6. Vaja: Hitrosti deformacij

Rado Flajs

---

1. Naloga

1.1. Naloga

Deformiranje telesa je podano s poljem pomikov $\vec{u}(x_1^0, x_2^0, x_3^0)$ v materialnih koordinatah x_1^0, x_2^0, x_3^0. Obravnavaj dva primera:

a)

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 & t & 0 \\ t & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{bmatrix} = \frac{1}{1-t^2} \begin{bmatrix} 1 & -t & 0 \\ -t & 1 & 0 \\ 0 & 0 & 1-t^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{1}$$

b)

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 & -t & 0 \\ t & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & -t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{bmatrix} = \frac{1}{1+t^2} \begin{bmatrix} 1 & t & 0 \\ -t & 1 & 0 \\ 0 & 0 & 1+t^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{2}$$

Za oba primera določi:

- hitrost poljubnega delca v materialnem in prostorskem opisu
- pospešek poljubnega delca v materialnem in prostorskem opisu
- materialni odvod $\dot{F}$ deformacijskega gradienta F v materialnem in prostorskem opisu
- hitrostni gradient L
- hitrost deformacijskega tenzorja D in spin W
- pojasni fizikalni pomen hitrosti deformacijskega tenzorja D in spina W
- materialni odvod $\dot{C}$ levega Cauchy Greenovega tenzorja C v materialnem in prostorskem opisu
- materialni odvod $\dot{B}$ desnega Cauchy Greenovega tenzorja B v materialnem in prostorskem opisu
- materialni odvod $\dot{U}$ tenzorja U v materialnem in prostorskem opisu
- materialni odvod $\dot{V}$ tenzorja V v materialnem in prostorskem opisu
- materialni odvod $\dot{R}$ rotacije R v materialnem in prostorskem opisu
- materialni odvod $\dot{E}$ Green Lagrangevega tenzorja E v materialnem in prostorskem opisu
- materialni odvod $\dot{e}$ Euler Almansijevega tenzorja e v materialnem in prostorskem opisu
- materialni odvod $(\dot{ds^2})$ spremembe kvadrata dolžine ds^2 v materialnem in prostorskem opisu
- materialni odvod $\dot{d\vec{S}}$ spremembe površine $d\vec{S}$ v materialnem in prostorskem opisu
- materialni odvod $(\dot{dV})$ spremembe volumna dV v materialnem in prostorskem opisu

(a) Prostorski prikaz deformiranja

(b) Tlorisni prikaz deformiranja

Slika 1: VR razcep

1.2. Rešitev

1.2.1. Grafični prikaz deformiranja telesa

1.2.2. Hitrost poljubnega delca v materialnem in prostorskem opisu I

Primer a).

$$
\begin{aligned}
v_1 &= \frac{Dx_1}{Dt} = x_2^0 = \frac{1}{1-t^2}(-t\,x_1 + x_2),\\
v_2 &= \frac{Dx_2}{Dt} = x_1^0 = \frac{1}{1-t^2}(x_1 - t\,x_2),\\
v_3 &= \frac{Dx_3}{Dt} = 0.
\end{aligned} \tag{3}
$$

Primer b).

$$
\begin{aligned}
v_1 &= \frac{Dx_1}{Dt} = -x_2^0 = \frac{1}{1+t^2}(t\,x_1 - x_2),\\
v_2 &= \frac{Dx_2}{Dt} = x_1^0 = \frac{1}{1+t^2}(x_1 + t\,x_2),\\
v_3 &= \frac{Dx_3}{Dt} = 0.
\end{aligned} \tag{4}
$$

1.2.3. Pospešek poljubnega delca v materialnem in prostorskem opisu

Primer a).

$$
\begin{aligned}
a_1 &= \frac{Dv_1}{Dt} = 0,\\
a_2 &= \frac{Dv_2}{Dt} = 0,\\
a_3 &= \frac{Dv_3}{Dt} = 0.
\end{aligned} \tag{5}
$$

Primer b).

$$
\begin{aligned}
a_1 &= \frac{Dv_1}{Dt} = 0,\\
a_2 &= \frac{Dv_2}{Dt} = 0,\\
a_3 &= \frac{Dv_3}{Dt} = 0.
\end{aligned} \tag{6}
$$

1.2.4. Hitrost poljubnega delca v materialnem in prostorskem opisu II

$$
\begin{aligned}
\vec{u} &\overset{a}{=} t\,(x_2^0, x_1^0, 0) = \left(\frac{t(t\,x_1 - x_2)}{t^2-1}, \frac{t(t\,x_2 - x_1)}{t^2-1}, 0\right),\\
\vec{u} &\overset{b}{=} t\,(-x_2^0, x_1^0, 0) = \frac{t}{1+t^2}\,(t\,x_1 - x_2, x_1 + t\,x_2, 0).
\end{aligned} \tag{7}
$$

Email address: rado.flajs@fgg.uni-lj.si (Rado Flajs)

Hitrost v prostorskem opisu:

$$\vec{v} = \frac{D\vec{r}}{Dt} = \frac{D\vec{u}}{Dt} = \frac{\partial \vec{u}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\,\vec{u} = \frac{\partial \vec{u}}{\partial t} + \sum_i \frac{\partial \vec{u}}{\partial x_i}\, v_i = \frac{\partial \vec{u}}{\partial t} + \frac{\partial \vec{u}}{\partial x_1}\, v_1 + \frac{\partial \vec{u}}{\partial x_2}\, v_2. \tag{8}$$

V enačbo (8) vstavimo izraza za hitrost v prostorskem opisu (3) in (4), izraz za pomik (7), izračunamo hitrost

$$\begin{aligned} \vec{v}_1 &= \frac{t\,x_1 - x_2}{t^2 - 1}\,\vec{e}_1 + \frac{-x_1 + t\,x_2}{t^2 - 1}\,\vec{e}_2, \\ \vec{v}_2 &= \frac{t\,x_1 - x_2}{t^2 + 1}\,\vec{e}_1 + \frac{x_1 + t\,x_2}{t^2 + 1}\,\vec{e}_2. \end{aligned} \tag{9}$$

in preverimo, da se leva stran enačbe (8) ujema za desno stranjo.

$$b = \frac{\partial \vec{u}}{\partial t} \overset{a}{=} \left(\frac{t^2\mathrm{x2} - 2t\mathrm{x1} + \mathrm{x2}}{(t^2-1)^2}, \frac{t^2\mathrm{x1} - 2t\mathrm{x2} + \mathrm{x1}}{(t^2-1)^2}, 0 \right)$$

$$A = \frac{\partial \vec{u}}{\partial \vec{x}} \overset{a}{=} \begin{bmatrix} \frac{t^2}{t^2-1} & -\frac{t}{t^2-1} & 0 \\ -\frac{t}{t^2-1} & \frac{t^2}{t^2-1} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\vec{v} = (I - A)^{-1}\, b$$

1.2.5. Hitrostni gradient L

$$L = \frac{\partial \vec{v}}{\partial \vec{r}} = \begin{bmatrix} \frac{\partial v_1}{\partial x_1} & \frac{\partial v_1}{\partial x_2} & \frac{\partial v_1}{\partial x_3} \\ \frac{\partial v_2}{\partial x_1} & \frac{\partial v_2}{\partial x_2} & \frac{\partial v_2}{\partial x_3} \\ \frac{\partial v_3}{\partial x_1} & \frac{\partial v_3}{\partial x_2} & \frac{\partial v_3}{\partial x_3} \end{bmatrix}. \tag{10}$$

$$\begin{aligned} L &\overset{a)}{=} \frac{1}{t^2-1} \begin{bmatrix} t & -1 & 0 \\ -1 & t & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ L &\overset{b)}{=} \frac{1}{t^2+1} \begin{bmatrix} t & -1 & 0 \\ 1 & t & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{11}$$

1.2.6. Materialni odvod $\dot{F}$ deformacijskega gradienta F

Materialni opis.

$$\begin{aligned} F &\overset{a)}{=} \begin{bmatrix} 1 & t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \frac{DF}{Dt} = \dot{F} \overset{a)}{=} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ F &\overset{b)}{=} \begin{bmatrix} 1 & -t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \frac{DF}{Dt} = \dot{F} \overset{b)}{=} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{12}$$

Prostorski opis.

$$\frac{DF}{Dt} = L\,F, \tag{13}$$

kjer je L hitrostni gradient.

$$\begin{aligned} \dot{F} = L\,F &\overset{a)}{=} \frac{1}{t^2-1} \begin{bmatrix} t & -1 & 0 \\ -1 & t & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \dot{F} = L\,F &\overset{b)}{=} \frac{1}{t^2+1} \begin{bmatrix} t & -1 & 0 \\ 1 & t & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{14}$$

1.2.7. Hitrost deformacijskega tenzorja D in spin W

$$
\begin{aligned}
L &= D + W,\\
D &= \frac{1}{2}(L + L^T),\\
W &= \frac{1}{2}(L - L^T).
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
D &\overset{a)}{=} \frac{1}{2}(L + L^T) = L = \frac{1}{t^2-1}\begin{bmatrix} t & -1 & 0\\ -1 & t & 0\\ 0 & 0 & 0\end{bmatrix},\\
D &\overset{b)}{=} \frac{1}{2}(L + L^T) = \frac{1}{t^2+1}\begin{bmatrix} t & 0 & 0\\ 0 & t & 0\\ 0 & 0 & 0\end{bmatrix}.
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
W &\overset{a)}{=} \frac{1}{2}(L - L^T) = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\\
W &\overset{b)}{=} \frac{1}{2}(L - L^T) = \frac{1}{t^2+1}\begin{bmatrix} 0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}.
\end{aligned}
\tag{17}
$$

1.2.8. Fizikalni pomen hitrosti deformacijskega tenzorja D in spina W

$$
(\dot{\ln \lambda}) = \vec{n}\cdot D\,\vec{n}. \tag{18}
$$

Primer 1.

Primer 2.

$$
\begin{aligned}
W\,\vec{m} &= \vec{\omega}\times\vec{m},\\
W\,\vec{m}_D &= \vec{\omega}\times\vec{m}_D = \dot{\vec{m}}_D, \quad \vec{m}_D \text{ lastni vektor } D.
\end{aligned}
\tag{19}
$$

Primer 3.

Primer 4.

1.2.9. Materialni odvod $\dot{C}$ levega Cauchy Greenovega tenzorja C

Materialni opis.

$$
\begin{aligned}
C &\overset{a)}{=} \begin{bmatrix} 1+t^2 & 2\,t & 0\\ 2\,t & 1+t^2 & 0\\ 0 & 0 & 1\end{bmatrix} \longrightarrow \dot{C} \overset{a)}{=} \begin{bmatrix} 2\,t & 2 & 0\\ 2 & 2\,t & 0\\ 0 & 0 & 0\end{bmatrix}\\
C &\overset{b)}{=} \begin{bmatrix} 1+t^2 & 0 & 0\\ 0 & 1+t^2 & 0\\ 0 & 0 & 1\end{bmatrix} \longrightarrow \dot{C} \overset{a)}{=} \begin{bmatrix} 2\,t & 0 & 0\\ 0 & 2\,t & 0\\ 0 & 0 & 0\end{bmatrix}
\end{aligned}
\tag{20}
$$

Prostorski opis.

$$
\begin{aligned}
\dot{C} &\overset{a)}{=} 2\,F^T D\,F = 2\begin{bmatrix} 1 & t & 0\\ t & 1 & 0\\ 0 & 0 & 1\end{bmatrix}^T \frac{1}{t^2-1}\begin{bmatrix} t & -1 & 0\\ -1 & t & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} 1 & t & 0\\ t & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix} 2\,t & 2 & 0\\ 2 & 2\,t & 0\\ 0 & 0 & 0\end{bmatrix},\\
\dot{C} &\overset{b)}{=} 2\,F^T D\,F = 2\begin{bmatrix} 1 & -t & 0\\ t & 1 & 0\\ 0 & 0 & 1\end{bmatrix}^T \frac{1}{t^2+1}\begin{bmatrix} t & 0 & 0\\ 0 & t & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} 1 & -t & 0\\ t & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix} 2\,t & 0 & 0\\ 0 & 2\,t & 0\\ 0 & 0 & 0\end{bmatrix}.
\end{aligned}
\tag{21}
$$

1.2.10. Materialni odvod $\dot{B}$ desnega Cauchy Greenovega tenzorja B
Materialni opis.

$$\begin{aligned} B &\overset{a)}{=} \begin{bmatrix} 1+t^2 & 2\,t & 0 \\ 2\,t & 1+t^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \dot{B} \overset{a)}{=} \begin{bmatrix} 2\,t & 2 & 0 \\ 2 & 2\,t & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ B &\overset{b)}{=} \begin{bmatrix} 1+t^2 & 0 & 0 \\ 0 & 1+t^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \dot{B} \overset{a)}{=} \begin{bmatrix} 2\,t & 0 & 0 \\ 0 & 2\,t & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{22}$$

Prostorski opis.

$$\dot{B} \overset{a)}{=} L\,B + B\,L^T = \frac{1}{t^2-1} \begin{bmatrix} t & -1 & 0 \\ -1 & t & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1+t^2 & 2\,t & 0 \\ 2\,t & 1+t^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1+t^2 & 2\,t & 0 \\ 2\,t & 1+t^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \frac{1}{t^2-1} \begin{bmatrix} t & -1 & 0 \\ -1 & t & 0 \\ 0 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 2\,t & 2 & 0 \\ 2 & 2\,t & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\dot{B} \overset{b)}{=} L\,B + B\,L^T = \begin{bmatrix} 2\,t & 0 & 0 \\ 0 & 2\,t & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{23}$$

1.2.11. Materialni odvod $\dot{U}$ tenzorja U
Materialni odvod $\dot{U}$ tenzorja U izračunamo iz enačbe

$$\dot{C} = \dot{U\,U} = U\,\dot{U} + \dot{U}\,U, \tag{24}$$

to je Sylvestrove enačbe

$$C = A\,X + X\,B. \tag{25}$$

V Mathematici lahko uporabimo ukaz `X = LyapunovSolve[A,B,C]`, v Matlabu ukaz `X = lyap(A,B,C)`, v Octaveju pa ukaz `X = syl(A,B,C)`. Z uporabo Mathematice dobimo

$$U \overset{a)}{=} \begin{bmatrix} 1 & t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \dot{U} \overset{a)}{=} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{26}$$

$$U \overset{b)}{=} \begin{bmatrix} \sqrt{1+t^2} & 0 & 0 \\ 0 & \sqrt{1+t^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \dot{U} \overset{b)}{=} \begin{bmatrix} \frac{t}{\sqrt{1+t^2}} & 0 & 0 \\ 0 & \frac{t}{\sqrt{1+t^2}} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{27}$$

Uporabnost rešitve Sylevstrove enačbe (24) je v tem, da lahko matriko $\dot{U}$ določimo za vsak delec posebej, Vse kar pri tem potrebujemo sta zgolj matriki $\dot{C}$ in U, ki pripadata temu delcu.

1.2.12. Materialni odvod $\dot{V}$ tenzorja V
Materialni odvod $\dot{U}$ tenzorja U izračunamo iz enačbe

$$\dot{B} = \dot{V\,V} = V\,\dot{V} + \dot{V}\,V, \tag{28}$$

to je Sylvestrove enačbe

$$C = A\,X + X\,B. \tag{29}$$

V Mathematici lahko uporabimo ukaz `X = LyapunovSolve[A,B,C]`, v Matlabu ukaz `X = lyap(A,B,C)`, v Octaveju pa ukaz `X = syl(A,B,C)`. Z uporabo Mathematice dobimo

$$V \overset{a)}{=} \begin{bmatrix} 1 & t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \dot{V} \overset{a)}{=} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{30}$$

$$V \overset{b)}{=} \begin{bmatrix} \sqrt{1+t^2} & 0 & 0 \\ 0 & \sqrt{1+t^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \dot{V} \overset{b)}{=} \begin{bmatrix} \frac{t}{\sqrt{1+t^2}} & 0 & 0 \\ 0 & \frac{t}{\sqrt{1+t^2}} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{31}$$

Uporabnost rešitve Sylevstrove enačbe (28) je v tem, da lahko matriko $\dot{V}$ določimo za vsak delec posebej, Vse kar pri tem potrebujemo sta zgolj matriki $\dot{B}$ in V, ki pripadata temu delcu.

1.2.13. Materialni odvod $\dot{R}$ rotacije R
Z uporabo RU ali VR razcepa določimo rotacijsko matriko R. Dobimo

$$R \overset{a)}{=} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{32}$$

$$R \overset{b)}{=} \begin{bmatrix} \frac{1}{\sqrt{t^2+1}} & -\frac{t}{\sqrt{t^2+1}} & 0 \\ \frac{t}{\sqrt{t^2+1}} & \frac{1}{\sqrt{t^2+1}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{33}$$

Odvajamo in dobimo

$$\dot{R} \overset{a)}{=} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{34}$$

$$\dot{R} \overset{b)}{=} \begin{bmatrix} -\frac{t}{(t^2+1)^{3/2}} & -\frac{1}{(t^2+1)^{3/2}} & 0 \\ \frac{1}{(t^2+1)^{3/2}} & -\frac{t}{(t^2+1)^{3/2}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{35}$$

1.2.14. Materialni odvod $\dot{E}$ Green Lagrangevega tenzorja E
Materialni opis.

Prostorski opis.

$$\begin{aligned} \dot{E} &\overset{a)}{=} \frac{1}{2}\dot{C} = F^T D F = \begin{bmatrix} 1 & t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \frac{1}{t^2-1} \begin{bmatrix} t & -1 & 0 \\ -1 & t & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} t & 1 & 0 \\ 1 & t & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \dot{E} &\overset{b)}{=} \frac{1}{2}\dot{C} = F^T D F = \begin{bmatrix} 1 & -t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \frac{1}{t^2+1} \begin{bmatrix} t & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} t & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{36}$$

1.2.15. Materialni odvod $\dot{e}$ Euler Almansijevega tenzorja e
Materialni opis.

Prostorski opis.

$$\begin{aligned} \dot{e} &\overset{a)}{=} -\frac{1}{2}(\dot{B^1}) - \frac{1}{2} B^{-1}\,\dot{B}\,B^{-1} = \begin{bmatrix} 1 & -t & 0 \\ t & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ \dot{e} &\overset{b)}{=} -\frac{1}{2}(\dot{B^1}) - \frac{1}{2} B^{-1}\,\dot{B}\,B^{-1} = \frac{1}{(1+t^2)^2} \begin{bmatrix} t & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{37}$$

1.2.16. Materialni odvod $(\dot{ds^2})$ spremembe kvadrata dolžine ds^2 v materialnem in prostorskem opisu
Materialni opis. Materialni odvod $(\dot{ds^2})$ spremembe kvadrata dolžine ds^2 v prostorskem opisu lahko izračunamo po ena'v cbah:

- primer a)

$$\frac{D(ds^2)}{Dt} = 2\,d\vec{r}^0 \cdot \dot{E}\,d\vec{r}^0 \overset{a)}{=} 2\,d\vec{r}^0 \cdot \begin{bmatrix} t & 1 & 0 \\ 1 & t & 0 \\ 0 & 0 & 0 \end{bmatrix} d\vec{r}^0. \tag{38}$$

- primer b)

$$\frac{D(ds^2)}{Dt} = 2\,d\vec{r}^0 \cdot \dot{E}\,d\vec{r}^0 \overset{b)}{=} 2\,d\vec{r}^0 \cdot \begin{bmatrix} t & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & 0 \end{bmatrix} d\vec{r}^0. \tag{39}$$

Prostorski opis. Prostorski odvod $(\dot{ds^2})$ spremembe kvadrata dolžine ds^2 v prostorskem opisu

- primer a)

$$\frac{D\,(ds^2)}{Dt} = 2\,\vec{dr}\cdot D\,\vec{dr} \overset{a)}{=} 2\,\vec{dr}\cdot\begin{bmatrix} t & 1 & 0 \\ 1 & t & 0 \\ 0 & 0 & 0 \end{bmatrix}\vec{dr}. \tag{40}$$

- primer b)

$$\frac{D\,(ds^2)}{Dt} = 2\,\vec{dr}\cdot D\,\vec{dr} \overset{b)}{=} 2\,\vec{dr}\cdot\begin{bmatrix} t & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & 0 \end{bmatrix}\vec{dr}. \tag{41}$$

1.2.17. Materialni odvod $\dot{\vec{dS}}$ spremembe površine $\vec{dS}$ v materialnem in prostorskem opisu
1.2.18. Materialni odvod $(\dot{dV})$ spremembe volumna dV v materialnem in prostorskem opisu