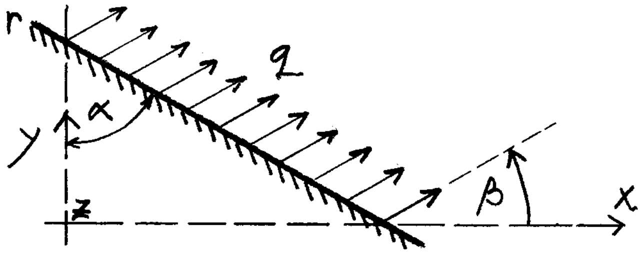
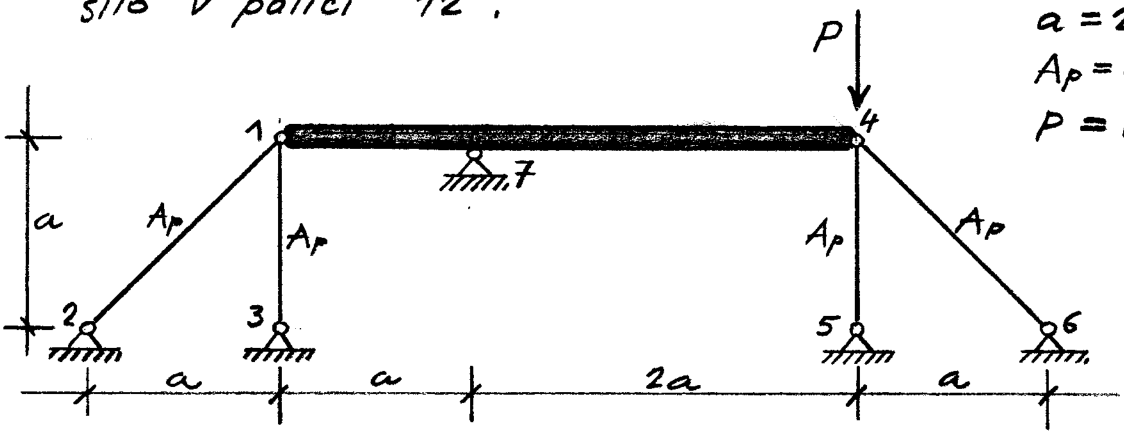
1. Na rob stene (RNS) deluje enakomerna zvezna obtežba $q$ kot kaže skica. Določi komponente tenzorja napetosti v koordinatnem sistemu $(x, y, z)$ tako, da bosta glavni normalni napetosti nasprotno enaki med seboj ($\sigma_{22} = -\sigma_{11}$)! Določi ravnini obeh glavnih normalnih napetosti!



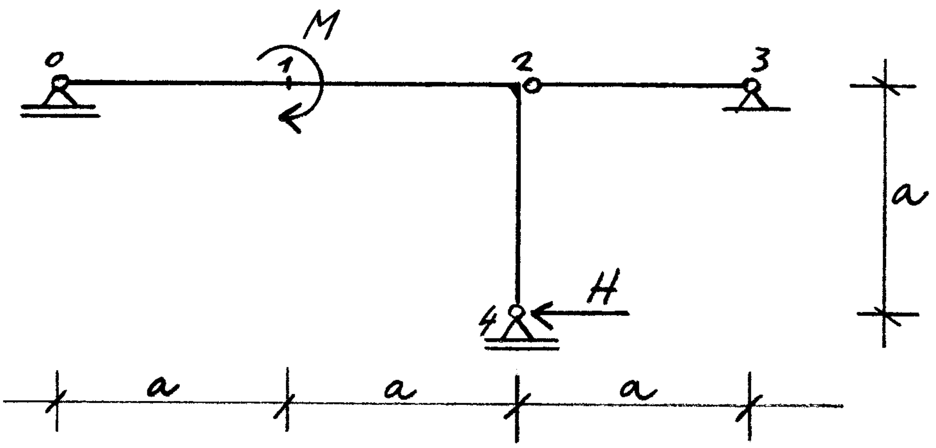

$q = 10$ MPa
$\alpha = 60°$ , $\beta = 30°$

2. Nosilec $\overline{14}$ je zelo tog v primerjavi s palicami. Določi in skiciraj potek notranjih sil vzdolž tega nosilca ter osno silo v palici $\overline{12}$!




$a = 2$ m
$A_p = 8$ cm$^2$
$P = 60$ kN

3. Z izrekom o virtualnem delu določi reakcije v podporah!

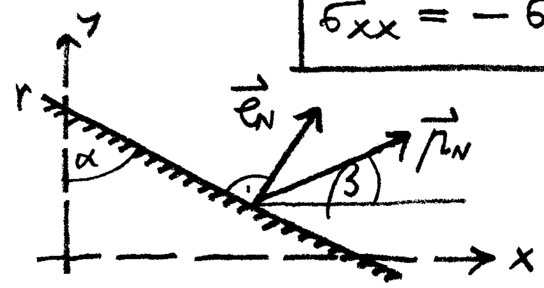


## Ad 1.)

$$\sigma_{22} = -\sigma_{11}$$

$$\frac{1}{2}(\sigma_{xx}+\sigma_{yy}) - \sqrt{\frac{1}{4}(\sigma_{xx}-\sigma_{yy}) + \sigma_{xy}^2} = -\left(\frac{1}{2}(\sigma_{xx}+\sigma_{yy}) + \sqrt{\frac{1}{4}(\sigma_{xx}-\sigma_{yy})^2 + \sigma_{xy}^2}\right)$$

$$\boxed{\sigma_{xx} = -\sigma_{yy}}$$




$$e_{Nx} = \cos\alpha = \frac{1}{2}\;;\quad e_{Ny} = \sin\alpha = \frac{\sqrt{3}}{2}$$

$$p_{Nx} = q\cos\beta = q\,\frac{\sqrt{3}}{2}$$

$$p_{Ny} = q\sin\beta = q\cdot\frac{1}{2}$$

$$p_{Nx} = \sigma_{xx}\,e_{Nx} + \sigma_{xy}\,e_{Ny} \longrightarrow \sigma_{xx}\cdot\frac{1}{2} + \sigma_{xy}\cdot\frac{\sqrt{3}}{2} = q\,\frac{\sqrt{3}}{2}$$

$$p_{Ny} = \sigma_{xy}\,e_{Nx} + \sigma_{yy}\,e_{Ny} \longrightarrow \sigma_{xy}\cdot\frac{1}{2} + \sigma_{yy}\cdot\frac{\sqrt{3}}{2} = q\,\frac{1}{2}$$

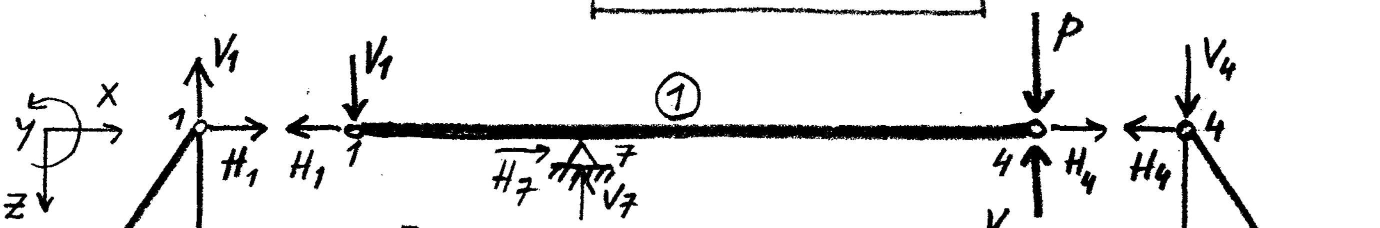
$$\left.\begin{aligned} \sigma_{xx} + \sqrt{3}\,\sigma_{xy} &= q\sqrt{3} \\ \sigma_{xy} - \sqrt{3}\,\sigma_{xx} &= q \end{aligned}\right\} \quad \boxed{\begin{gathered}\sigma_{xy} = q\,, \quad \sigma_{xx} = -\sigma_{yy} = 0 \\ \sigma_{11} = -\sigma_{22} = q\end{gathered}}$$

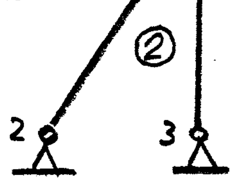
$$\operatorname{tg} 2\alpha_\sigma = \frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}} = \infty \longrightarrow 2\alpha_\sigma = 90^\circ$$

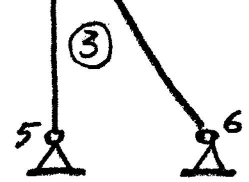
$$\boxed{\alpha_\sigma = 45^\circ\,,\ 135^\circ}$$

## Ad 2.)

$$\Sigma X = 0 \;\ldots\; \boxed{-H_1 + H_7 + H_4 = 0}$$




$$\Sigma M^7 = 0 \;\ldots\; V_1 a + 2V_4 a - 2Pa = 0$$

$$\boxed{V_1 + 2V_4 = 2P}$$

$$\Sigma Z = 0 \;\ldots\; \boxed{V_1 - V_4 - V_7 + P = 0}$$

(1)

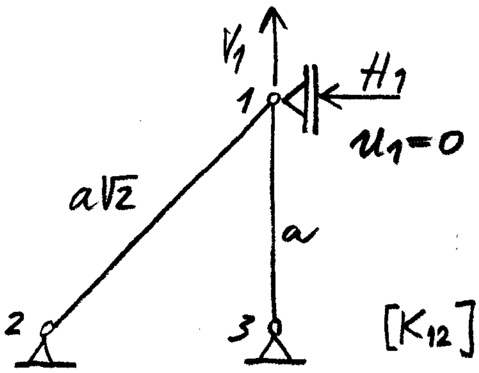

$$\vec{u}_7 = \vec{0}$$
$$\vec{\omega}_7 = \omega_y \vec{e}_y$$
$$\vec{r}_1 = -a\,\vec{e}_x$$
$$\vec{r}_4 = 2a\,\vec{e}_x$$

$$\vec{u}_1 = \vec{u}_7 + \vec{\omega}_7 \times \vec{r}_1 = \omega_y \vec{e}_y \times (-a\,\vec{e}_x) \longrightarrow \boxed{\vec{u}_1 = a\omega_y\,\vec{e}_z}$$

$$\vec{u}_4 = \vec{u}_7 + \vec{\omega}_7 \times \vec{r}_4 = \omega_y \vec{e}_y \times 2a\,\vec{e}_x \longrightarrow \boxed{\vec{u}_4 = -2a\omega_y\,\vec{e}_z}$$

$$\boxed{\begin{array}{ll} u_1 = 0\,, & w_1 = a\omega_y \\ u_4 = 0\,, & w_4 = -2a\omega_y \end{array}}$$

(2)




$$k_{12} = \frac{EA_p}{a\sqrt{2}} \qquad k_{13} = \frac{EA_p}{a}$$

$$[K_{12}] = \frac{EA_p}{a\sqrt{2}} \begin{array}{c|cc} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \hline -\frac{\sqrt{2}}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{1}{2} \end{array} = \frac{EA_p}{a} \begin{bmatrix} \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{bmatrix}$$

$$[K_{13}] = \frac{EA_p}{a} \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 1 \end{array} = \frac{EA_p}{a} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[K_{11}] = \frac{EA_p}{a} \begin{bmatrix} -\frac{\sqrt{2}}{4} & +\frac{\sqrt{2}}{4} \\ +\frac{\sqrt{2}}{4} & -(1+\frac{\sqrt{2}}{4}) \end{bmatrix}$$

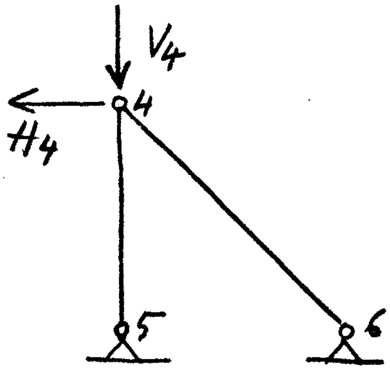
$$1: \quad [K_{11}] \begin{Bmatrix} u_1 \\ w_1 \end{Bmatrix} + \begin{Bmatrix} -H_1 \\ -V_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$-\frac{EA_p}{a} \begin{bmatrix} \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}}{4} & 1+\frac{\sqrt{2}}{4} \end{bmatrix} \begin{Bmatrix} 0 \\ w_1 \end{Bmatrix} = \begin{Bmatrix} H_1 \\ V_1 \end{Bmatrix}$$

$$\frac{EA_p}{a}\cdot\frac{\sqrt{2}}{4}\cdot w_1 = H_1 \longrightarrow \boxed{H_1 = \frac{EA_p\sqrt{2}}{4}\,\omega_y} \quad \ldots(a)$$

$$-\frac{EA_p}{a}\,\frac{4+\sqrt{2}}{4}\cdot w_1 = V_1 \longrightarrow \boxed{V_1 = -\frac{EA_p\,(4+\sqrt{2})}{4}\,\omega_y} \quad \ldots(b)$$

3




$$[K_{44}] = -\frac{EA_p}{a}\begin{bmatrix}\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4}\\ \frac{\sqrt{2}}{4} & \frac{4+\sqrt{2}}{4}\end{bmatrix}$$

4:
$$[K_{44}]\begin{Bmatrix}u_4\\ w_4\end{Bmatrix} + \begin{Bmatrix}-H_4\\ V_4\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$$

$$-\frac{EA_p}{a}\begin{bmatrix}\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4}\\ \frac{\sqrt{2}}{4} & \frac{4+\sqrt{2}}{4}\end{bmatrix}\begin{Bmatrix}0\\ w_4\end{Bmatrix} = \begin{Bmatrix}H_4\\ -V_4\end{Bmatrix}$$

$$-\frac{EA_p}{a}\cdot\frac{\sqrt{2}}{4}\,w_4 = H_4 \longrightarrow \boxed{H_4 = -\frac{EA_p\sqrt{2}}{2}\,\omega_y} \quad \ldots(c)$$

$$-\frac{EA_p}{a}\cdot\frac{4+\sqrt{2}}{4}\,w_4 = -V_4 \longrightarrow \boxed{V_4 = -\frac{EA_p\,(4+\sqrt{2})}{2}\,\omega_y} \quad \ldots(d)$$

$$a) \leftrightarrow c) \longrightarrow \frac{4H_1}{EA_p\sqrt{2}} = -\frac{2H_4}{EA_p\sqrt{2}} \longrightarrow \boxed{H_4 = -2H_1}$$

$$b) \leftrightarrow d) \longrightarrow -\frac{4V_1}{EA_p(4+\sqrt{2})} = -\frac{2V_4}{EA_p(4+\sqrt{2})} \longrightarrow \boxed{V_4 = 2V_1}$$

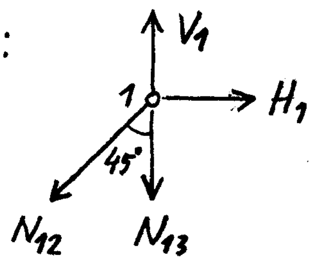
$$V_1 + 2V_4 = 2P \longrightarrow \boxed{\begin{array}{l} V_1 = \frac{2P}{5} = 24\ kN \\ V_4 = \frac{4P}{5} = 48\ kN\end{array}}$$

a) ↔ b) $$\frac{4H_1}{EA_p\sqrt{2}} = -\frac{4V_1}{EA_p(4+\sqrt{2})}$$

$$H_1 = -V_1\frac{\sqrt{2}}{4+\sqrt{2}} \longrightarrow \boxed{H_1 = -6{,}27\ kN}$$

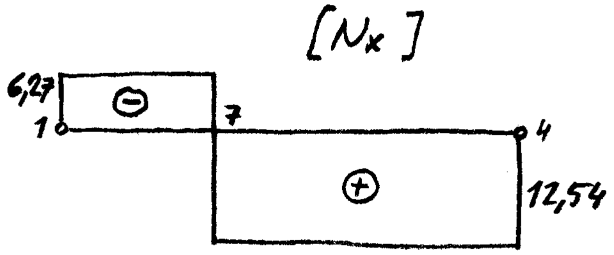
$$H_4 = V_4\frac{\sqrt{2}}{4+\sqrt{2}} \longrightarrow \boxed{H_4 = 12{,}54\ kN}$$

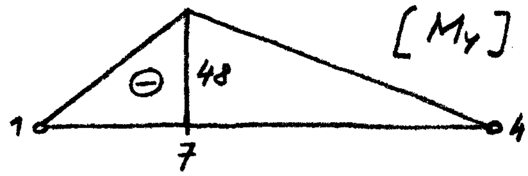
②:

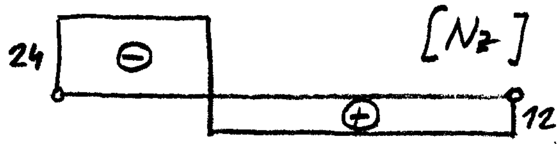



$$H_1 = N_{12}\frac{\sqrt{2}}{2}$$

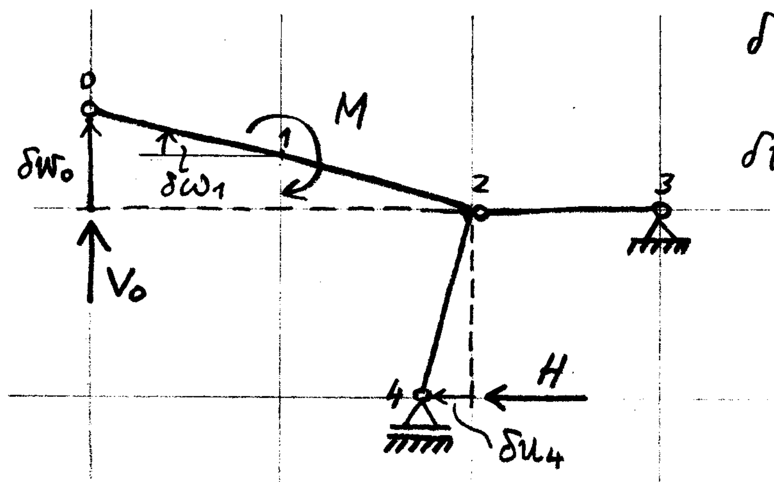
$$N_{12} = \frac{2}{\sqrt{2}}H_1 \rightarrow \boxed{N_{12} = -8{,}87\ kN}$$




## Ad 3.)

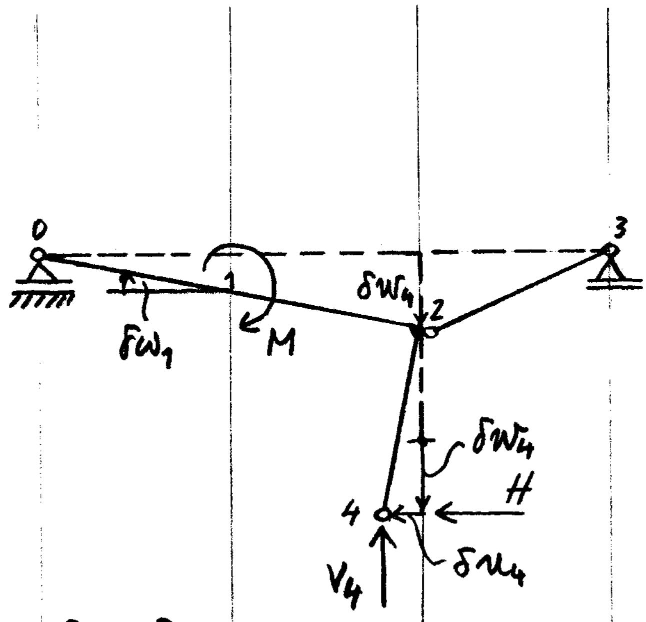



$$\delta W = V_0\,\delta w_0 + M\,\delta\omega_0 + H\,\delta u_4 = 0$$

$$\delta w_0\left(V_0 + \frac{M}{2a} + \frac{H}{2}\right) = 0$$

$$\boxed{V_0 = -\frac{M}{2a} - \frac{H}{2}}$$

$$\delta\omega_1 = \frac{\delta w_0}{2a}\ ,\quad \delta u_4 = \frac{1}{2}\,\delta w_0$$



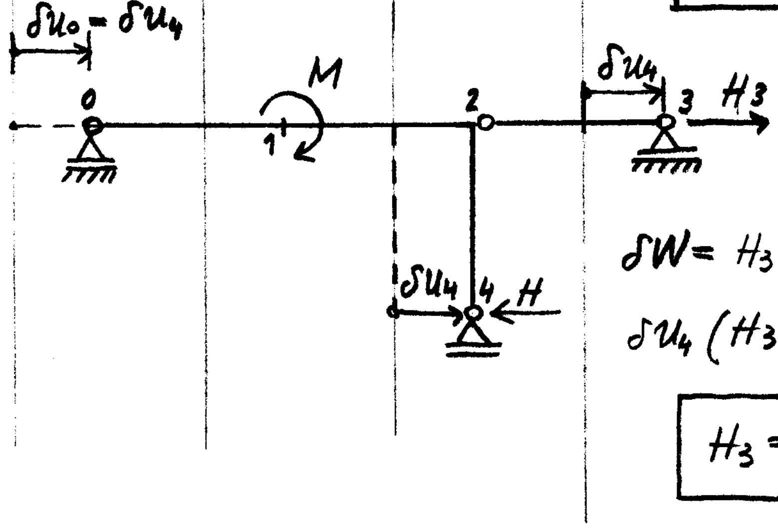

$$\delta \omega_1 = \frac{\delta w_4}{2a}$$

$$\delta u_4 = \frac{\delta w_4}{2}$$

$$\delta W = -V_4 \delta w_4 + M \delta \omega_1 + H \delta u_4 = 0$$

$$\delta w_4 \left( -V_4 + \frac{M}{2a} + \frac{H}{2} \right) = 0$$

$$\boxed{V_4 = \frac{M}{2a} + \frac{H}{2}}$$




$$\delta W = H_3 \delta u_4 - H \delta u_4 = 0$$

$$\delta u_4 (H_3 - H) = 0$$

$$\boxed{H_3 = H}$$