2. Homework in Nonlinear Mechanics, 25. 10. 2013

Deadline, 8. 11. 2013

`VSi` is i–th digit of **your** registration number. For registration number 26102734 are `VS6=7`, `VS8=4`.

TASK 1: The figure shows two six-node triangular finite elements. Consider both the left and the right triangular six-node finite elements. ssume that element dimensions are $a = (\texttt{VS7} + 20)\,\mathrm{mm}$ and $h = (\texttt{VS8} + 20)\,\mathrm{mm}$.

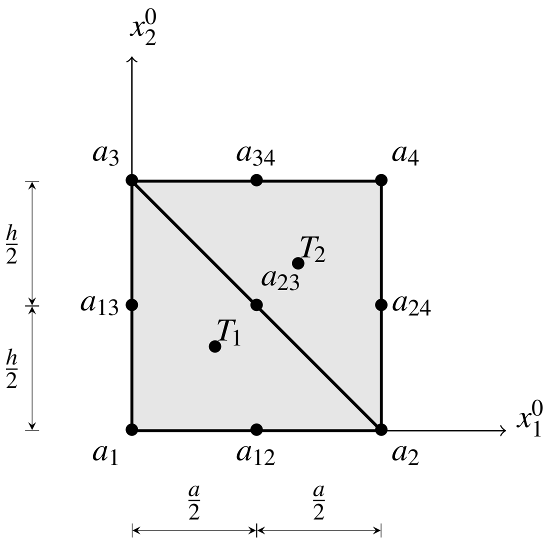

Figure 1: The left and the right triangular six-node finite elements

Displacement field $\vec{u}(x_1^0, x_2^0) = u(x_1^0, x_2^0)\vec{e}_1 + v(x_1^0, x_2^0)\vec{e}_2$ within each triangular finite element is given in the material coordinates by

$$u(x_1^0, x_2^0) = c_1 + c_2 x_1^0 + c_3 x_2^0 + c_4 x_1^0 x_2^0 + c_5 \left(x_1^0\right)^2 + c_6 \left(x_2^0\right)^2,$$
$$v(x_1^0, x_2^0) = b_1 + b_2 x_1^0 + b_3 x_2^0 + b_4 x_1^0 x_2^0 + b_5 \left(x_1^0\right)^2 + b_6 \left(x_2^0\right)^2.$$

The displacements of the nodes $a_1, a_2, \ldots, a_{34}$ are known:

$$\vec{u}(a_1) = (0\vec{e}_1 + 0\vec{e}_2)\,\mathrm{mm}, \quad \vec{u}(a_{12}) = (0\vec{e}_1 + 0\vec{e}_2)\,\mathrm{mm}, \quad \vec{u}(a_2) = (0\vec{e}_1 + 0\vec{e}_2)\,\mathrm{mm},$$
$$\vec{u}(a_{13}) = (0\vec{e}_1 + 0\vec{e}_2)\,\mathrm{mm}, \quad \vec{u}(a_{23}) = (2\vec{e}_1 + 1\vec{e}_2)\,\mathrm{mm}, \quad \vec{u}(a_{24}) = (3\vec{e}_1 + 1.5\vec{e}_2)\,\mathrm{mm},$$
$$\vec{u}(a_3) = (0\vec{e}_1 + 0\vec{e}_2)\,\mathrm{mm}, \quad \vec{u}(a_{34}) = (3\vec{e}_1 + 1.5\vec{e}_2)\,\mathrm{mm}, \quad \vec{u}(a_4) = (4\vec{e}_1 + 2\vec{e}_2)\,\mathrm{mm}.$$

Determine:

1. polar decomposition RU of deformation gradient F at point $T_1(x_1^0 = \frac{a}{3}, x_2^0 = \frac{h}{3}, x_3^0 = 0\,\mathrm{cm})$;
2. polar decomposition VR of deformation gradient F at point T_1;
3. unit vectors, whose directions are keep unchanged during the mapping U,
4. unit vectors, whose directions are keep unchanged during the mapping V,
5. left Cauchy tensor $C = F^T F$ at point T_1,
6. right Cauchy tensor $B = F F^T$ at point T_1,

7. Green Lagrange deformation tensor E at point T_1,
8. Euler Almansi deformation tensor e at point T_1,
9. differentials of areas $\vec{dS}$ at point T_1 for $\vec{dS}^0 = dS^0\vec{e}_1$, $\vec{dS}^0 = dS^0\vec{e}_2$ and $\vec{dS}^0 = dS^0\vec{e}_3$,
10. differential of volume dV at point T_1,
11. differentials of lenghts $\vec{ds}$ at point T_1 for $\vec{ds}^0 = ds^0\vec{e}_1$, $\vec{ds}^0 = ds^0\vec{e}_2$ and $\vec{ds}^0 = ds^0\vec{e}_3$,
12. exact value of lenght deformation at point T_1 in the direction $\frac{\sqrt{2}}{2}$‘$\vec{e}_1 + \frac{\sqrt{2}}{2}\vec{e}_2$.