Geometrijske karakteristike prereza

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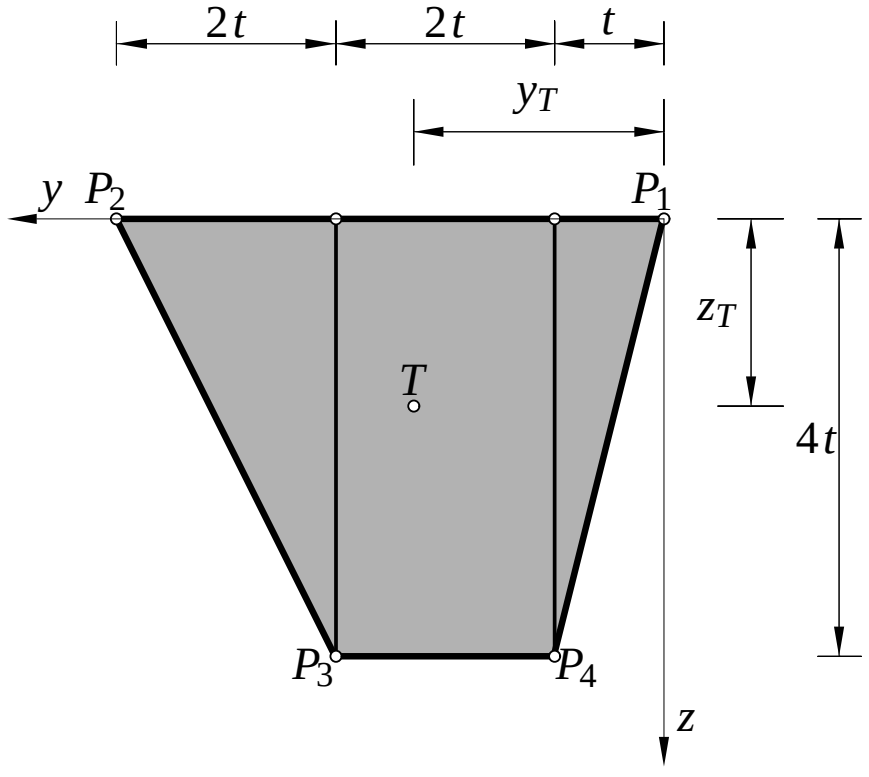
Ploščina prereza

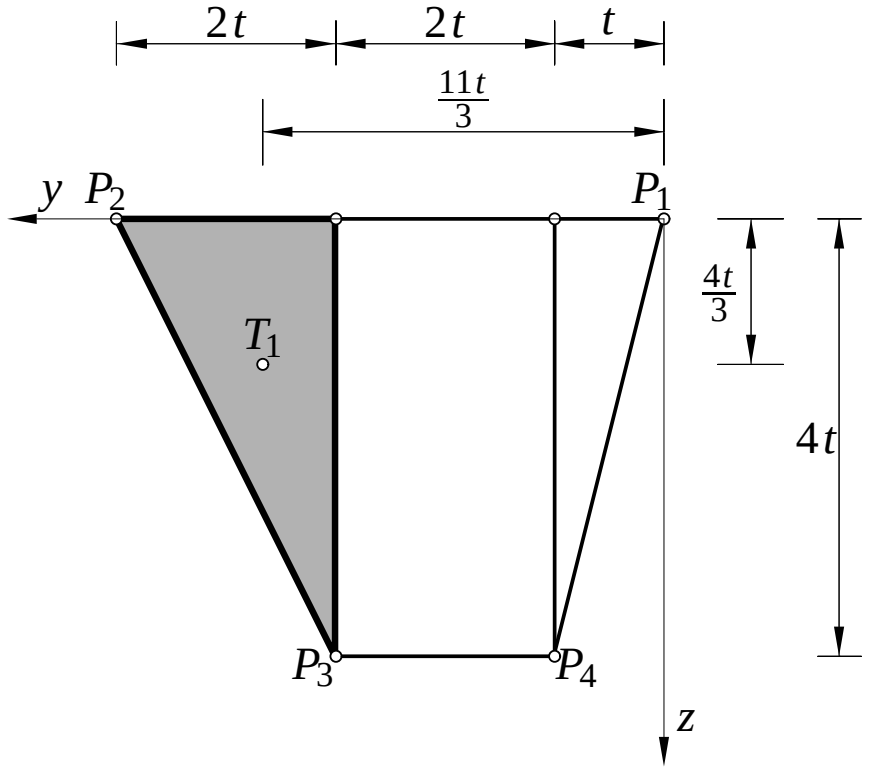
$A_x = 14\,t^2$,



$A_x = A_{x1} + A_{x2} + A_{x3}$

Statični moment S_y okrog osi y

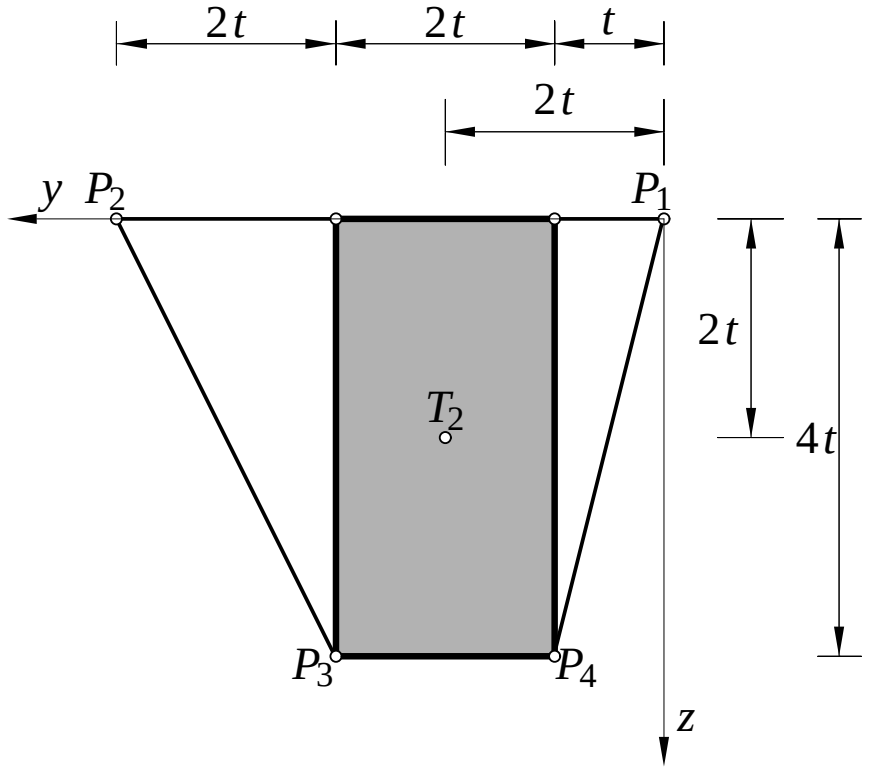
$A_x = 14\,t^2,$
$S_y = \frac{2\,t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t +$



$S_{y1} = A_{x1} \cdot z_{T1}$

Statični moment S_y okrog osi y

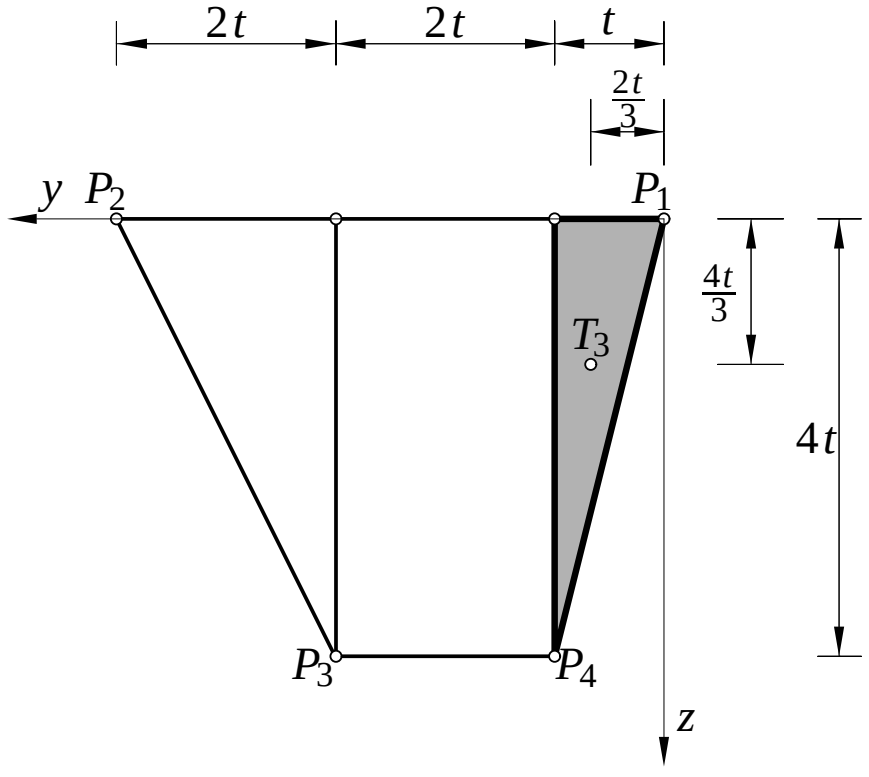
$A_x = 14\,t^2,$
$S_y = \frac{2\,t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t + 2\,t \cdot 4\,t \cdot 2\,t +$



$S_{y2} = A_{x2} \cdot z_{T2}$

Statični moment S_y okrog osi y

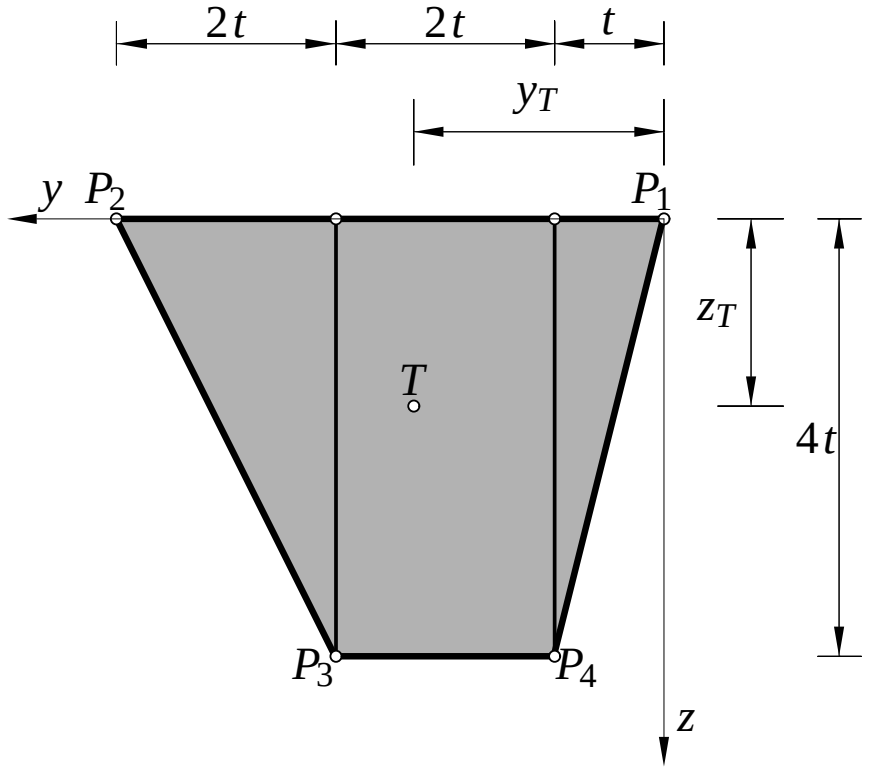
$A_x = 14\,t^2,$
$S_y = \frac{2\,t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t + 2\,t \cdot 4\,t \cdot 2\,t + \frac{t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t =$



$S_{y3} = A_{x3} \cdot z_{T3}$

Statični moment S_y okrog osi y

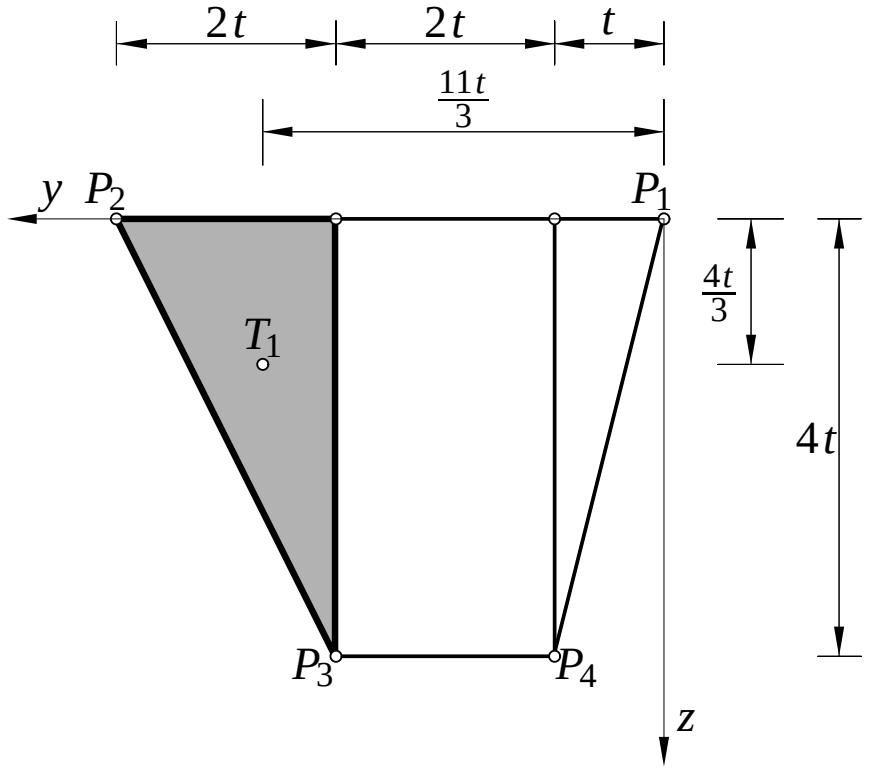
$A_x = 14\,t^2,$
$S_y = \frac{2\,t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t + 2\,t \cdot 4\,t \cdot 2\,t + \frac{t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t = 24\,t^3,$



$S_y = S_{y1} + S_{y2} + S_{y3}$

Statični moment S_z okrog osi z

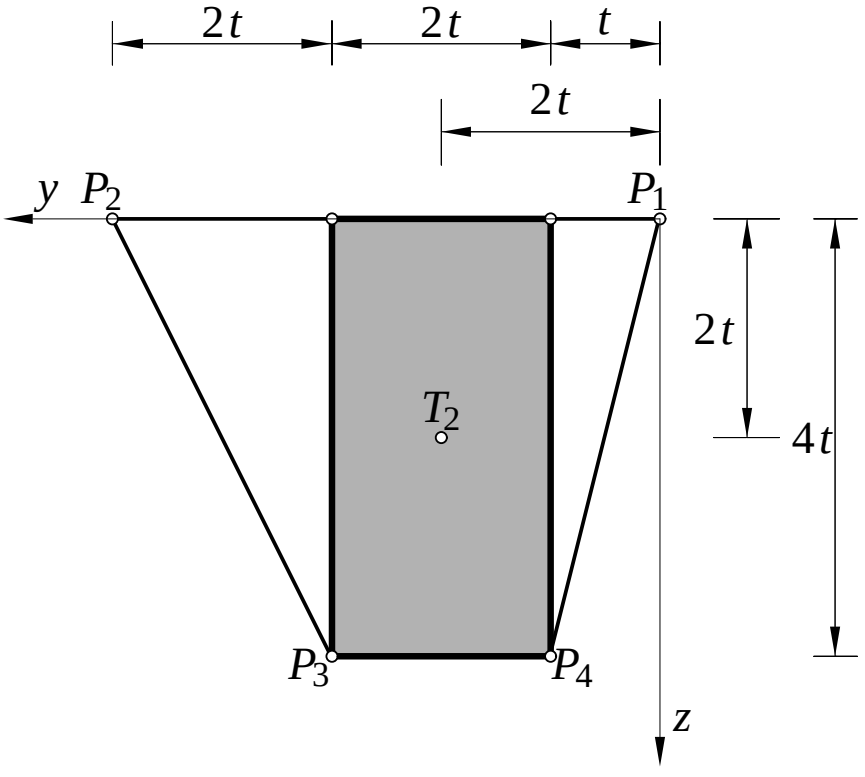
$A_x = 14\,t^2,$
$S_y = \frac{2\,t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t + 2\,t \cdot 4\,t \cdot 2\,t + \frac{t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t = 24\,t^3,$
$S_z = \frac{2\,t \cdot 4\,t}{2} \cdot \frac{11}{3}\,t +$



$S_{z1} = A_{x1} \cdot y_{T1}$

Statični moment S_z okrog osi z

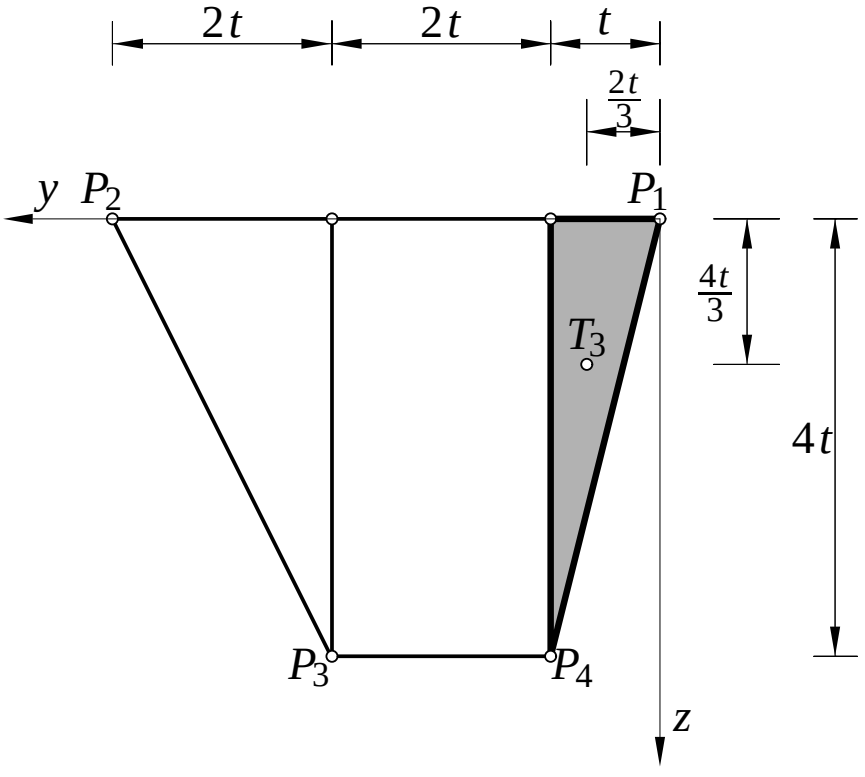
$A_x = 14\,t^2,$
$S_y = \frac{2\,t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t + 2\,t \cdot 4\,t \cdot 2\,t + \frac{t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t = 24\,t^3,$
$S_z = \frac{2\,t \cdot 4\,t}{2} \cdot \frac{11}{3}\,t + 2\,t \cdot 4\,t \cdot 2\,t +$



$S_{z2} = A_{x2} \cdot y_{T2}$

Statični moment S_z okrog osi z

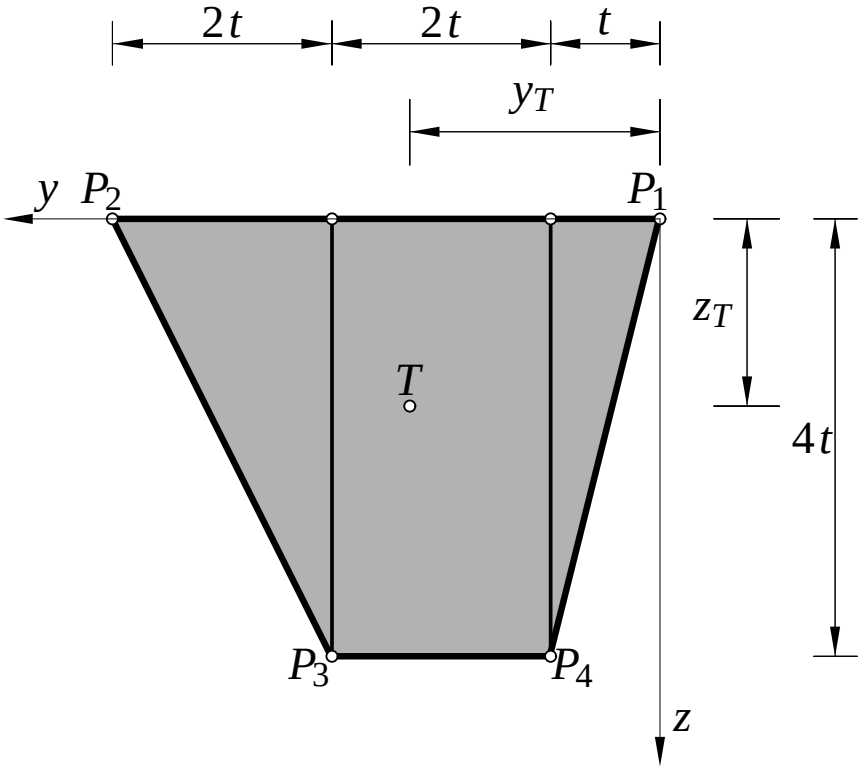
$A_x = 14\,t^2,$
$S_y = \frac{2\,t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t + 2\,t \cdot 4\,t \cdot 2\,t + \frac{t \cdot 4\,t}{2} \cdot \frac{4}{3}\,t = 24\,t^3,$
$S_z = \frac{2\,t \cdot 4\,t}{2} \cdot \frac{11}{3}\,t + 2\,t \cdot 4\,t \cdot 2\,t + \frac{t \cdot 4\,t}{2} \cdot \frac{2}{3}\,t =$



$S_{z3} = A_{x3} \cdot y_{T3}$

Statični moment S_z okrog osi z

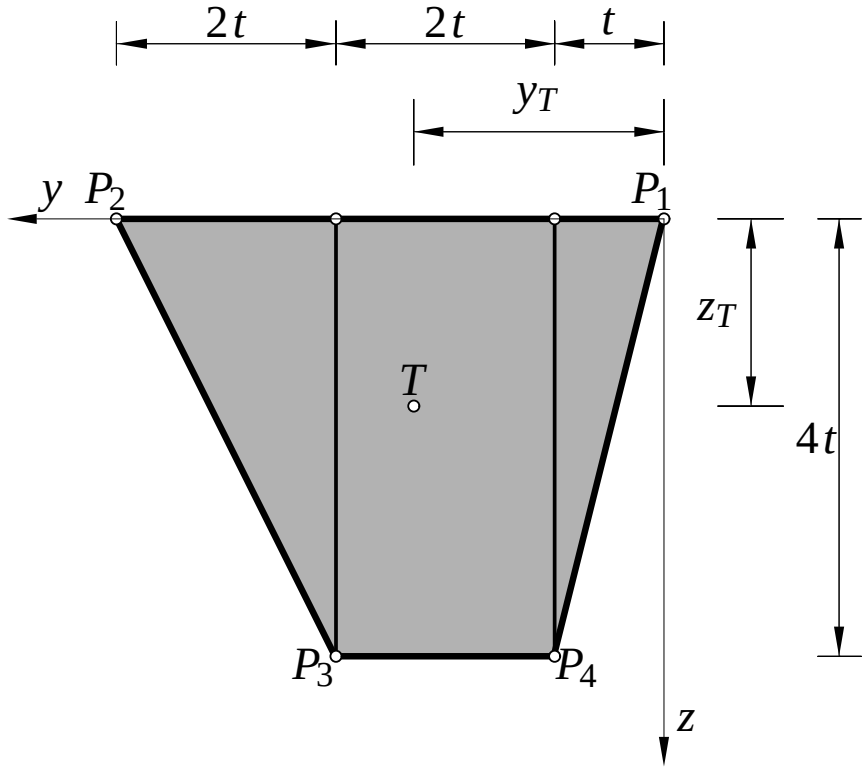
$A_x = 14\,t^2,$
$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$
$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$



$S_z = S_{z1} + S_{z2} + S_{z3}$

Koordinati y_T in z_T težišča prereza

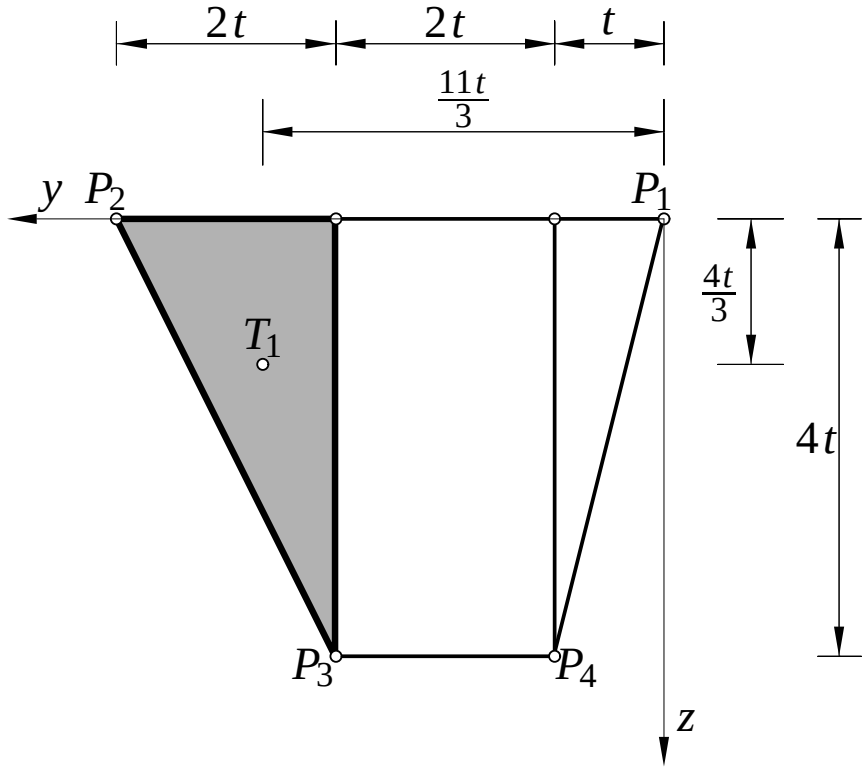
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$



$$y_T = \frac{S_z}{A_x}, z_T = \frac{S_y}{A_x}$$

Vztrajnostni moment $I_{yy} \equiv I_y$ okrog osi y

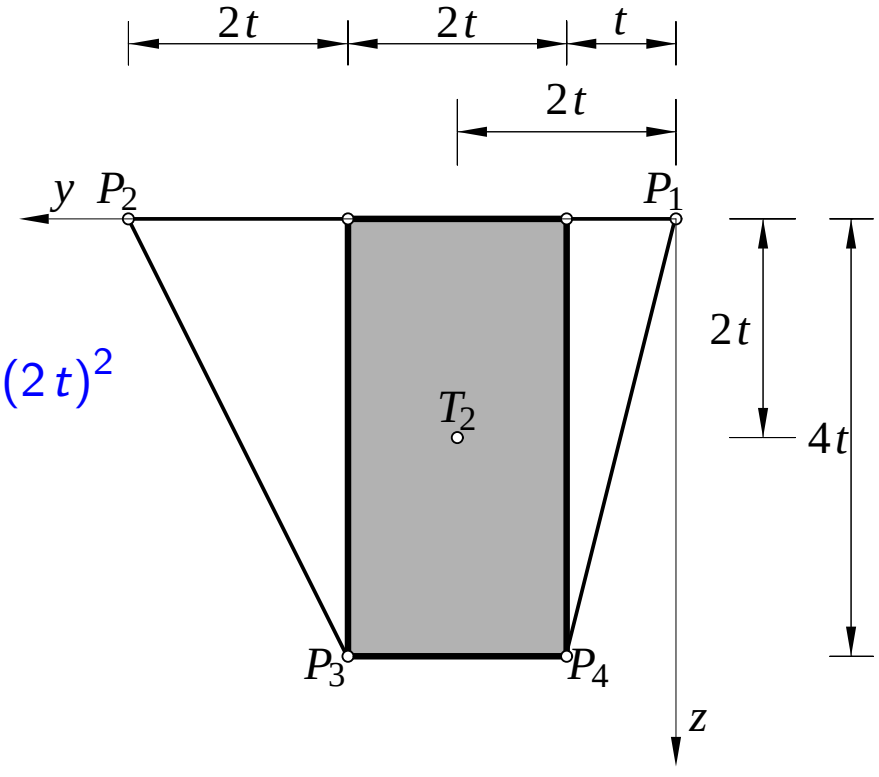
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 +$$



Steinerjev stavek: $I_{y1} = I_{y1}^{T1} + A_{x1}\cdot z_{T1}^2$

Vztrajnostni moment $I_{yy} \equiv I_y$ okrog osi y

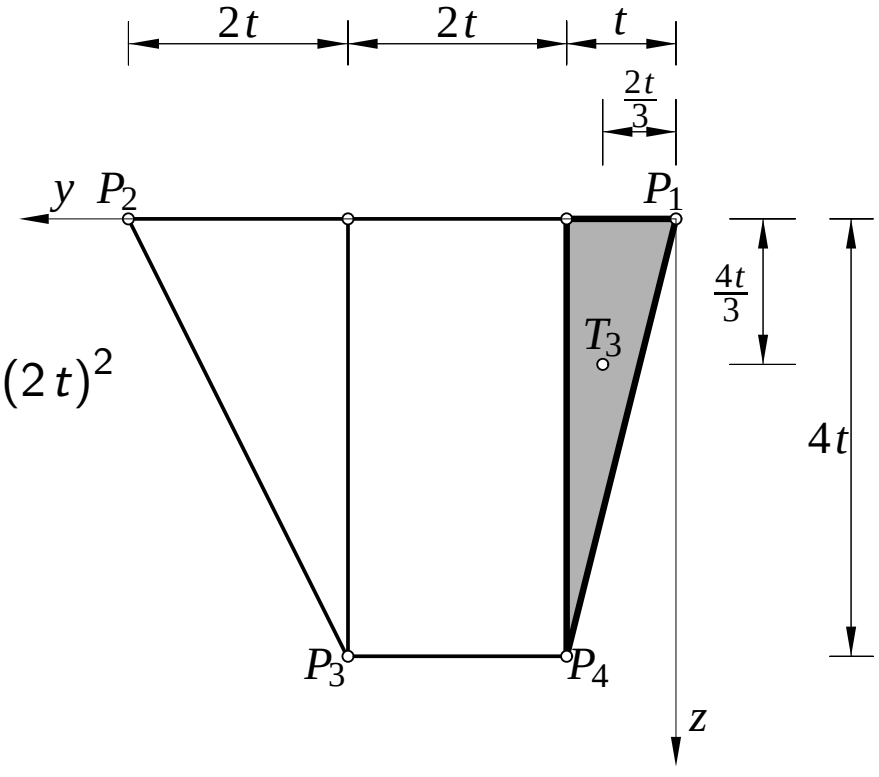
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2$$
$$+$$



Steinerjev stavek: $I_{y2} = I_{y2}^{T2} + A_{x2}\cdot z_{T2}^2$

Vztrajnostni moment $I_{yy} \equiv I_y$ okrog osi y

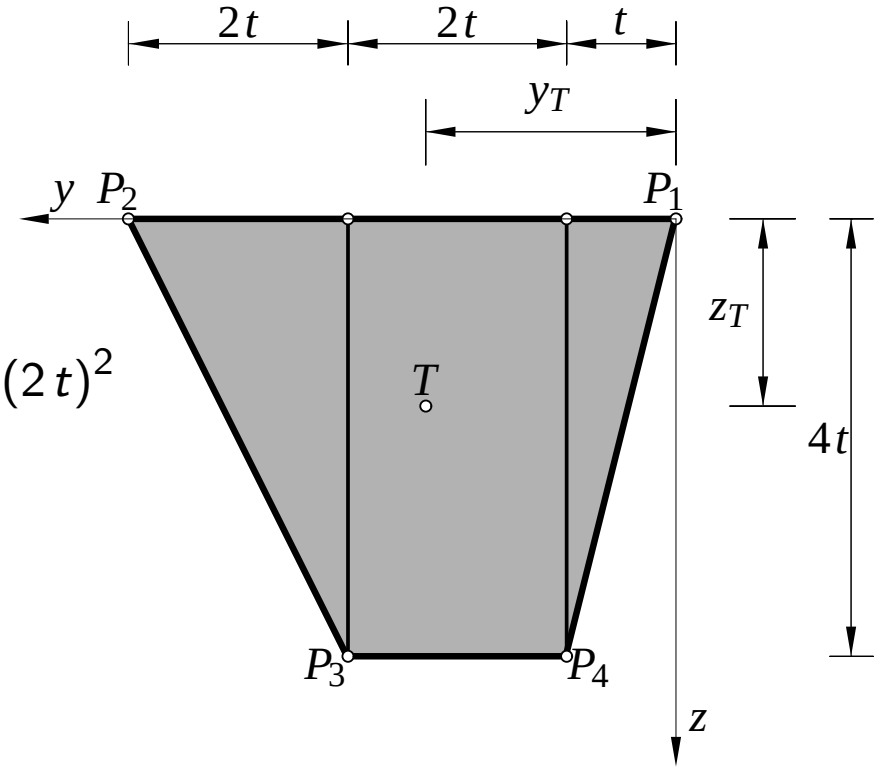
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2$$
$$+ \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 =$$



Steinerjev stavek: $I_{y3} = I_{y3}^{T3} + A_{x3}\cdot z_{T3}^2$

Vztrajnostni moment $I_{yy} \equiv I_y$ okrog osi y

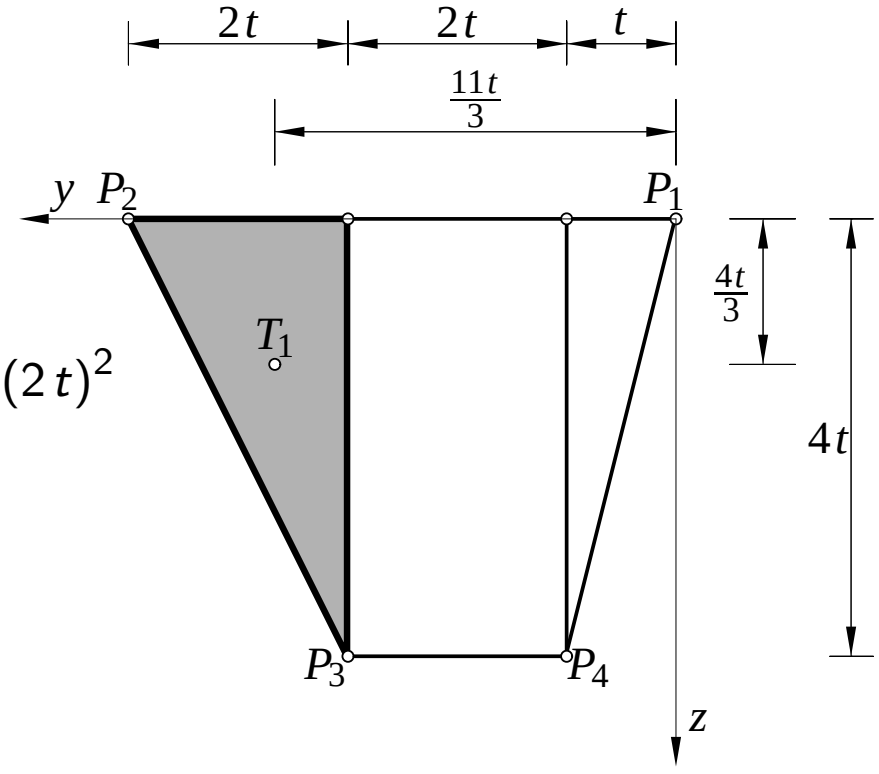
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 = 58.6667\,t^4$$



$$I_y = I_{y1} + I_{y2} + I_{y3}$$

Vztrajnostni moment $I_{zz} \equiv I_z$ okrog osi z

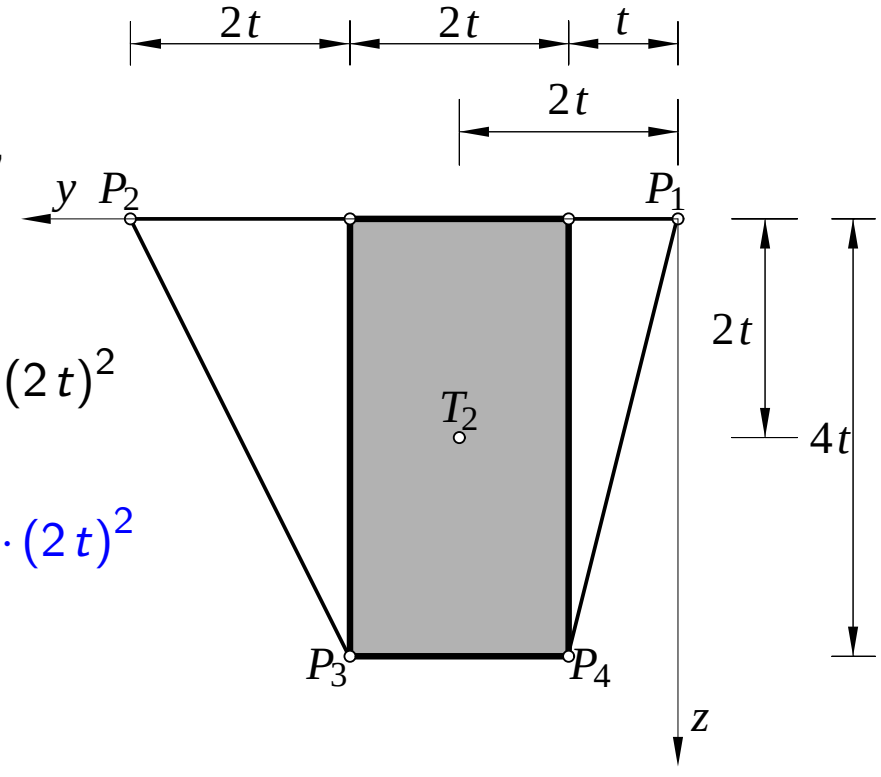
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2$$
$$+ \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 = 58.6667\,t^4$$
$$I_z = \frac{(2\,t)^3\cdot 4\,t}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{11\,t}{3}\right)^2 +$$



Steinerjev stavek: $I_{z1} = I_{z1}^{T1} + A_{x1}\cdot y_{T1}^2$

Vztrajnostni moment $I_{zz} \equiv I_z$ okrog osi z

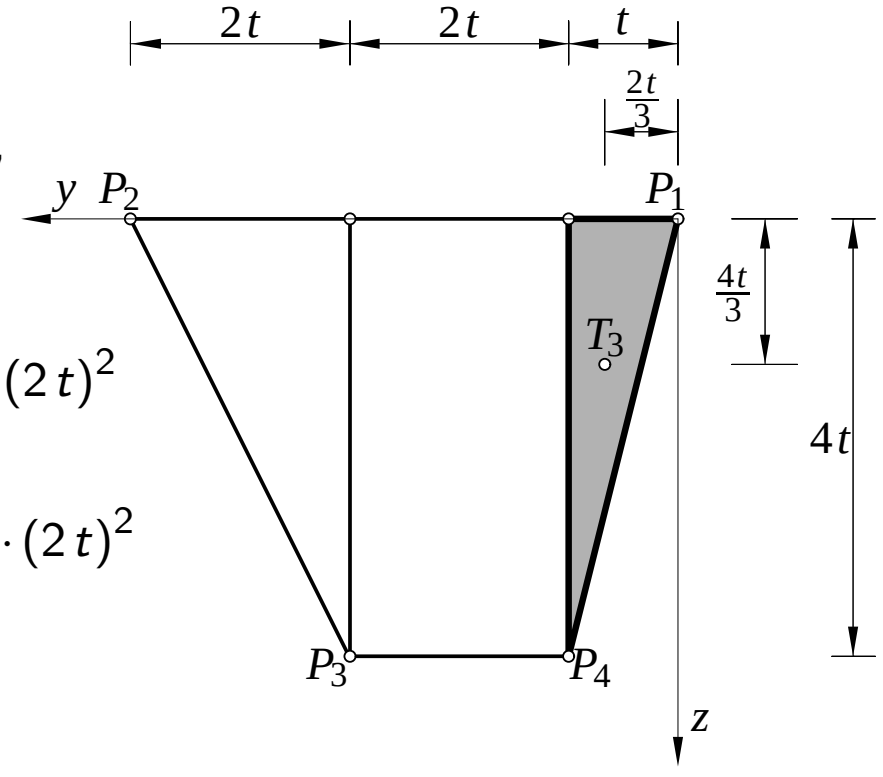
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 = 58.6667\,t^4$$
$$I_z = \frac{(2\,t)^3\cdot 4\,t}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{11\,t}{3}\right)^2 + \frac{(2\,t)^3\cdot 4\,t}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \;+$$



Steinerjev stavek: $I_{z2} = I_{z2}^{T2} + A_{x2}\cdot y_{T2}^2$

Vztrajnostni moment $I_{zz} \equiv I_z$ okrog osi z

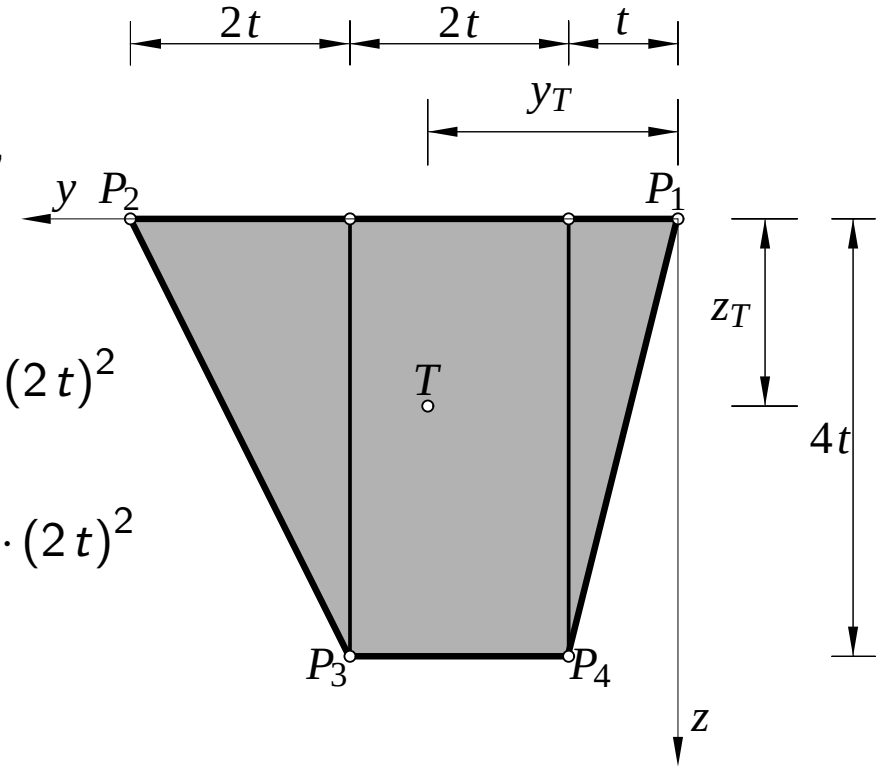
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 = 58.6667\,t^4$$
$$I_z = \frac{(2\,t)^3\cdot 4\,t}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{11\,t}{3}\right)^2 + \frac{(2\,t)^3\cdot 4\,t}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{(1\,t)^3\cdot 4\,t}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{2\,t}{3}\right)^2 =$$



Steinerjev stavek: $I_{z3} = I_{z3}^{T3} + A_{x3}\cdot y_{T3}^2$

Vztrajnostni moment $I_{zz} \equiv I_z$ okrog osi z

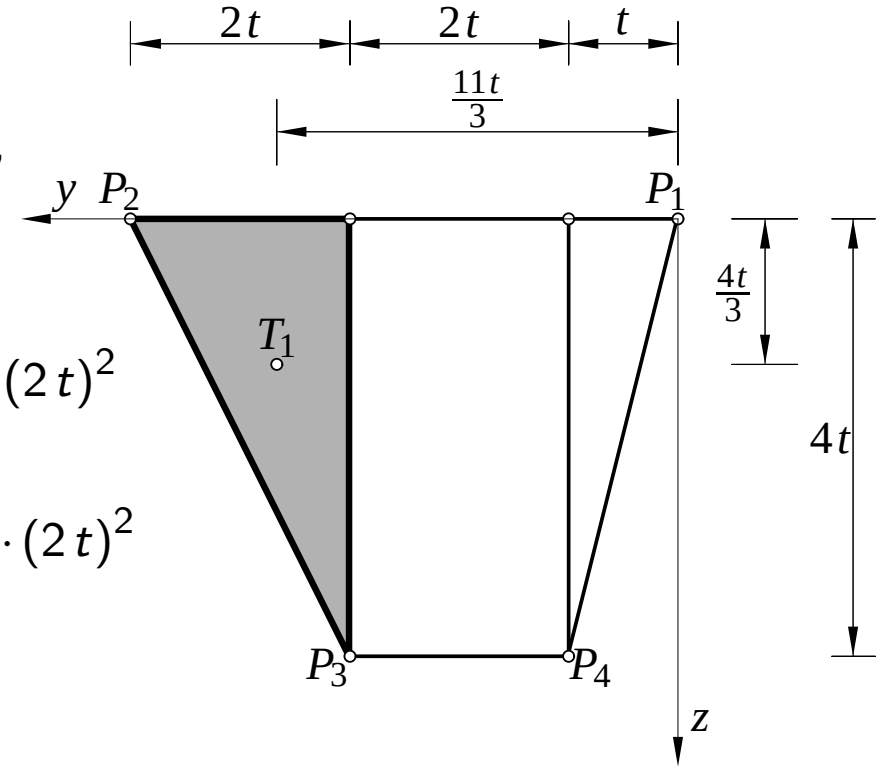
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 = 58.6667\,t^4$$
$$I_z = \frac{(2\,t)^3\cdot 4\,t}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{11\,t}{3}\right)^2 + \frac{(2\,t)^3\cdot 4\,t}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{(1\,t)^3\cdot 4\,t}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{2\,t}{3}\right)^2 = 90.3333\,t^4$$



$$I_z = I_{z1} + I_{z2} + I_{z3}$$

Deviacijski vztrajnostni moment I_{yz} okrog osi y in z

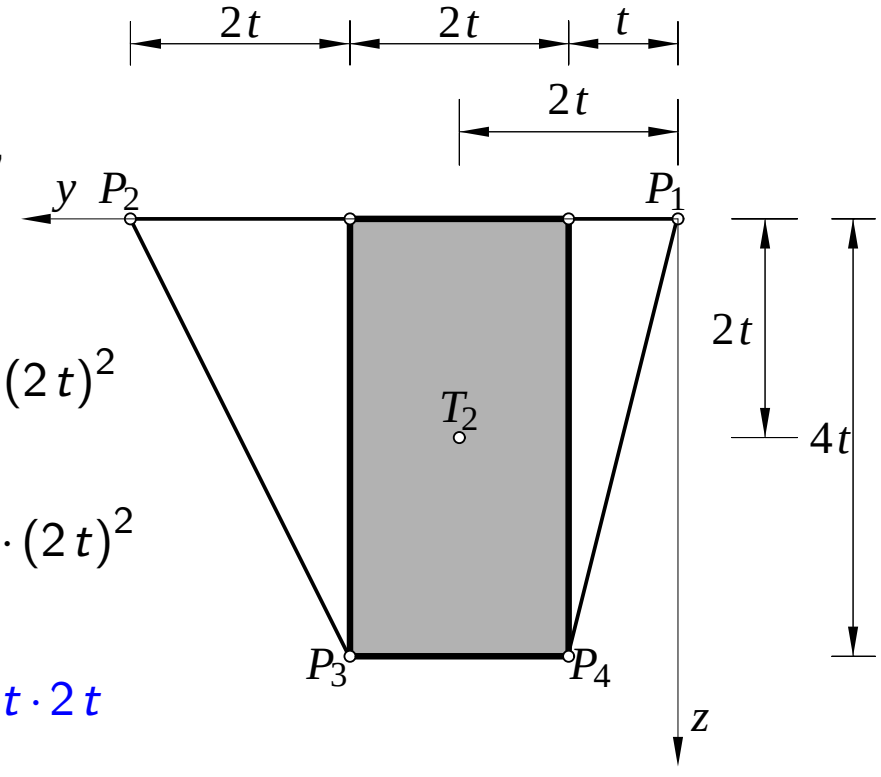
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 = 58.6667\,t^4$$
$$I_z = \frac{(2\,t)^3\cdot 4\,t}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{11\,t}{3}\right)^2 + \frac{(2\,t)^3\cdot 4\,t}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{(1\,t)^3\cdot 4\,t}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{2\,t}{3}\right)^2 = 90.3333\,t^4$$
$$I_{yz} = \frac{(2\,t)^2\cdot(4\,t)^2}{72} - \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11\,t}{3}\cdot\frac{4\,t}{3} +$$



Steinerjev stavek: $I_{yz1} = I_{yz1}^{T1} - A_{x1}\cdot y_{T1}\cdot z_{T1}$

Deviacijski vztrajnostni moment I_{yz} okrog osi y in z

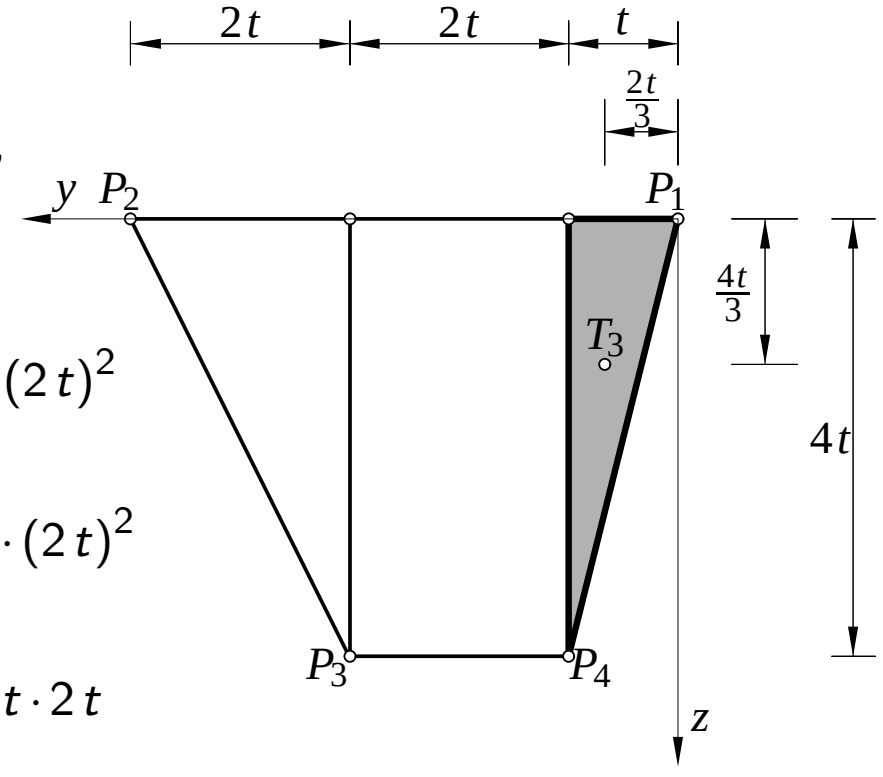
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 = 58.6667\,t^4$$
$$I_z = \frac{(2\,t)^3\cdot 4\,t}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{11\,t}{3}\right)^2 + \frac{(2\,t)^3\cdot 4\,t}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{(1\,t)^3\cdot 4\,t}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{2\,t}{3}\right)^2 = 90.3333\,t^4$$
$$I_{yz} = \frac{(2\,t)^2\cdot(4\,t)^2}{72} - \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11\,t}{3}\cdot\frac{4\,t}{3} + 0 - 2\,t\cdot 4\,t\cdot 2\,t\cdot 2\,t + +$$



Steinerjev stavek: $I_{yz2} = I_{yz2}^{T2} - A_{x2}\cdot y_{T2}\cdot z_{T2}$

Deviacijski vztrajnostni moment I_{yz} okrog osi y in z

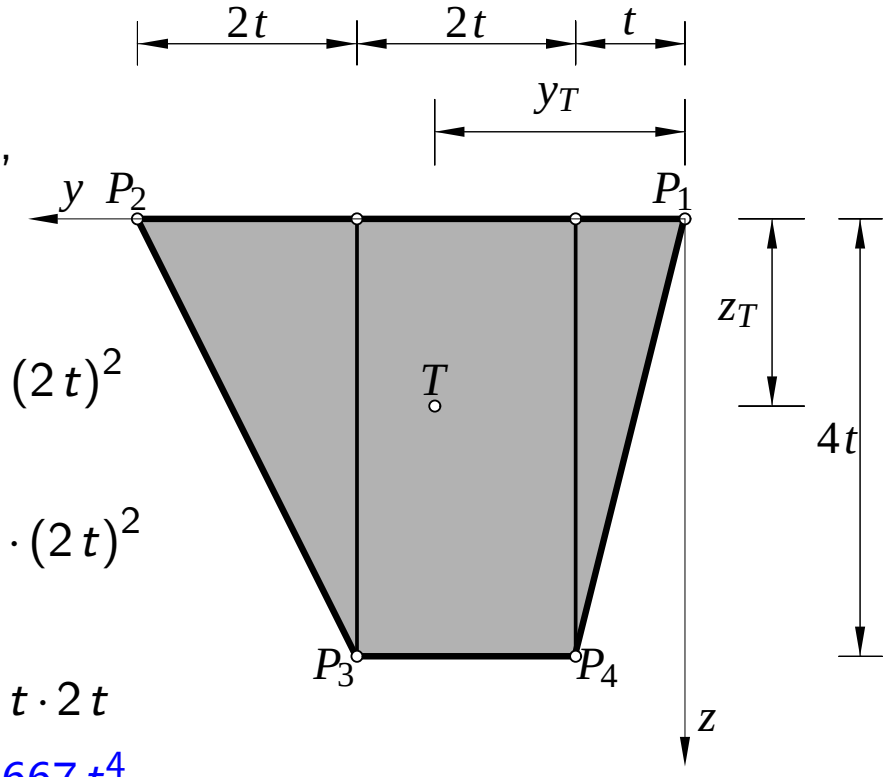
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 = 58.6667\,t^4$$
$$I_z = \frac{(2\,t)^3\cdot 4\,t}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{11\,t}{3}\right)^2 + \frac{(2\,t)^3\cdot 4\,t}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{(1\,t)^3\cdot 4\,t}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{2\,t}{3}\right)^2 = 90.3333\,t^4$$
$$I_{yz} = \frac{(2\,t)^2\cdot(4\,t)^2}{72} - \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11\,t}{3}\cdot\frac{4\,t}{3} + 0 - 2\,t\cdot 4\,t\cdot 2\,t\cdot 2\,t + (-1)\cdot\frac{(1\,t)^2\cdot(4\,t)^2}{72} - \frac{1\,t\cdot 4\,t}{2}\cdot\frac{2\,t}{3}\cdot\frac{4\,t}{3} =$$



Steinerjev stavek: $I_{yz3} = I_{yz3}^{T3} - A_{x3}\cdot y_{T3}\cdot z_{T3}$. Negativni predznak v prvem členu se pojavi zato, ker smo trikotnik prezrcalili okrog osi z (ali os y zamenjali z osjo $-y$) glede na lik v tabeli.

Deviacijski vztrajnostni moment I_{yz} okrog osi y in z

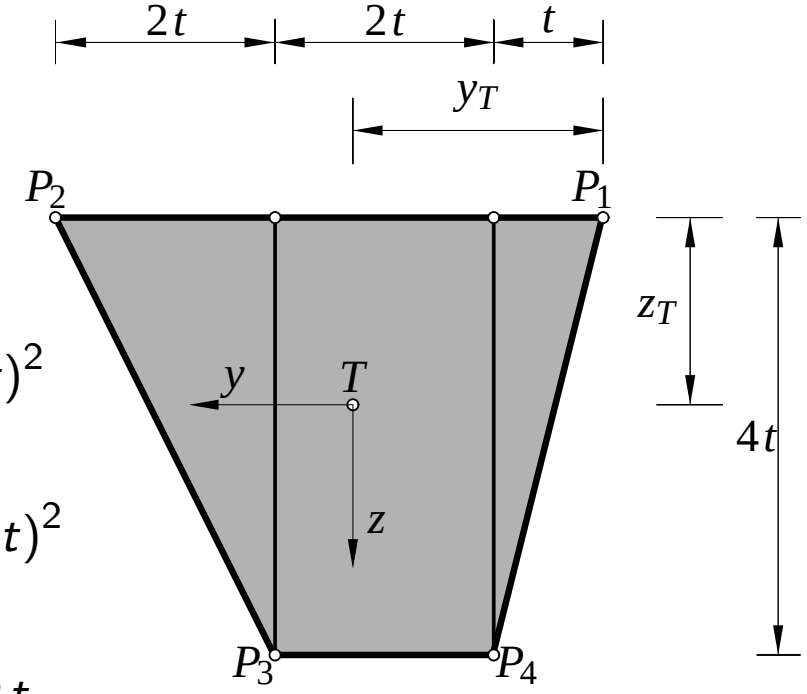
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 = 58.6667\,t^4$$
$$I_z = \frac{(2\,t)^3\cdot 4\,t}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{11\,t}{3}\right)^2 + \frac{(2\,t)^3\cdot 4\,t}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{(1\,t)^3\cdot 4\,t}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{2\,t}{3}\right)^2 = 90.3333\,t^4$$
$$I_{yz} = \frac{(2\,t)^2\cdot(4\,t)^2}{72} - \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11\,t}{3}\cdot\frac{4\,t}{3} + 0 - 2\,t\cdot 4\,t\cdot 2\,t\cdot 2\,t + (-1)\cdot\frac{(1\,t)^2\cdot(4\,t)^2}{72} - \frac{1\,t\cdot 4\,t}{2}\cdot\frac{2\,t}{3}\cdot\frac{4\,t}{3} = -52.6667\,t^4$$



$$I_{yz} = I_{yz1} + I_{yz2} + I_{yz3}$$

Vztrajnostni momenti $I_{yy}^T \equiv I_y^T$, $I_{zz}^T \equiv I_z^T$, I_{yz}^T okrog osi y in z skozi težišče

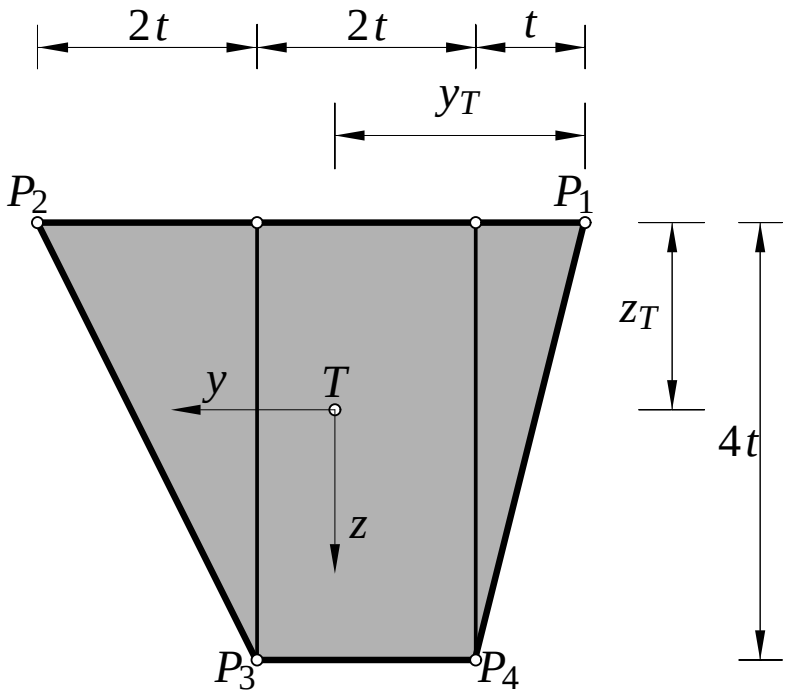
$$A_x = 14\,t^2,$$
$$S_y = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{4}{3}\,t = 24\,t^3,$$
$$S_z = \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11}{3}\,t + 2\,t\cdot 4\,t\cdot 2\,t + \frac{t\cdot 4\,t}{2}\cdot\frac{2}{3}\,t = 32\,t^3,$$
$$y_T = \frac{S_z}{A_x} = 2.2857\,t,$$
$$z_T = \frac{S_y}{A_x} = 1.7143\,t,$$
$$I_y = \frac{2\,t\cdot(4\,t)^3}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 + \frac{2\,t\cdot(4\,t)^3}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{1\,t\cdot(4\,t)^3}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{4\,t}{3}\right)^2 = 58.6667\,t^4$$
$$I_z = \frac{(2\,t)^3\cdot 4\,t}{36} + \frac{2\,t\cdot 4\,t}{2}\cdot\left(\frac{11\,t}{3}\right)^2 + \frac{(2\,t)^3\cdot 4\,t}{12} + 2\,t\cdot 4\,t\cdot(2\,t)^2 + \frac{(1\,t)^3\cdot 4\,t}{36} + \frac{1\,t\cdot 4\,t}{2}\cdot\left(\frac{2\,t}{3}\right)^2 = 90.3333\,t^4$$
$$I_{yz} = \frac{(2\,t)^2\cdot(4\,t)^2}{72} - \frac{2\,t\cdot 4\,t}{2}\cdot\frac{11\,t}{3}\cdot\frac{4\,t}{3} + 0 - 2\,t\cdot 4\,t\cdot 2\,t\cdot 2\,t + (-1)\cdot\frac{(1\,t)^2\cdot(4\,t)^2}{72} - \frac{1\,t\cdot 4\,t}{2}\cdot\frac{2\,t}{3}\cdot\frac{4\,t}{3} = -52.6667\,t^4$$
$$I_y^T = I_y - z_T^2\,A_x = 17.5238\,t^4$$
$$I_z^T = I_z - y_T^2\,A_x = 17.1905\,t^4$$
$$I_{yz}^T = I_{yz} + y_T\,z_T\,A_x = 2.1905\,t^4.$$



Steinerjev stavek

Vztrajnostni tenzor J^T

$$J^T = \begin{bmatrix} 17.5238 & 2.1905 \\ 2.1905 & 17.1905 \end{bmatrix} \cdot t^4,$$



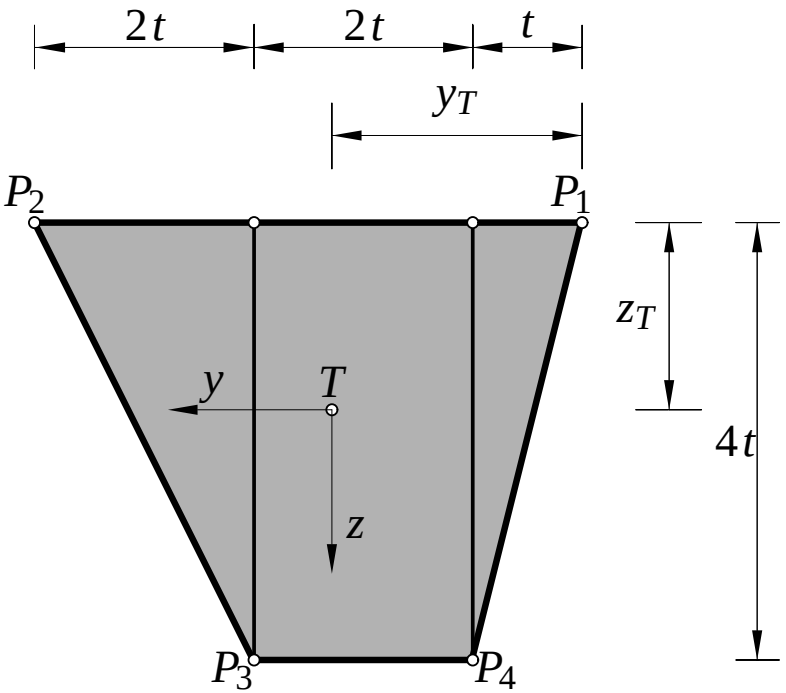
$$J^T = \begin{bmatrix} I_{yy}^T & I_{yz}^T \\ I_{yz}^T & I_{zz}^T \end{bmatrix}$$

J^T je matrika, ki predstavlja vztrajnostni tenzor v kartezičnem koordinatnem sistemu y, z.

Glavna vztrajnostna momenta $I_\eta \equiv I_{\eta\eta}$ in $I_\zeta \equiv I_{\zeta\zeta}$

$$J^T = \begin{bmatrix} 17.5238 & 2.1905 \\ 2.1905 & 17.1905 \end{bmatrix} \cdot t^4,$$

$$I_\eta = 19.5540 \cdot t^4, \quad I_\zeta = 15.1603 \cdot t^4,$$



Glavna vztrajnostna momenta sta lastni vrednosti J^T. Izračunamo ju po enačbi

$$I_{1,2} = \frac{I_y^T + I_z^T}{2} \pm \sqrt{\left(\frac{I_y^T - I_z^T}{2}\right)^2 + {I_{yz}^T}^2}$$

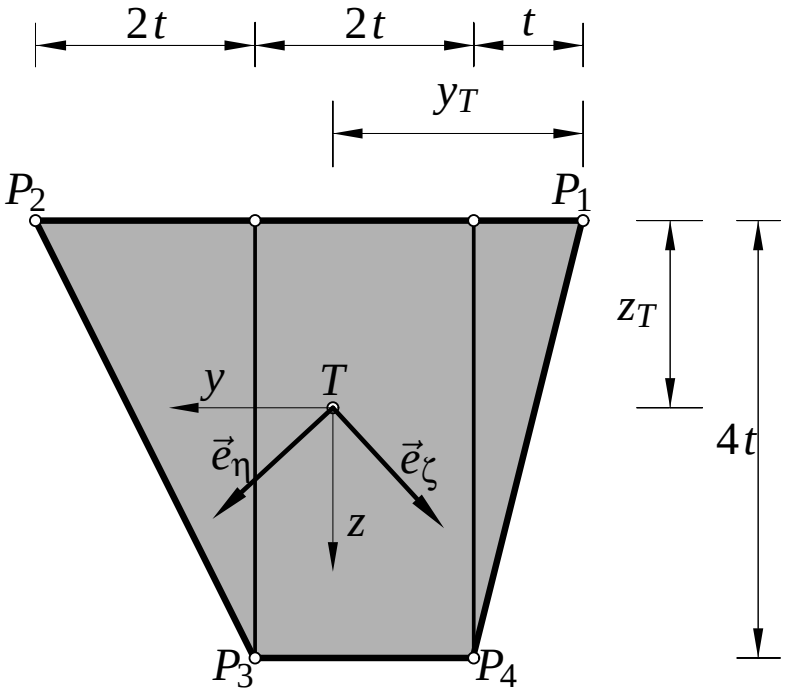
.

Glavni vztrajnostni osi $\vec{e}_\eta$ in $\vec{e}_\zeta$ sta lastna vektorja matrike J^T

$$J^T = \begin{bmatrix} 17.5238 & 2.1905 \\ 2.1905 & 17.1905 \end{bmatrix} \cdot t^4,$$

$$I_\eta = 19.5540 \cdot t^4, \quad I_\zeta = 15.1603 \cdot t^4,$$

$$\begin{aligned} \vec{e}_\eta &= 0.7334\,\vec{e}_y + 0.6798\,\vec{e}_z, \\ \vec{e}_\zeta &= -0.6798\,\vec{e}_y + 0.7334\,\vec{e}_z. \end{aligned}$$



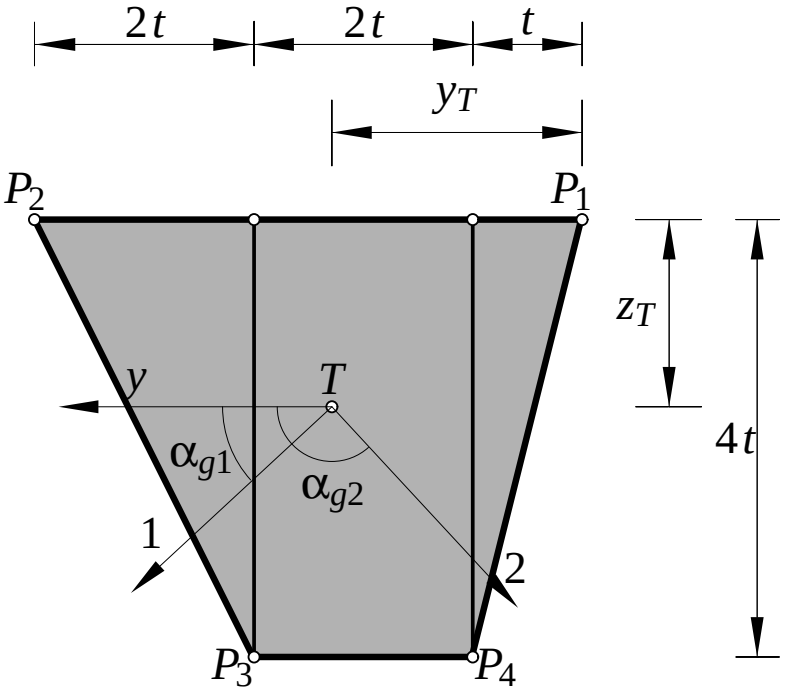
Glavni vztrajnostni osi sta lastna vektorja J^T. Lastnih vektorjev je neskončno mnogo. Izmed vseh izberemo dva pravokotna enotska bazna lastna vektorja.

Glavni vztrajnostni osi $\vec{e}_\eta$ in $\vec{e}_\zeta$ oklepata z osjo y kota α_g

$$J^T = \begin{bmatrix} 17.5238 & 2.1905 \\ 2.1905 & 17.1905 \end{bmatrix} \cdot t^4,$$

$$I_\eta = 19.5540 \cdot t^4, \quad I_\zeta = 15.1603 \cdot t^4,$$

$$\begin{aligned} \vec{e}_\eta &= 0.7334\,\vec{e}_y + 0.6798\,\vec{e}_z, \\ \vec{e}_\zeta &= -0.6798\,\vec{e}_y + 0.7334\,\vec{e}_z. \end{aligned}$$



Kot α_g je kot, ki ga glavni osi $\vec{e}_\eta$ in $\vec{e}_\zeta$ oklepata z osjo y. Izračunamo ga po enačbi

$$\tan(2\alpha_g) = \frac{2\,I_{yz}^T}{I_y^T - I_z^T}.$$

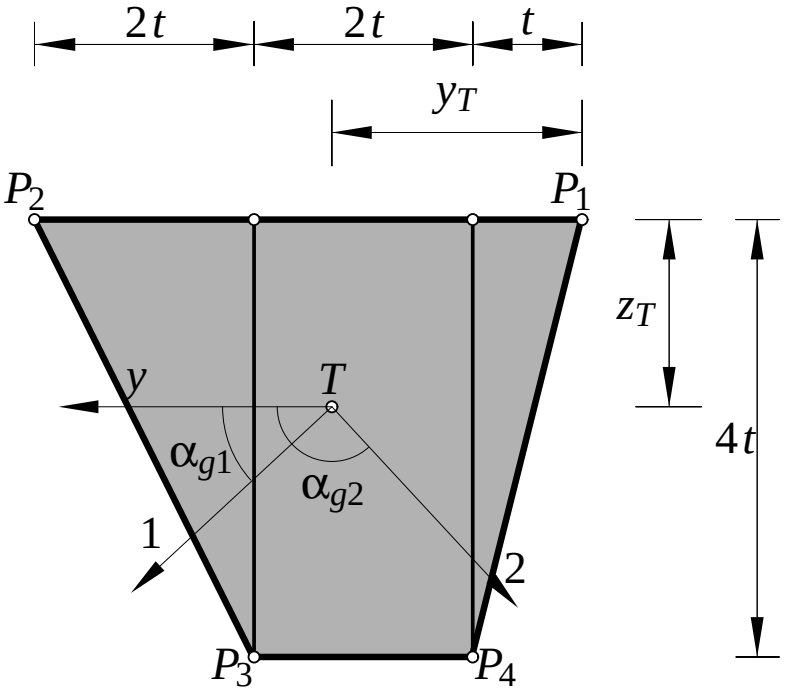
Enačba ima v splošnem dve rešitvi α_{g1} in α_{g2}.

Glavni vztrajnostni osi $\vec{e}_\eta$ in $\vec{e}_\zeta$ oklepata z osjo y kota α_g

$$J^T = \begin{bmatrix} 17.5238 & 2.1905 \\ 2.1905 & 17.1905 \end{bmatrix} \cdot t^4,$$

$$I_\eta = 19.5540 \cdot t^4, \quad I_\zeta = 15.1603 \cdot t^4,$$

$$\begin{aligned} \vec{e}_\eta &= 0.7334\,\vec{e}_y + 0.6798\,\vec{e}_z, \\ \vec{e}_\zeta &= -0.6798\,\vec{e}_y + 0.7334\,\vec{e}_z. \end{aligned}$$



Po enačbi

$$I_{\xi\xi}(\alpha) = \frac{I_y^T + I_z^T}{2} + \frac{I_y^T - I_z^T}{2}\cos(2\alpha) + I_{yz}^T \sin(2\alpha)$$

dobimo $I_\eta = I_1 = I_{\xi\xi}(\alpha_{g1})$ in $I_\zeta = I_2 = I_{\xi\xi}(\alpha_{g2})$.