8. Homeworks from Nonlinear Mechanics, 20. 12. 2013

Deadline, 3. 1. 2013

`VSi` is i–th digit of **your** registration number. For registration number 26102734 are `VS6`=7, `VS8`=4.

NALOGA 1: Stress state of the particle is given by the components of Cauchy stress tensor σ_{ij} in the Cartesian coordinate system (x,y,z).

$$[\sigma_{ij}] = \begin{bmatrix} \frac{t}{4} & \frac{t}{4} & \frac{t}{2\sqrt{2}} \\ \frac{t}{4} & \frac{t}{4} & \frac{t}{2\sqrt{2}} \\ \frac{t}{2\sqrt{2}} & \frac{t}{2\sqrt{2}} & \frac{t}{2} \end{bmatrix}$$

Stress $t = (\texttt{VS7}+1)\,\mathrm{MPa}$.
Determine:

a) components of stress tensor $\sigma_{\alpha\beta}$ in a Cartesian coordinate system (ξ,η,ζ), which is obtained by rotation of the coordinate system (x,y,z) around an axis $\vec{e}_\phi = \frac{1}{\sqrt{3}}\vec{e}_1 + \frac{1}{\sqrt{3}}\vec{e}_2 + \frac{1}{\sqrt{3}}\vec{e}_3$ for angle $\phi = 30^\circ$,
b) the principal normal stresses and normal principal planes,
c) extreme shear stresses with the corresponding planes,
e) resulting shear and normal stresses at octahedron's planes.

What kind of the stress state is presented by the matrix $[\sigma_{ij}]$?

NALOGA 2: Rectangular cuboid with nodes $A(x_1^0=0,x_2^0=0,x_3^0=0)$, $B(x_1^0=b,x_2^0=0,x_3^0=0)$, $C(x_1^0=b,x_2^0=h,x_3^0=0)$, $D(x_1^0=0,x_2^0=h,x_3^0=0)$, $A'(x_1^0=0,x_2^0=0,x_3^0=l)$, $B'(x_1^0=b,x_2^0=0,x_3^0=l)$, $C'(x_1^0=b,x_2^0=h,x_3^0=l)$, $D'(x_1^0=0,x_2^0=h,x_3^0=l)$ firstly rotates around the axis $\vec{e}_3$ for 45°, then stretched in the directions $\frac{\sqrt{2}}{2}(\vec{e}_1+\vec{e}_2)$ and $\frac{\sqrt{2}}{2}(-\vec{e}_1+\vec{e}_2)$ for a and stretched in the direction of $\vec{e}_3$ for c. After deformation in the body is presented stress state described by components σ_{ij} of Cauchy stress tensor $\boldsymbol{\sigma}$.

$$[\sigma_{ij}] = \begin{bmatrix} t & t & 0 \\ t & t & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Data: $a = \frac{\texttt{VS8}+1}{100}\,\mathrm{cm}$, $c = \frac{\texttt{VS7}+1}{100}\,\mathrm{cm}$, $t = 10\,\mathrm{MPa}$, $b = h = 10\,\mathrm{cm}$, $l = 20\,\mathrm{cm}$.
Determine deformation gradient F and Jacobian J.
Determine stress tensors P, S, τ and T_B.