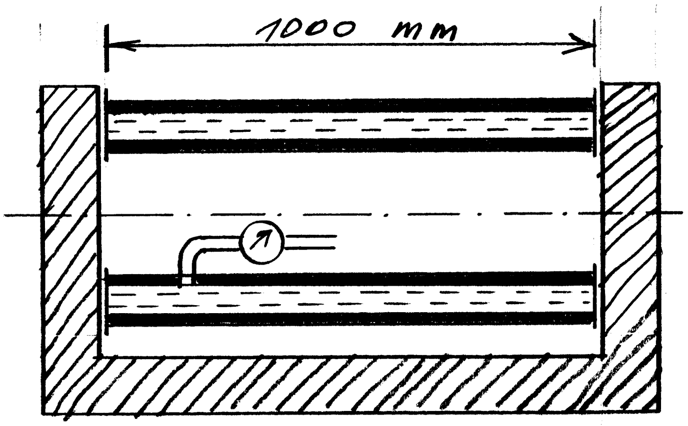
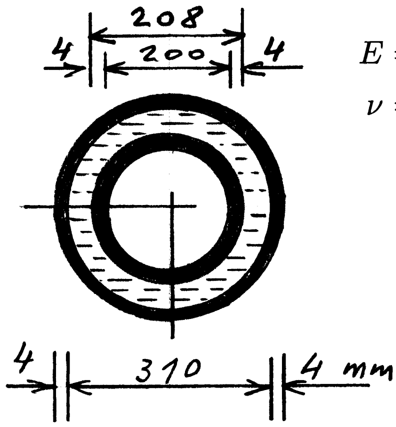
1. Napetostno stanje telesa je opisano s komponentami tenzorja napetosti v koordinatnem sistemu (x, y, z). V notranjosti telesa si zamislimo sferično ploskev (kroglo) s središčem $S(2, 2, 0)$ in polmerom $r = 6\,\mathrm{cm}$. V točki $T(6, 4, z > 0)$, ki leži na površju opisane krogle, določi:
   a. Rezultirajoči vektor napetosti , ki pripada tangencialni ravnini krogle v točki T, njegovo normalno in strižno komponento ter enotski vektor $\mathbf{e}_t$ smeri strižne komponente!
   b. Specifično spremembo dolžine normale ter spremembo pravega kota med normalo in smerjo $\mathbf{e}_t$!
   c. Specifično prostorninsko obtežbo $\boldsymbol{v}$, pri kateri je telo v ravnotežju!

$$[\sigma_{ij}] = \begin{bmatrix} 2xy & x & 0 \\ x & 0 & yz \\ 0 & yz & 2y^2 \end{bmatrix} \qquad \begin{aligned} E &= 20\,000\,\mathrm{kN/cm^2} \\ \nu &= 0.25 \end{aligned}$$

(35 točk)

2. V vmesni prostor med dvema bakrenima cevema z debelino stene $\delta = 4\,\mathrm{mm}$ načrpamo nestisljivo hladilno tekočino. Koliko tekočine porabimo, da znaša hidrostatični tlak $p = 6\,\mathrm{MPa}$? Kolikšne so tedaj normalne napetosti v tangencialni smeri v obeh ceveh? (Tesnila ob nepodajnih priključnih ploščah omogočajo neovirano deformiranje cevi. Vzdolžne normalne napetosti so zanemarljive.)



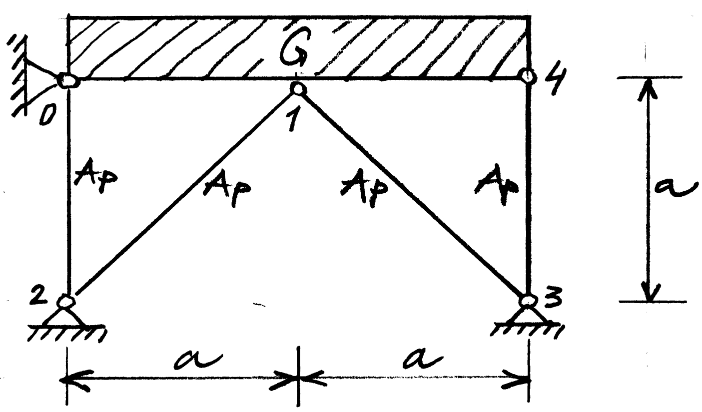


$E = 100\,000\,\mathrm{MPa}$

$\nu = 0.3$

(35 točk)

3. Absolutno toga homogena greda teže G je podprta, kot kaže skica. Določi osne sile v podpornih palicah! Za koliko moramo spremeniti temperaturo palice $\overline{34}$, da v palicah $\overline{12}$ in $\overline{13}$ ne bo napetosti?



$E = 200\,000\,\mathrm{MPa}$

$\alpha_T = 1.25 \cdot 10^{-5}/\mathrm{K}$

$a = 2\,\mathrm{m}$

$A_p = 40\,\mathrm{cm^2}$

$G = 0.4\,\mathrm{MN}$

(35 točk)

MTT IZPIT 21. 12. 99

Ad 1.) $r^2 = (6-2)^2 + (4-2)^2 + z^2 = 6^2$

$z = \sqrt{36-16-4} = 4$ cm $\rightarrow \boxed{T(6,4,4)}$

a)

$$[\sigma_{ij}]_T = \begin{bmatrix} 48 & 6 & 0 \\ 6 & 0 & 16 \\ 0 & 16 & 32 \end{bmatrix}$$

$$\vec{n} = (6-2)\vec{e}_x + (4-2)\vec{e}_y + (4-0)\vec{e}_z = 4\vec{e}_x + 2\vec{e}_y + 4\vec{e}_z$$

$$\vec{e}_n = \frac{\vec{n}}{n} \rightarrow \boxed{\vec{e}_n = \frac{1}{3}(2\vec{e}_x + \vec{e}_y + 2\vec{e}_z)}$$

$$\begin{Bmatrix} \sigma_{nx} \\ \sigma_{ny} \\ \sigma_{nz} \end{Bmatrix} = \frac{1}{3}\begin{bmatrix} 48 & 6 & 0 \\ 6 & 0 & 16 \\ 0 & 16 & 32 \end{bmatrix} \cdot \begin{Bmatrix} 2 \\ 1 \\ 2 \end{Bmatrix} = \frac{1}{3}\begin{Bmatrix} 102 \\ 44 \\ 80 \end{Bmatrix} = \begin{Bmatrix} 34 \\ 14{,}67 \\ 26{,}67 \end{Bmatrix}$$

$$\vec{\sigma}_n = 34\,\vec{e}_x + 14{,}67\,\vec{e}_y + 26{,}67\,\vec{e}_z$$

$$\sigma_{nn} = \vec{\sigma}_n \cdot \vec{e}_n = \boxed{45{,}33 \ \mathrm{kN/cm^2}}$$

$$\sigma_{nt}\,\vec{e}_t = \vec{\sigma}_n - \sigma_{nn}\,\vec{e}_n = \frac{1}{3}(202\,\vec{e}_x + 44\,\vec{e}_y + 80\,\vec{e}_z) -$$

$$- \frac{408}{9} \cdot \frac{1}{3}(2\vec{e}_x + \vec{e}_y + 2\vec{e}_z)$$

$$\sigma_{nt}\,\vec{e}_t = (34 - 30{,}22)\vec{e}_x + (14{,}67 - 15{,}11)\vec{e}_y + (26{,}67 - 30{,}22)\vec{e}_z$$

$$\sigma_{nt}\,\vec{e}_t = 3{,}78\,\vec{e}_x - 0{,}44\,\vec{e}_y - 3{,}56\,\vec{e}_z$$

$$\sigma_{nt} = \sqrt{3{,}78^2 + 0{,}44^2 + 3{,}56^2} \rightarrow \boxed{\sigma_{nt} = 5{,}207 \ \mathrm{kN/cm^2}}$$

$$\vec{e}_t = \frac{\sigma_{nt}\,\vec{e}_t}{\sigma_{nt}} \rightarrow \boxed{\vec{e}_t = 0{,}726\,\vec{e}_x - 0{,}085\,\vec{e}_y - 0{,}683\,\vec{e}_z}$$

Kontrola: $\vec{e}_n \cdot \vec{e}_t = 0$ ✓

$$\sigma_{nt} = \sqrt{\vec{\sigma}_n \cdot \vec{\sigma}_n - \sigma_{nn}^2} = 5{,}207 \ \mathrm{kN/cm^2}$$ ✓

$$\boxed{\vec{e}_t = \frac{1}{\sigma_{nt}}(\vec{\sigma}_n - \sigma_{nn}\,\vec{e}_n)}$$

b)

$I_1^\sigma = 48 + 32 = 80 \ kN/cm^2$

$$\varepsilon_{nn} = \frac{1+\nu}{E}\sigma_{nn} - \frac{\nu}{E} I_1^\sigma = \frac{1}{20000}(1{,}25 \cdot 45{,}33 - 0{,}25 \cdot 80)$$

$$\boxed{\varepsilon_{nn} = 0{,}00183}$$

$$\varepsilon_{nt} = \frac{1+\nu}{E}\sigma_{nt} = \frac{1{,}25}{20\,000} \cdot 5{,}207 \rightarrow \boxed{\varepsilon_{nt} = 0{,}00033}$$

$$D_{nt} \cong 2\varepsilon_{nt} \longrightarrow \boxed{D_{nt} = 0{,}00065}$$

c)

$$\begin{aligned} 2y + v_x &= 0 &\rightarrow\ & v_x = -2y \\ 1+y + v_y &= 0 &\rightarrow\ & v_y = -(1+y) \\ z + v_z &= 0 &\rightarrow\ & v_z = -z \end{aligned}$$

$$T: \boxed{\vec{v} = -8\,\vec{e}_x - 5\,\vec{e}_y - 4\,\vec{e}_z}$$

Ad 2) $D_1 = 208$ mm , $D_2 = 310$ mm

$$V = \frac{\pi l}{4}(D_2^2 - D_1^2) \quad \ldots \quad V' = \frac{\pi l}{4}(D_2'^2 - D_1'^2)$$

Notranja cev : $\sigma_{ss}^1 = -\frac{p D_1}{2\delta}$, $\sigma_{rr}^1 = -p$

$$\varepsilon_{ss}^1 = \frac{1}{E}(\sigma_{ss}^1 - \nu\sigma_{rr}^1) = -\frac{p}{E}\left(\frac{D_1}{2\delta} - \nu\right)$$

$$\varepsilon_{ss}^1 = -\frac{p}{E}\left(\frac{D_1}{2\delta} - \nu\right) \longrightarrow \boxed{\varepsilon_{ss}^1 = -0{,}00154}$$

Zunanja cev : $\sigma_{ss}^2 = \frac{p D_2}{2\delta}$, $\sigma_{rr}^2 = -p$

$$\varepsilon_{ss}^2 = \frac{1}{E}(\sigma_{ss}^2 - \nu\sigma_{rr}^2) = \frac{p}{E}\left(\frac{D_2}{2\delta} - \nu\right)$$

$$\boxed{\varepsilon_{ss}^2 = 0{,}00231}$$

$$D_1' = D_1(1+\varepsilon_{ss}^1) = 208 \cdot (1-0{,}00154)$$

$$\underline{D_1' = 207{,}679 \text{ mm}}$$

$$D_2' = D_2(1+\varepsilon_{ss}^2) = 310 \cdot (1+0{,}0023)$$

$$\underline{D_2' = 310{,}715 \text{ mm}}$$

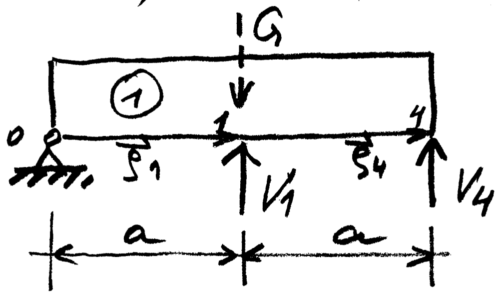
$$V' = \frac{\pi \cdot 1000}{4}\left(310{,}715^2 - 207{,}679^2\right) = 41.950.661 \text{ mm}^3$$

$$\boxed{V' = 41{,}951 \text{ litrov}}$$

$$\sigma_{ss}^n = -\frac{6 \cdot 208}{2 \cdot 4} \rightarrow \boxed{\sigma_{ss}^n = -156 \text{ MPa}}$$

$$\sigma_{ss}^z = \frac{6 \cdot 310}{2 \cdot 4} \rightarrow \boxed{\sigma_{ss}^z = 232{,}5 \text{ MPa}}$$

Ad 3.)



$$\Sigma M^0 = 0 \rightarrow V_1 a + V_4 \cdot 2a - Ga = 0$$

$$\boxed{V_1 + 2V_4 = G}$$

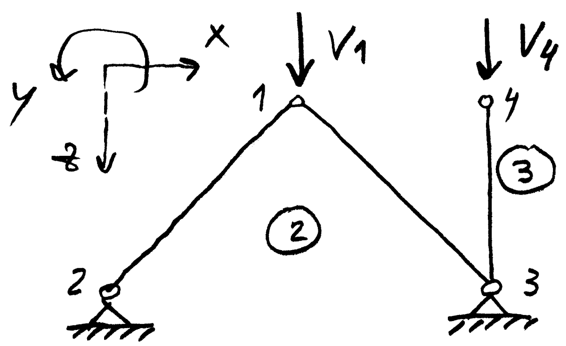
$$\vec{u}_0 = \vec{0}, \quad \vec{\omega}_0 = \omega_y \vec{e}_y$$

$$\vec{\rho}_1 = a\vec{e}_x, \quad \vec{\rho}_4 = 2a\,\vec{e}_x$$

$$\vec{u}_1 = \vec{u}_0 + \vec{\omega}_0 \times \vec{\rho}_1 = \omega \vec{e}_y \times a\vec{e}_x \rightarrow \vec{u}_1 = -a\omega_y \vec{e}_z$$

$$\vec{u}_4 = -2a\omega_y \vec{e}_z$$

$w_1^{(1)} = -a\omega_y$	$w_4^{(1)} = -2a\omega_y$



$$\boxed{w_4^{(3)} = V_4 \frac{a}{EA_p}}$$

$$w_4^{(3)} = \frac{V_4}{4000} = 0{,}00025\, V_4$$

② : $l_{12} = l_{13} = a\sqrt{2}$

$$k_{12} = k_{13} = \frac{E A_D}{a\sqrt{2}} = \frac{20000 \cdot 40}{200\sqrt{2}} = 2828{,}4\ kN/cm$$

$$[K_{12}] = 2828{,}4 \begin{array}{c|cc} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \hline -\frac{\sqrt{2}}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{1}{2} \end{array} \rightarrow [K_{12}] = \begin{bmatrix} 1414.2 & -1414.2 \\ -1414.2 & 1414.2 \end{bmatrix}$$

$$[K_{11}] = \begin{bmatrix} -2828{,}4 & 0 \\ 0 & -2828{,}4 \end{bmatrix} \qquad [K_{13}] = \begin{bmatrix} 1414.2 & 1414.2 \\ 1414.2 & 1414.2 \end{bmatrix}$$

$$[K_{11}] \begin{Bmatrix} u_1 \\ w_1 \end{Bmatrix} + \begin{Bmatrix} 0 \\ V_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} -2828{,}4 & 0 \\ 0 & -2828{,}4 \end{bmatrix} \begin{Bmatrix} u_1 \\ w_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -V_1 \end{Bmatrix} \rightarrow u_1^{②} = 0, \quad \boxed{w_1^{②} = \frac{V_1}{2828{,}4}}$$

$$w_1^{①} = w_1^{②} \rightarrow -200\ \omega_y = \frac{V_1}{2828{,}4} = 0{,}0003536\ V_1$$

$$w_4^{①} = w_4^{③} \rightarrow -400\ \omega_y = 0{,}00025\ V_4$$

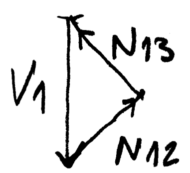
$$\omega_y = -\frac{V_1}{200} \cdot 0{,}0003536 = -\frac{V_4}{400} \cdot 0{,}00025$$

$$1{,}7678\ V_1 = 0{,}6250\ V_4$$

$$V_1 = 0{,}3536\ V_4 = G - 2V_4 \rightarrow 2{,}3536\ V_4 = G$$

$$\boxed{V_4 = 0{,}4249\ G} \rightarrow \boxed{V_4 = 0{,}1700\ MN}$$

$$V_1 = G - 2V_4 \rightarrow \boxed{V_1 = 0{,}1502\ G} \rightarrow \boxed{V_1 = 0{,}0600\ MN}$$



$$N_{12} = N_{13} = -\frac{V_1}{\sqrt{2}} = \underline{\underline{-0{,}0424\ MN}}$$

$$\boxed{\Delta l_{34} = 0} \; : \qquad \Delta l_{34} = -\tilde{V}_4 \frac{a}{EA_p} + \alpha_T \Delta T a$$

$$\tilde{V}_4 = \frac{G}{2} \quad \rightarrow \quad G \frac{a}{2EA_p} = \alpha_T \Delta T a$$

$$\boxed{\Delta T = \frac{G}{2EA_p \alpha_T}} \quad \rightarrow \quad \Delta T = \frac{400 \cdot 10^5}{2 \cdot 20000 \cdot 40 \cdot 1{,}25}$$

$$\boxed{\Delta T = 20 \;\; K}$$