

h) Upogibnica enkrat statično nedoločena nosilca obteženega z momentom  $M$

Pomik in zasuk v polju 1:

$$w_1 = \frac{Mx^2}{4L^3EI_y} (a^3 + 3a^2b - 2b^3 + (a-2L)ax),$$

$$\omega_{y1} = -\frac{Mx}{4L^3EI_y} (2(a^3 + 3a^2b - 2b^3) + 3(a-2L)ax)$$

Pomik in zasuk v polju 2:

$$w_2 = \frac{Ma}{4L^3EI_y} (2aL^3 - 4L^3x - 3L(a-2L)x^2 + (a-2L)x^3),$$

$$\omega_{y2} = -\frac{Ma}{4L^3EI_y} (4L^3 + 6L(a-2L)x - 3(a-2L)x^2).$$

Pomik in zasuk pri  $x = a$  ter zasuk pri  $x = L$ :

$$w(a) = \frac{Ma^2b}{4L^3EI_y} (a^2 - 2b^2), \quad \omega_y(L) = -\frac{Ma}{4LEI_y} (2b - a).$$

$$\omega_y(a) = \frac{Ma}{4L^3EI_y} (a^3 + 4b^3), \quad \omega_y(L) = -\frac{Ma}{4LEI_y} (2b - a).$$

Pri  $a_{mej} = 0.582925L$  je  $|w_{1,ekst}| = |w_{2,ekst}|$ .

Ekstremni pomik, če je  $a > a_{mej}$ :

$$x_{ekst} = \frac{2L(2b^2 - 2ab - a^2)}{3(a-2L)}, \quad w_{ekst} = \frac{M}{27a^2(a+2b)^2EI_y} (a^2 + 2ab - 2b^2)^3.$$

Ekstremni pomik, če je  $a < a_{mej}$ :

$$x_{ekst} = L \left( 1 - \sqrt{\frac{2b-a}{3(2b+a)}} \right), \quad w_{ekst} = -\frac{Ma}{18LEI_y} \sqrt{\frac{3(2b-a)^3}{2b+a}}.$$

g) Upogibnica enkrat statično nedoločena nosilca obteženega s silo  $F$

Pomik in zasuk v polju 1:

$$w_1 = \frac{Fbx^2}{12L^3EI_y} (3aL(L+b) - x(3L^2 - b^2)),$$

$$\omega_{y1} = -\frac{Fbx}{4L^3EI_y} (2aL(L+b) - x(3L^2 - b^2)).$$

Pomik in zasuk v polju 2:

$$w_2 = \frac{F}{12L^3EI_y} (3abLx^2(L+b) - bx^3(3L^2 - b^2) + 2L^3(x-a)^3),$$

$$\omega_{y2} = -\frac{F}{4L^3EI_y} (2abLx(L+b) - bx^3(3L^2 - b^2) + 2L^3(x-a)^2).$$

Pomik pri  $x = a$  ter zasuk pri  $x = L$ :

$$w(a) = \frac{Fa^3b^2}{12L^3EI_y} (3L+b),$$

$$\omega_y(a) = \frac{Fa^2b}{4L^3EI_y} (a^2 - 2b^2), \quad \omega_y(L) = \frac{Fa^2b}{4LEI_y}.$$

Največji pomik, če je  $a > (2 - \sqrt{2})L = 0.5858L$ :

$$x_{ekst} = \frac{3aL(L+b)}{3L^2 - b^2}, \quad w_{ekst} = \frac{Fa^3b}{3EI_y} (3L^2 - b^2)^2.$$

Največji pomik, če je  $a < (2 - \sqrt{2})L$ :

$$x_{ekst} = L \left( 1 - \sqrt{\frac{b}{b+2L}} \right), \quad w_{ekst} = \frac{Fa^2b}{6EI_y} \sqrt{\frac{b}{b+2L}}.$$

e) Upogibnica prostoležečega nosilca obteženega z momentom  $M$

Pomik in zasuk v polju 1:

$$w_1 = \frac{Mx}{6LEI_y} (L^2 - 3b^2 - x^2), \quad \omega_{y1} = -\frac{M}{6LEI_y} (L^2 - 3b^2 - 3x^2).$$

Pomik in zasuk v polju 2:

$$w_2 = \frac{M}{6LEI_y} (x(L^2 - 3b^2 - x^2) + 3L(x-a)^2),$$

$$\omega_{y2} = -\frac{M}{6LEI_y} (L^2 - 3b^2 - 3x^2 + 6L(x-a)).$$

Pomik pri  $x = a$  ter zasuki pri  $x = 0$ ,  $x = a$  in  $x = L$ :

$$w(a) = \frac{Mab}{3LEI_y} (a-b), \quad \omega_y(0) = -\frac{M}{6LEI_y} (L^2 - 3b^2),$$

$$\omega_y(a) = \frac{M}{3LEI_y} (a^2 - ab + b^2), \quad \omega_y(L) = -\frac{M}{6LEI_y} (L^2 - 3a^2).$$

Če je  $a = L\sqrt{3}/3$ , potem je  $\omega_y(L) = 0$ .

Ekstremni pomik, če je  $a > b$ :

$$x_{ekst} = \sqrt{\frac{L^2 - 3b^2}{3}}, \quad w_{ekst} = \frac{M}{3LEI_y} \sqrt{\left(\frac{L^2 - 3b^2}{3}\right)^3}.$$

Ekstremni pomik, če je  $a < b$ :

$$x_{ekst} = L \left( 1 - \sqrt{\frac{L^2 - 3a^2}{3L^2}} \right), \quad w_{ekst} = -\frac{M}{3LEI_y} \sqrt{\left(\frac{L^2 - 3a^2}{3}\right)^3}.$$

Upogibnica prostoležečega nosilca obteženega s silo  $F$

Pomik in zasuk v prvem polju:

$$w_1 = \frac{1}{6} \frac{Fbx}{LEI_y} (-x^2 + L^2 - b^2), \quad \omega_{y1} = -\frac{1}{6} \frac{Fb}{LEI_y} (-3x^2 + L^2 - b^2)$$

Pomik in zasuk v drugem polju:

$$w_2 = \frac{1}{6} \frac{F(-x+L)a}{LEI_y} (L^2 - 2Lb - 2Lx + x^2 + b^2)$$

$$\omega_{y2} = -\frac{1}{6} \frac{Fa}{LEI_y} (3L^2 - 6Lx - 2Lb + 3x^2 + b^2)$$

Pomik pri in zasuk pri  $x = a$ :

$$w(a) = \frac{1}{3} \frac{Fb^2a^2}{LEI_y}, \quad \omega_y(a) = \frac{1}{3} \frac{bFa(L-2b)}{LEI_y}$$

Zasuka pri  $x = 0$  in  $x = L$ :

$$\omega_y(0) = -\frac{1}{6} \frac{bFa(L+b)}{LEI_y}, \quad \omega_y(L) = \frac{1}{6} \frac{Fb(2L-b)a}{LEI_y}$$

f) Upogibnica prostoležečega nosilca obteženega s konstantno linijsko obtežbo  $\mathcal{P}_z$

Pomik in zasuk:

$$w = \frac{\mathcal{P}_z x}{24EI_y} (L^3 - 2Lx^2 + x^3), \quad \omega_y = -\frac{\mathcal{P}_z}{24EI_y} (L^3 - 6Lx^2 + 4x^3).$$

Največji pomik ter zasuka pri  $x = 0$  in  $x = L$ :

$$x_{ekst} = \frac{L}{2}, \quad w_{ekst} = \frac{5\mathcal{P}_z L^4}{384EI_y}, \quad \omega_y(0) = -\frac{\mathcal{P}_z L^3}{24EI_y}, \quad \omega_y(L) = \frac{\mathcal{P}_z L^3}{24EI_y}.$$