9. Homeworks from Nonlinear Mechanics, 3. 1. 2014

Deadline, 10. 1. 2014

`VSi` is i–th digit of **your** registration number. For registration number 26102734 are `VS6=7`, `VS8=4`.

TASK 1: The body is loaded by components of the vector of the specifics mass load $b_i = g\,\delta_{i1}, \quad i = 1,2,3$, with known gravity acceleration g. The stress state in the body is determined by components σ_{ij} of Cauchy stress tensor σ.

$$[\sigma_{ij}] = \begin{bmatrix} x_2^2 + x_3^2 & x_1 x_2 & x_1 x_3 \\ x_1 x_2 & x_1^2 + x_3^2 & x_2 x_3 \\ x_1 x_3 & x_2 x_3 & x_1^2 + x_2^2 \end{bmatrix}.$$

- Determine the acceleration vector field in the body.

TASK 2: Consider the deformation ob the body described with equations

$$x \equiv x_1 = x_1^0 - c\,t\,x_1^0$$
$$y \equiv x_2 = x_2^0$$
$$z \equiv x_3 = x_3^0$$

Consider:

- Jaumann derivative:

$$\overset{\nabla}{\sigma} = \dot{\sigma} - W\sigma + \sigma W = C : D = \lambda\,\mathrm{tr}(D)\,I + 2\mu D.$$

- Truesdell derivative:

$$\overset{\circ}{\sigma} = \dot{\sigma} - L\sigma - \sigma L^T + \mathrm{tr}(L)\,\sigma = C : D = \lambda\,\mathrm{tr}(D)\,I + 2\mu D.$$

- Green–Naghdi derivative:

$$\overset{\triangle}{\sigma} = \dot{\sigma} - \dot{R}R^T\sigma + \sigma\dot{R}R^T = C : D = \lambda\,\mathrm{tr}(D)\,I + 2\mu D.$$

Assume the following boundary conditions:

$$\sigma_{xx}^0 = 0,$$
$$\sigma_{xy}^0 = 0,$$
$$\sigma_{yy}^0 = 0,$$
$$\sigma_{xz}^0 = 0,$$
$$\sigma_{yz}^0 = 0,$$
$$\sigma_{zz}^0 = 0.$$

Sketch the Cauchy stress components as function of the time. Data:
$c = \frac{(\mathtt{VS7}+1)}{1000\,\mathrm{s}}$, $E = (100\,000 + \mathtt{VS8}\,10\,000)\,\mathrm{MPa}$, $\nu = \frac{1}{3}$.