1. Homework in Nonlinear Mechanics, 11. 10. 2013

Deadline, 25. 10. 2013

`VSi` is i–th digit of **your** registration number. For registration number 26102734 are `VS6=7`, `VS8=4`.

TASK 1: The figure shows two six-node triangular finite elements. Consider both the left and the right triangular six-node finite elements. Assume that element dimensions are $a = (\texttt{VS7} + 20)\,\text{mm}$ and $h = (\texttt{VS8} + 20)\,\text{mm}$.

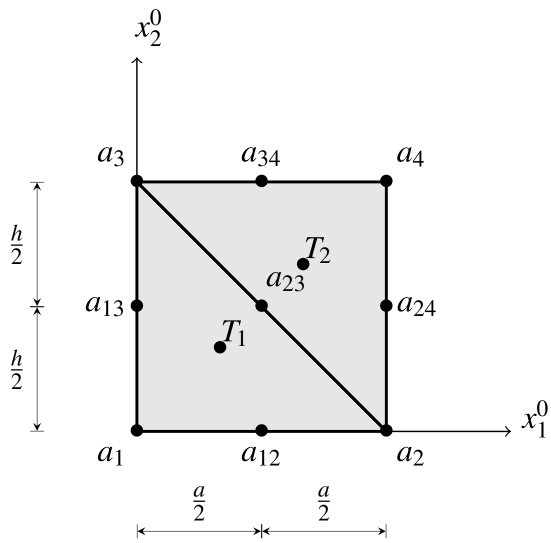

Figure 1: The left and the right triangular six-node finite elements

Displacement field $\vec{u}(x_1^0, x_2^0) = u(x_1^0, x_2^0)\vec{e}_1 + v(x_1^0, x_2^0)\vec{e}_2$ within each triangular finite element is given in the material coordinates by

$$u(x_1^0, x_2^0) = c_1 + c_2 x_1^0 + c_3 x_2^0 + c_4 x_1^0 x_2^0 + c_5 \left(x_1^0\right)^2 + c_6 \left(x_2^0\right)^2,$$
$$v(x_1^0, x_2^0) = b_1 + b_2 x_1^0 + b_3 x_2^0 + b_4 x_1^0 x_2^0 + b_5 \left(x_1^0\right)^2 + b_6 \left(x_2^0\right)^2.$$

The displacements of the nodes a_1, a_2, ..., a_{34} are known:

$$\vec{u}(a_1) = (0\vec{e}_1 + 0\vec{e}_2)\,\text{mm}, \quad \vec{u}(a_{12}) = (0\vec{e}_1 + 0\vec{e}_2)\,\text{mm}, \quad \vec{u}(a_2) = (0\vec{e}_1 + 0\vec{e}_2)\,\text{mm},$$
$$\vec{u}(a_{13}) = (0\vec{e}_1 + 0\vec{e}_2)\,\text{mm}, \quad \vec{u}(a_{23}) = (2\vec{e}_1 + 1\vec{e}_2)\,\text{mm}, \quad \vec{u}(a_{24}) = (3\vec{e}_1 + 1.5\vec{e}_2)\,\text{mm},$$
$$\vec{u}(a_3) = (0\vec{e}_1 + 0\vec{e}_2)\,\text{mm}, \quad \vec{u}(a_{34}) = (3\vec{e}_1 + 1.5\vec{e}_2)\,\text{mm}, \quad \vec{u}(a_4) = (4\vec{e}_1 + 2\vec{e}_2)\,\text{mm}.$$

Determine:

1. and draw the deformed state;
2. constants c_1, c_2, c_3, c_4, c_5, c_6, b_1, b_2, b_3, b_4, b_5 and b_6, such that the displacements of the nodes a_1, a_2, ..., a_{34} will be equal to the prescribed values;
3. the displacement vector of particle $T_1(x_1^0 = \frac{a}{3}, x_2^0 = \frac{h}{3})$;
4. express the spatial coordinates with the material coordinates;
5. deformation gradient F at particle T_1;

6. the inverse of the deformation gradient F at particle T_1 using two different methods;
7. deformed base vectors $\vec{g}_1$, $\vec{g}_2$ and $\vec{g}_3$ at particle T_1;
8. tensor of small deformations ε at particle T_1;
9. polar decomposition RU of the deformation gradient F at particle T_1.

TASK 2: We introduce the linear functions λ_1, λ_2 and λ_3 of material coordinates (x_1^0, x_2^0), where we assume that the function λ_i is at the i-th node of triangle a_i equal 1 and at the remaining two corners it is equal 0.

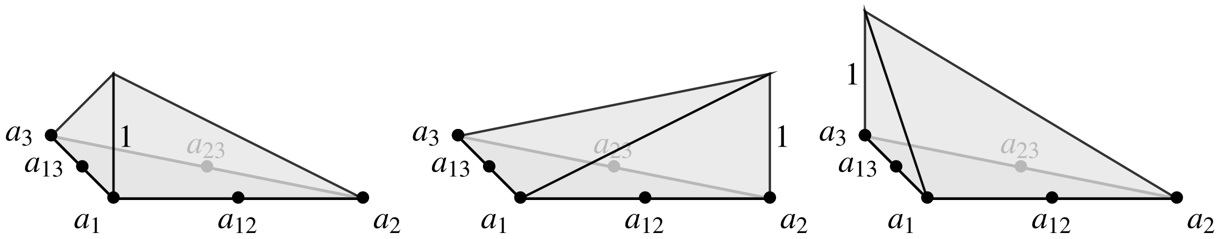

Figure 2: Linear functions λ_1, λ_2 and λ_3 on the lower triangle.

The displacement fields u and v can be described with functions

$$u = \sum_i \lambda_i (2\lambda_i - 1)\, u(a_i) + 4 \sum_{i<j} \lambda_i \lambda_j\, u(a_{ij}),$$
$$v = \sum_i \lambda_i (2\lambda_i - 1)\, v(a_i) + 4 \sum_{i<j} \lambda_i \lambda_j\, v(a_{ij}).$$

1. Sketch the deformed state. Assume the same nodal displacements as in the first task.
2. Are the obtained displacement fields u and v the same as in the first task?
3. Are the resulting interpolation displacement functions u and v continuous along the edge a_2–a_3?
4. Determine the deformation gradient at node a_{23} using the displacement interpolation functions on the left triangle and using the displacement interpolation functions on the right triangle and compare the results. Are the resulting interpolation functions of displacements u and v continuously differentiable along the edge a_2-a_3?