3. Homework in Nonlinear Mechanics, 8. 11. 2013

Deadline, 15. 11. 2013

`VSi` is i–th digit of **your** registration number. For registration number 26102734 are `VS6`=7, `VS8`=4.

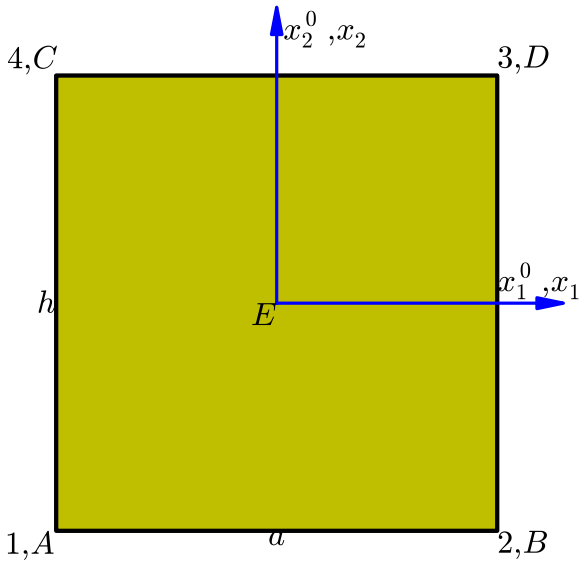
TASK 1: The figure shows four-node quadrilateral finite element of dimensions $a = 5\text{dm}$ and $h = 5\text{dm}$. Displacement field $\vec{u}(x_1^0, x_2^0) = u(x_1^0, x_2^0)\vec{e}_1 + v(x_1^0, x_2^0)\vec{e}_2$ within quadrilateral finite element is given in the material coordinates by

$$u\left(x_1^0, x_2^0\right) = \frac{u_1\left(x_1^0 - \frac{a}{2}\right)\left(x_2^0 - \frac{h}{2}\right)}{ah} - \frac{u_2\left(x_1^0 + \frac{a}{2}\right)\left(x_2^0 - \frac{h}{2}\right)}{ah} + \frac{u_3\left(x_1^0 + \frac{a}{2}\right)\left(x_2^0 + \frac{h}{2}\right)}{ah} - \frac{u_4\left(x_1^0 - \frac{a}{2}\right)\left(x_2^0 + \frac{h}{2}\right)}{ah}.$$

and

$$v\left(x_1^0, x_2^0\right) = \frac{v_1\left(x_1^0 - \frac{a}{2}\right)\left(x_2^0 - \frac{h}{2}\right)}{ah} - \frac{v_2\left(x_1^0 + \frac{a}{2}\right)\left(x_2^0 - \frac{h}{2}\right)}{ah} + \frac{v_3\left(x_1^0 + \frac{a}{2}\right)\left(x_2^0 + \frac{h}{2}\right)}{ah} - \frac{v_4\left(x_1^0 - \frac{a}{2}\right)\left(x_2^0 + \frac{h}{2}\right)}{ah}.$$

with known nodal displacements $\vec{u}_1 = \vec{0}$, $\vec{u}_2 = \vec{0}$, $\vec{u}_3 = \vec{0}$, $\vec{u}_4 = (u_4, v_4)$, $u_4 = (\texttt{VS7} + 10)\,\text{cm}$, $v_4 = (\texttt{VS8} + 10)\,\text{cm}$.



a) Express the spatial coordinates with the material coordinates and calculate deformation gradient $F(x_1^0 = 0, x_2^0 = 0)$.
b) Is the deformation state homogenuous?
b) Calculate components of Green-Lagrange tenzor $E_{ij}(x_1^0 = 0, x_2^0 = 0)$ and components of Euler-Almansi tenzor $e_{ij}(x_1 = 0, x_2 = 0)$ for indices $i, j = 1, 2, 3$.
c) Calculate principal normal deformations $E_{ii}(x_1^0 = 0, x_2^0 = 0)$ and $e_{ii}(x_1 = 0, x_2 = 0)$ for $i = 1, 2, 3$ and principal directions.
d) Calculate the changes of the right angles *BAC* and *DEC*.

e) Calculate the area change of the quadrilateral.
f) Calculate the perimeter change of the quadrilateral.
g) Calculate the area change of the circle of radius $r = 10$cm with center $E(x_1^0 = 0, x_2^0 = 0)$.
g) Calculate the circumference change of the circle of radius $r = 10$cm with center $E(x_1^0 = 0, x_2^0 = 0)$.