2\delta}$ , $\sigma_{rr}^1 = -p$

$$\varepsilon_{ss}^1 = \frac{1}{E}(\sigma_{ss}^1 - \nu \sigma_{rr}^1) = -\frac{p}{E}\left(\frac{D_1}{2\delta} - \nu\right)$$

$$\varepsilon_{ss}^1 = -\frac{p}{E}\left(\frac{D_1}{2\delta} - \nu\right) \rightarrow \boxed{\varepsilon_{ss}^1 = -0{,}00154}$$

Zunanja cev: $\sigma_{ss}^2 = \frac{p D_2}{2\delta}$ , $\sigma_{rr}^2 = -p$

$$\varepsilon_{ss}^2 = \frac{1}{E}(\sigma_{ss}^2 - \nu \sigma_{rr}^2) = \frac{p}{E}\left(\frac{D_2}{2\delta} - \nu\right)$$

$$\boxed{\varepsilon_{ss}^2 = 0{,}00231}$$

$$D_1' = D_1(1 + \varepsilon_{ss}^1) = 208 \cdot (1 - 0{,}00154)$$

$$\underline{D_1' = 207{,}679 \text{ mm}}$$

$$D_2' = D_2(1 + \varepsilon_{ss}^2) = 310 \cdot (1 + 0{,}0023)$$

$$\underline{D_2' = 310{,}715 \text{ mm}}$$

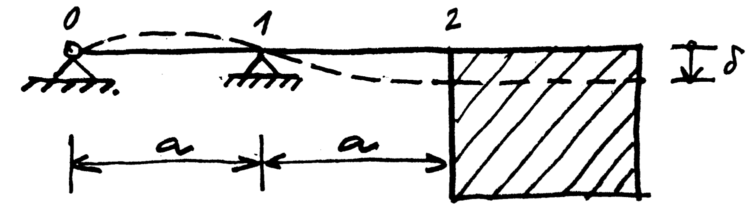
$$V' = \frac{\pi \cdot 1000}{4}(310{,}715^2 - 207{,}679^2) = 41.950.661 \text{ mm}^3$$

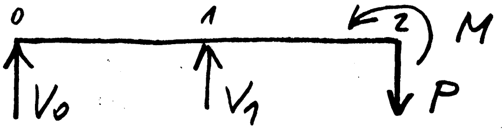
$$\boxed{V' = 41{,}951 \text{ litrov}}$$

$$\sigma_{ss}^n = -\frac{6 \cdot 208}{2 \cdot 4} \rightarrow \boxed{\sigma_{ss}^n = -156 \text{ MPa}}$$

$$\sigma_{ss}^z = \frac{6 \cdot 310}{2 \cdot 4} \rightarrow \boxed{\sigma_{ss}^z = 232{,}5 \text{ MPa}}$$

## Ad 3.




$$V_0 = \frac{M}{a} - P$$
$$V_1 = 2P - \frac{M}{a}$$

$$M_y = V_0 x + V_1 \langle x-a \rangle = \left(\frac{M}{a} - P\right) x + \left(2P - \frac{M}{a}\right) \langle x-a \rangle$$
$$M_y = -P\left(x - 2\langle x-a \rangle\right) + \frac{M}{a}\left(x - \langle x-a \rangle\right) = -EI_y w''$$

$$EI_y w'' = P\left(x - 2\langle x-a \rangle\right) - \frac{M}{a}\left(x - \langle x-a \rangle\right)$$
$$EI_y w' = \frac{P}{2}\left(x^2 - 2\langle x-a \rangle^2\right) - \frac{M}{2a}\left(x^2 - \langle x-a \rangle^2\right) + C_1$$
$$EI_y w = \frac{P}{6}\left(x^3 - 2\langle x-a \rangle^3\right) - \frac{M}{6a}\left(x^3 - \langle x-a \rangle^3\right) + C_1 x + C_2$$

$$x = 0 \;\ldots\; w = 0 \rightarrow \boxed{C_2 = 0}$$
$$x = a \;\ldots\; w = 0 \rightarrow \frac{P}{6} a^3 - \frac{M}{6a} a^3 + C_1 a = 0$$

$$\boxed{C_1 = -\frac{Pa^2}{6} + \frac{Ma}{6}}$$

$$\omega_y = -\frac{1}{EI_y}\left[\frac{P}{2}\left(x^2 - 2\langle x-a \rangle^2\right) - \frac{M}{2a}\left(x^2 - \langle x-a \rangle^2\right) - \frac{Pa^2}{6} + \frac{Ma}{6}\right]$$

$$\boxed{\omega_y = \frac{P}{6EI_y}\left(a^2 - 3x^2 + 6\langle x-a \rangle^2\right) - \frac{M}{6aEI_y}\left(a^2 - 3x^2 + 3\langle x-a \rangle^2\right)}$$

$$\boxed{w = \frac{P}{6EI_y}\left(x^3 - 2\langle x-a \rangle^3 - a^2 x\right) - \frac{M}{6aEI_y}\left(x^3 - \langle x-a \rangle^3 - a^2 x\right)}$$

$x = 2a \ \ldots \ \omega_y = 0$

$$\omega_y(2a) = \frac{P}{6EI_y}\left(a^2 - 12a^2 + 6a^2\right) - \frac{M}{6aEI_y}\left(a^2 - 12a^2 + 3a^2\right)$$

$$\omega_y(2a) = -\frac{5Pa^2}{6EI_y} + \frac{4Ma}{3EI_y} = 0 \rightarrow \boxed{M = P \cdot \frac{5a}{8}}$$

$$w(2a) = \frac{P}{6EI_y}\left(8a^3 - 2a^3 - 2a^3\right) - \frac{M}{6aEI_y}\left(8a^3 - a^3 - 2a^3\right)$$

$$\boxed{w(2a) = P\frac{2a^3}{3EI_y} - M\frac{5a^2}{6EI_y}}$$

$$w(2a) = P\frac{2a^3}{3EI_y} - P\frac{5a}{8} \cdot \frac{5a^2}{6EI_y}$$

$$\boxed{w(2a) = P\frac{7a^3}{48EI_y}} \rightarrow \boxed{w(2a) = \delta}$$

$$\boxed{P = \delta\frac{48EI_y}{7a^3}} \longrightarrow \boxed{M = \delta\frac{30EI_y}{7a^2}}$$

$$\boxed{V_0 = -\delta\frac{18EI_y}{7a^3}} \qquad \boxed{V_1 = \delta\frac{66EI_y}{7a^3}}$$