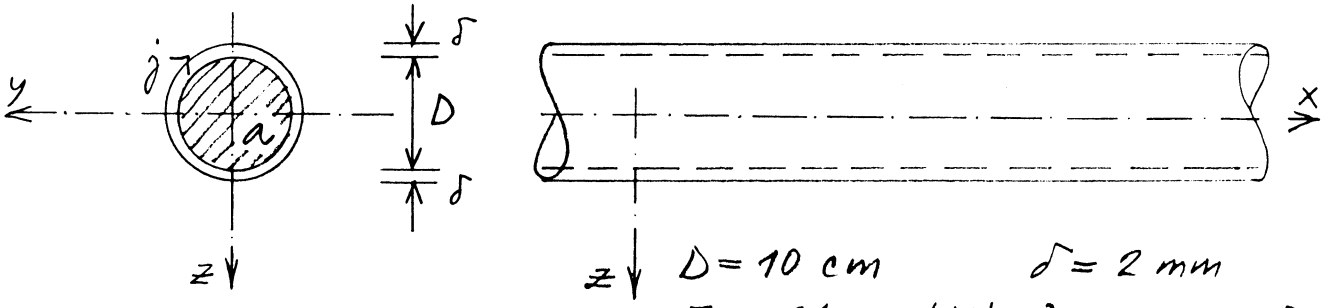
MTT 2. KOLOKVIJ 19. 1. 1999

1. Aluminijasta palica premera 10 cm je tesno, vendar brez napetosti obdana z odprto jekleno cevjo z debelino stene 2 mm. Določi napetosti v palici in cevi, če opisani sestav enakomerno segrejemo za 100 K ! Določi tudi novi premer aluminijastega jedra. Trenje med palico in cevjo je zanemarljivo.



$D = 10$ cm $\quad \delta = 2$ mm

$E_j = 21000\ kN/cm^2, \quad \nu_j = 0,3$

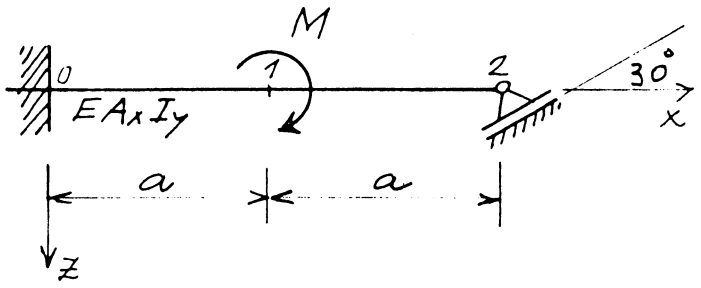
$E_a = 12000\ kN/cm^2, \quad \nu_a = 0,34$

$\alpha_j = 1.2 \cdot 10^{-5}/K$

$\alpha_a = 2,4 \cdot 10^{-5}/K$

40%

2.



Določi vektor pomika točke 2 glede na koordinatni sistem (x, z)!

Določi in skiciraj diagrame notranjih sil!

$E = 20\,000\,\mathrm{kN/cm^2}$

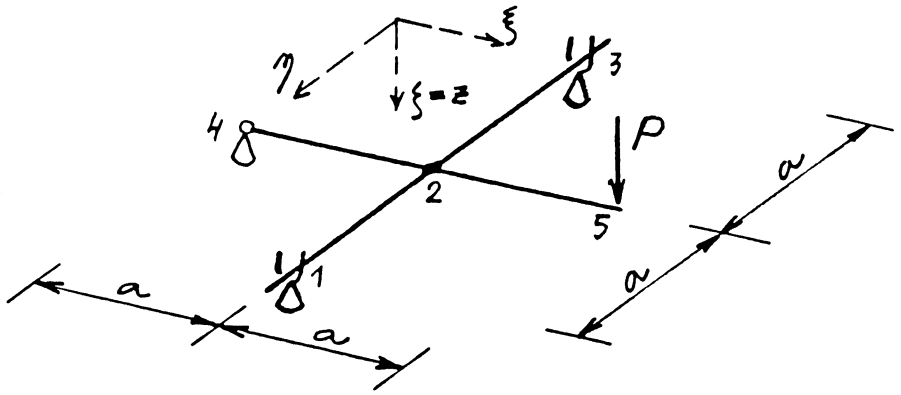
$a = 4\,\mathrm{m}$

$A_x = 40\,\mathrm{cm^2}$

$I_y = 4000\,\mathrm{cm^4}$

40%.

Določi in skiciraj notranje sile! Za koliko se spremeni navpični pomik točke 5, če nosilec $\overline{45}$ v točki 2 ni togo povezan z nosilcem $\overline{13}$, temveč je nanj le prosto položen? V podporah 1 in 3 sta preprečena navpična pomika in torzijska zasuka. $(EI_y = GI_x)$.



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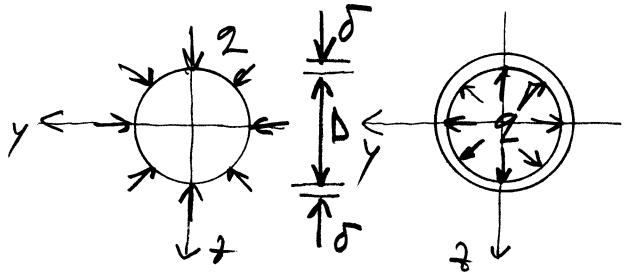
Ad 1.)



Valj: $\sigma_{yy}^a = \sigma_{zz}^a = -q$

Cev: $\sigma_{rr}^j = -q$

$\sigma_{ss}^j = \frac{q\Delta}{2\delta}$

$$\boxed{\varepsilon_{yy}^a = \varepsilon_{ss}^j}$$

$$\boxed{\sigma_{xx}^a = \sigma_{xx}^j = 0}$$

$$\varepsilon_{yy}^a = \frac{1}{E_a}\left(\sigma_{yy}^a - \nu_a \sigma_{zz}^a\right) + \alpha_a \Delta T$$

$$\boxed{\varepsilon_{yy}^a = \frac{-q}{E_a}(1-\nu_a) + \alpha_a \Delta T}$$

$$\varepsilon_{ss}^j = \frac{1}{E_j}\left(\sigma_{ss}^j - \nu_j \sigma_{rr}^j\right) + \alpha_j \Delta T$$

$$\boxed{\varepsilon_{ss}^j = \frac{q}{E_j}\left(\frac{\Delta}{2\delta} + \nu_j\right) + \alpha_j \Delta T}$$

$$\varepsilon_{yy}^a = \varepsilon_{ss}^j \rightarrow \quad -\frac{q}{E_a}(1-\nu_a) + \alpha_a \Delta T = \frac{q}{E_j}\left(\frac{\Delta}{2\delta} + \nu_j\right) + \alpha_j \Delta T$$

$$\boxed{q = \frac{(\alpha_a - \alpha_j) E_a E_j}{E_a\left(\frac{\Delta}{2\delta} + \nu_j\right) + E_j(1-\nu_a)} \Delta T}$$

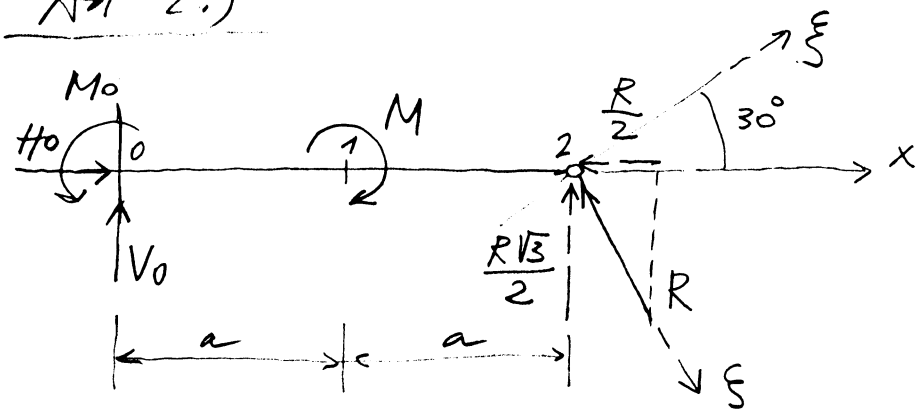
$$q = \frac{(2{,}4 - 1{,}2)\cdot 10^{-5} \cdot 12000 \cdot 21000}{12000\left(\frac{10}{2\cdot 0{,}2} + 0{,}3\right) + 21000(1-0{,}34)} \cdot 100$$

$$\boxed{q = 0{,}953 \ \text{kN/cm}^2}$$

$$\varepsilon_{yy}^a = -\frac{0{,}953}{12000}(1-0{,}34) + 2{,}4\cdot 10^{-5}\cdot 100$$

$$\boxed{\varepsilon_{yy}^a = 0{,}00235} \qquad \Delta' = \Delta(1+\varepsilon_{yy}^a) \rightarrow \boxed{\Delta' = 10{,}0235 \ \text{cm}}$$

Aufg. 2.)



$$H_0 = \frac{R}{2}$$

$$V_0 = -\frac{R\sqrt{3}}{2}$$

$$M_0 = M - Ra\sqrt{3}$$

$$M_y = -M_0 + V_0 x + M\langle x-a\rangle^0 = -M + Ra\sqrt{3} - \frac{R\sqrt{3}}{2}x + M\langle x-a\rangle^0$$

$$EI_y w'' = M\left(1-\langle x-a\rangle^0\right) - R\frac{\sqrt{3}}{2}(2a-x)$$

$$EI_y w' = M\left(x-\langle x-a\rangle\right) - R\frac{\sqrt{3}}{2}\left(2ax - \frac{x^2}{2}\right) + C_1$$

$$EI_y w = \frac{M}{2}\left(x^2 - \langle x-a\rangle^2\right) - R\frac{\sqrt{3}}{2}\left(ax^2 - \frac{x^3}{6}\right) + C_1 x + C_2$$

$x = 0 \ldots w = 0,\ w' = 0 \rightarrow C_1 = C_2 = 0$

$x = 2a:$ $\quad w = \frac{M}{2EI_y}\left(4a^2 - a^2\right) - \frac{R\sqrt{3}}{2EI_y}\left(4a^3 - \frac{8a^3}{6}\right)$

$$u_B(2) = w_2 = M\frac{3a^2}{2EI_y} - R\frac{4\sqrt{3}a^3}{3EI_y}$$

$$u_x(2) - u_2 = -R\frac{a}{EA_x}$$

Vrch. 2: $\quad u_\xi = u_2 e_{\xi x} + w_2 e_{\xi z} = 0$

$e_{\xi x} = \frac{1}{2}$; $\quad e_{\xi z} = \frac{\sqrt{3}}{2}$

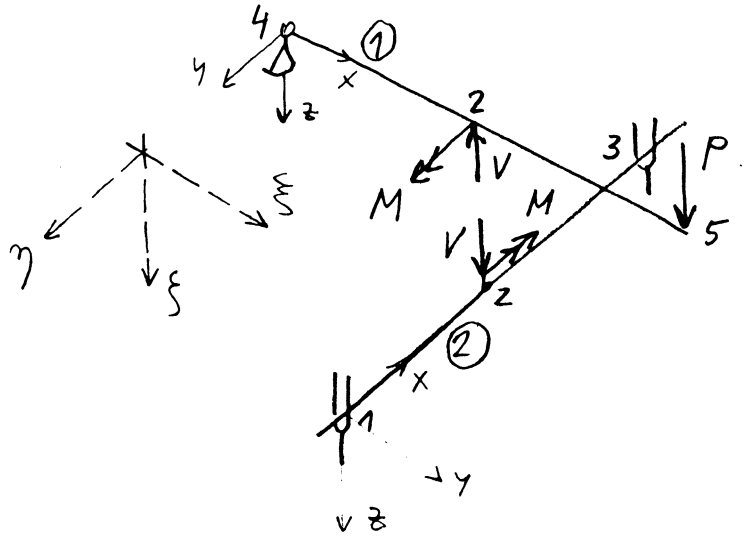
$$-R\frac{a}{EA_x}\cdot\frac{1}{2} + M\frac{3a^2}{2EI_y}\cdot\frac{\sqrt{3}}{2} - R\frac{4\sqrt{3}a^3}{3EI_y}\cdot\frac{\sqrt{3}}{1} = 0$$

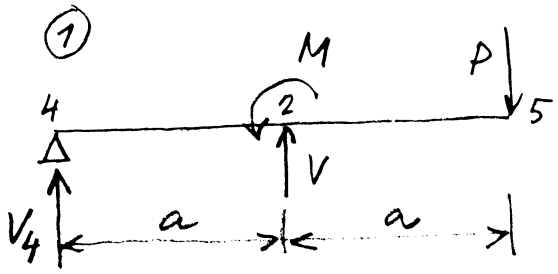
$$R = M\frac{3\sqrt{3}\,aA_x}{2(I_y + 4a^2A_x)} \rightarrow R = 0{,}001624\,M$$

$u_2 = -0{,}812\,M\cdot 10^{-6}$; $\quad w_2 = 0{,}469\,M\cdot 10^{-6}$

$$\vec{u}_2 = 10^{-6}M\left(-0{,}812\,\vec{e}_x + 0{,}469\,\vec{e}_z\right)$$

Ad 3.)

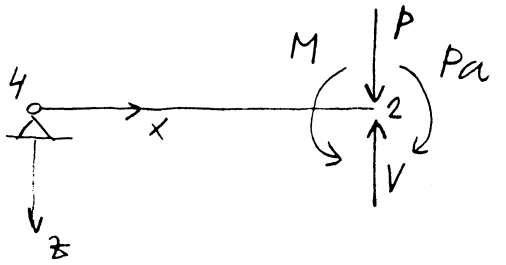


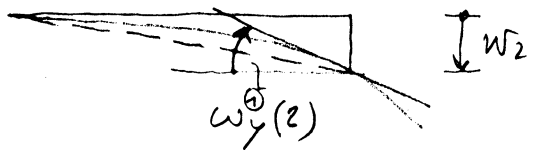


$$V_4 = P - V$$

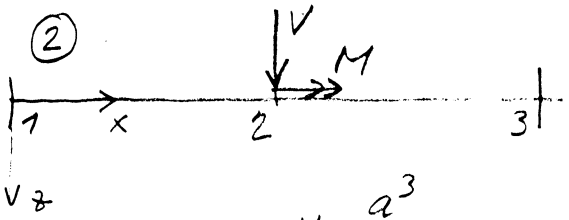
$$M = a(2P - V)$$

①





②



$$w_2 = V \frac{a^3}{6EI_y}$$

$$\omega_x^{①}(2) = M \frac{a}{2GI_x}$$

$$\boxed{EI_y = GI_x = 1}$$

$$\omega_y^{①}(2) = -\frac{w_2}{a} + (M - Pa)\frac{a}{3EI_y}$$

$$\omega_y^{①}(2) = -V\frac{a^2}{6EI_y} + M\frac{a}{3EI_y} - P\frac{a^2}{3EI_y}$$

$$\boxed{\omega_y^{①}(2) = -\omega_x^{②}(2)}$$

$$-V\frac{a}{6} + M\cdot\frac{1}{3} - P\frac{a}{3} = -M\cdot\frac{1}{2} \longrightarrow M\cdot\frac{5}{6} - V\frac{a}{6} = P\frac{a}{3}$$

$$\left.\begin{aligned} 5M - Va &= 2Pa \\ \text{R.P.:}\quad M + Va &= 2Pa \end{aligned}\right\} \quad 6M = 4Pa \longrightarrow \boxed{M = P\frac{2a}{3}}$$

$$\boxed{V = P\cdot\frac{4}{3}}$$

$$w_5 = w_2 - a\,\omega_y^{①}(2) + P\frac{a^3}{3EI_y}$$

$$w_5 = P\cdot\frac{4}{3}\cdot\frac{a^3}{6EI_y} + P\cdot\frac{2a}{3}\cdot\frac{a}{2GI_x}\cdot a + P\frac{a^3}{3EI_y}$$

$$w_5 = \frac{Pa^3}{EI_y}\left(\frac{4}{18} + \frac{2}{6} + \frac{1}{3}\right) \longrightarrow \boxed{w_5 = P\frac{8a^3}{9EI_y}}$$

Probe Alk: $\boxed{M = 0} \longrightarrow \boxed{V = 2P}$

$$\omega_y^{①}(2) = -2P\frac{a^2}{6EI_y} - P\frac{a^2}{3EI_y} = -P\frac{2a^2}{3EI_y}$$

$$w_5 = 2P\frac{a^3}{6EI_y} + P\frac{2a^3}{3EI_y} + P\frac{a^3}{3EI_y} \longrightarrow \boxed{w_5 = P\frac{4a^3}{3EI_y}}$$