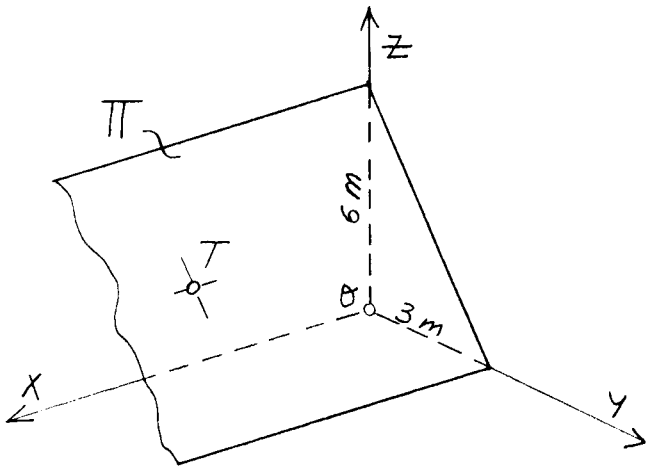
1. Napetostno stanje telesa je podano s komponentami σ_{ij} tenzorja napetosti v koordinatnem sistemu x, y, z. V točki $T(5, y, 2)$, ki leži v ravnini Π, določi:

   a. glavne normalne napetosti in glavne normalne deformacije,

   b. kot β med normalo ravnine Π in normalo ravnine, v kateri deluje največja glavna normalna napetost,

   c. strižno napetost v ravnini Π ter enotski vektor $\vec{e}_t$, ki določa smer te strižne napetosti.



$$E = 200\,000\,\text{MPa}$$
$$\nu = 0.3$$

$$[\sigma_{ij}] = \begin{bmatrix} 4(2y^2+z) & 5z & 5zx \\ 5z & -zx & 5z^2-2xy \\ 5zx & 5z^2-2xy & 10x \end{bmatrix}$$

[MPa]

---

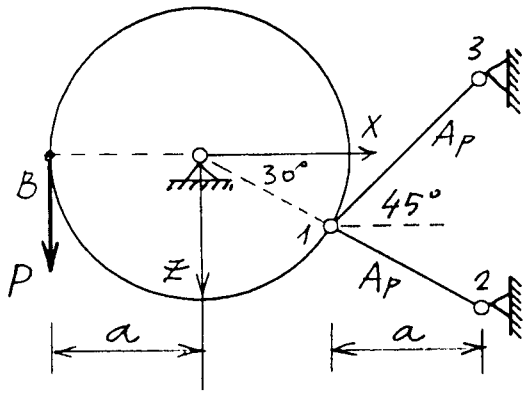
2. Z jeklenim merilnim trakom, ki je bil umerjen pri temperaturi 20°C, želimo pri temperaturi 36°C izmeriti razdaljo med točkama A in B. Trak nategnemo s silo $N = 200\,\text{N}$ in med točkama A in B odčitamo odmerek $L = 26,3290\,\text{m}$. Trak ima pravokoten prečni prerez dimenzij $16 \times 0.5\,\text{mm}$, modul elastičnosti je 250 000 MPa, temperaturni razteznostni koeficient pa $\alpha_T = 1.25 \cdot 10^{-5}/K$.

   a. Določi dolžino deformirane enote razdelbe traku ter dejansko razdaljo L_0 med točkama A in B!

   b. Pri kateri temperaturi bi bil odčitani odmerek L prava razdalja med točkama A in B? (Trak je tudi v tem primeru napet s silo $N = 200\,\text{N}$.)

---

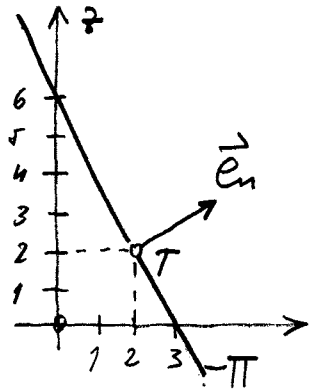
3. Absolutno toga homogena in enakomerno debela krožna plošča je v točki B obtežena z navpično točkovno silo P. Plošča je v središču nepomično vrtljivo podprta, v točki 1 pa je členkasto pritrjena na palici $\overline{12}$ in $\overline{13}$. Določi zasuk plošče okrog središča in osno silo v palici $\overline{13}$! Rezultate izrazi v odvisnosti od sile P!



$$E = 20\,000\,\text{kN/cm}^2$$
$$a = 120\,\text{cm}$$
$$A_p = 4\,\text{cm}^2$$

MTT 1. KOLOKVIJ 22. 12. 98

Ad 1.)



$y_T = 4 \cdot \frac{3}{6} = 2\,m$

$T(5,2,2)$:

$$[\sigma_{ij}]_T = \begin{bmatrix} 40 & 10 & 50 \\ 10 & -10 & 0 \\ 50 & 0 & 50 \end{bmatrix} \text{ MPa}$$

a) $I_1 = 40 - 10 + 50 = 80$

$I_2 = -500 - 500 - 500 = -1500$

$I_3 = 0$

$$\sigma^3 - 80\sigma^2 - 1500\sigma = 0 \rightarrow \sigma(\sigma^2 - 80\sigma - 1500) = 0$$

$\boxed{\sigma_{11} = 0}$, $\sigma_{22,33} = \frac{80}{2} \pm \frac{1}{2}\sqrt{80^2 + 4 + 1500}$

$\boxed{\sigma_{22} = 95{,}7 \text{ MPa}}$ $\boxed{\sigma_{33} = -15{,}7 \text{ MPa}}$

$$\varepsilon_{11} = \frac{1+\nu}{E}\sigma_{11} - \frac{\nu}{E} I_1^{\sigma} \rightarrow \boxed{\varepsilon_{11} = -1{,}2 \cdot 10^{-4}}$$

$$\varepsilon_{22} = \frac{1+\nu}{E}\sigma_{22} - \frac{\nu}{E} I_1^{\sigma} \rightarrow \boxed{\varepsilon_{22} = 5{,}02 \cdot 10^{-4}}$$

$$\varepsilon_{33} = \frac{1+\nu}{E}\sigma_{33} - \frac{\nu}{E} I_1^{\sigma} \rightarrow \boxed{\varepsilon_{33} = -2{,}22 \cdot 10^{-4}}$$

MTT 1. KOLOKVIJ 22.12.98

Ad 2.) $A = 16 \cdot 0{,}5 = 8\ mm^2$

$$\varepsilon = \frac{N}{EA} + \alpha_T \Delta T = \frac{200}{2{,}5 \cdot 10^5 \cdot 8} + 1{,}25 \cdot 10^{-5} \cdot 16$$

$$\varepsilon = (10 + 20) \cdot 10^{-5} \rightarrow \boxed{\varepsilon = 3 \cdot 10^{-4}}$$

a) Dolzina deformirane enote:

$$e' = 1 + \varepsilon \rightarrow \boxed{e' = 1{,}0003}$$

Deformirana razdalja $\overline{AB}$: $L_0 = L\,\varepsilon'$

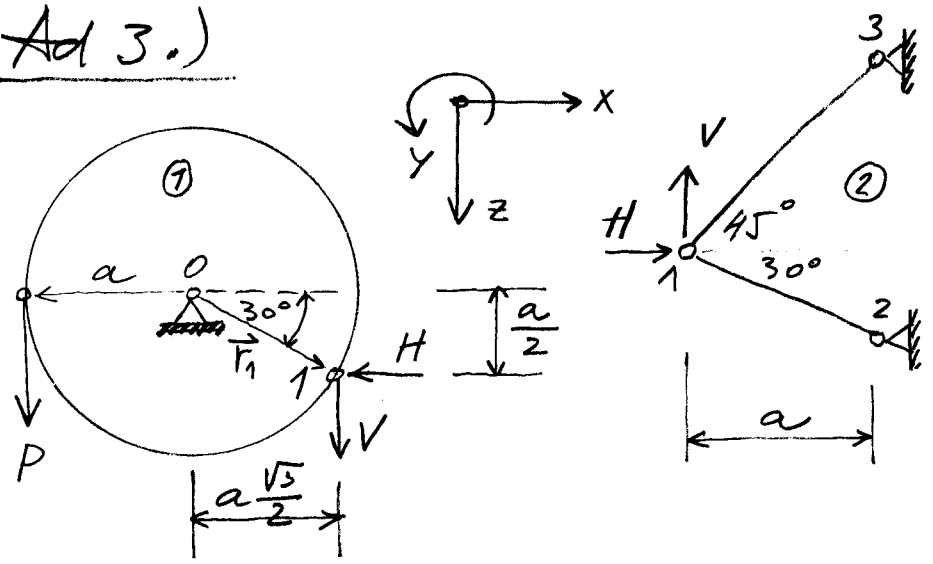
$$L_0 = 26{,}3290 \cdot 1{,}0003 \rightarrow \boxed{L_0 = 26{,}3369\ m}$$

b) $\varepsilon = \frac{N}{EA} + \alpha_T \Delta T = 0 \rightarrow \boxed{\Delta T = -\frac{N}{EA\alpha_T}}$

$$\Delta T = -\frac{200}{2{,}5 \cdot 10^5 \cdot 8 \cdot 1{,}25 \cdot 10^{-5}}$$

$$\Delta T = -8^\circ \longrightarrow T_0 = 20 - 8 \rightarrow \boxed{T_0 = 12^\circ C}$$

Ad 3.)



① $\Sigma M^{\circ} = 0 \rightarrow Pa - Va\frac{\sqrt{3}}{2} - H\frac{a}{2} = 0$

$$\boxed{V\sqrt{3} + H = 2P}$$

$\vec{u}_0 = \vec{0}$, $\vec{\omega}_0 = \omega_0 \vec{e}_y$, $\vec{r}_1 = \frac{a\sqrt{3}}{2}\vec{e}_x + \frac{a}{2}\vec{e}_z$

$\vec{u}_1 = \vec{u}_0 + \vec{\omega}_0 \times \vec{r}_1 = \omega_0 \vec{e}_y \times \left(\frac{a\sqrt{3}}{2}\vec{e}_x + \frac{a}{2}\vec{e}_z\right)$

$$\boxed{\vec{u}_1 = \omega_0 \frac{a}{2}\vec{e}_x - \omega_0 \frac{a\sqrt{3}}{2}\vec{e}_z} \rightarrow \boxed{w_1 = -u_1\sqrt{3}}$$

② $l_{12} = \frac{2a}{\sqrt{3}} \rightarrow k_{12} = EA_p \frac{\sqrt{3}}{2a} \rightarrow k_{12} = 577{,}4 \ \frac{kN}{cm}$

$l_{13} = a\sqrt{2} \rightarrow k_{13} = EA_p \frac{1}{a\sqrt{2}} \rightarrow k_{13} = 471{,}4 \ \frac{kN}{cm}$

$[K_{12}] = k_{12}$

	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\sqrt{3}}{2}$	$\frac{3}{4}$	$\frac{\sqrt{3}}{4}$
$\frac{1}{2}$	$\frac{\sqrt{3}}{4}$	$\frac{1}{4}$

$\rightarrow [K_{12}] =$

433,0	250,0
250,0	144,3

$[K_{13}] = k_{13}$

	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

$\rightarrow [K_{13}] =$

235,7	-235,7
-235,7	235,7

$[K_{11}] = -[K_{12}] - [K_{13}] \rightarrow [K_{11}] =$

-668,7	-14,3
-14,3	-380,0

$$\begin{bmatrix} -668{,}7 & -14{,}3 \\ -14{,}3 & -380{,}0 \end{bmatrix} \begin{Bmatrix} u_1 \\ w_1 \end{Bmatrix} = \begin{Bmatrix} -H \\ V \end{Bmatrix}$$

$$u_1 = 10^{-6}\,(1496{,}61\,H + 56{,}31\,V) = 60\,\omega_0$$
$$w_1 = 10^{-6}\,(56{,}31\,H + 2633{,}42\,V) = -60\sqrt{3}\,\omega_0$$

$H = 2P - \sqrt{3}\,V$;

$$\omega_0 = 10^{-6}\,[24{,}9435\,(2P - \sqrt{3}V) + 0{,}9384\,V] =$$
$$= 10^{-6}\,[0{,}5418\,(2P - \sqrt{3}\,V) + 25{,}3401]$$

$$\boxed{V = 0{,}7320\,P \quad , \quad H = 0{,}7320\,P}$$

$$\boxed{\omega_0 = 18{,}9469 \cdot 10^{-6}}$$

$$\boxed{u_1 = 1136{,}812 \cdot 10^{-6}\ \text{cm}}$$
$$\boxed{w_1 = -1969{,}016 \cdot 10^{-6}\ \text{cm}}$$

$$N_{13} = 471{,}4\,[-1136{,}812 \cdot \tfrac{\sqrt{2}}{2} - 1969{,}016\,\tfrac{\sqrt{2}}{2}] \cdot 10^{-6}$$

$$\boxed{N_{13} = -1{,}0353\,P}$$

Kontrola :

$\Sigma M^0 = 0$

$$Pa + N_{13}\,\frac{\sqrt{2}}{2}\left(\frac{a}{2} + a\,\frac{\sqrt{3}}{2}\right) = 0$$

$$N_{13} = -\frac{4}{\sqrt{2}\,(1+\sqrt{3})}\,P$$

$$\boxed{N_{13} = -1{,}0353\,P}$$