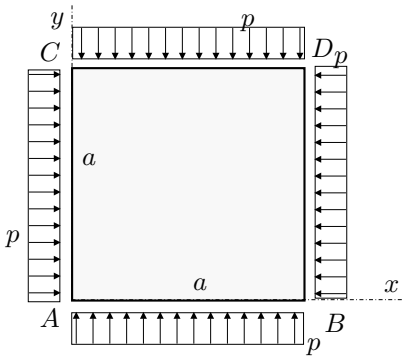
TRDNOST (VSŠ) - 1. KOLOKVIJ (17. 12. 2003)

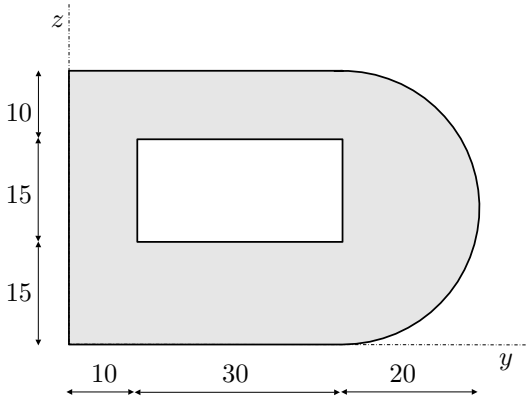
Pazljivo preberite besedilo vsake naloge! Naloge so točkovane enakovredno (vsaka 25%)! Pišite čitljivo! Uspešno reševanje!

1. Deformiranje telesa je podano s poljem pomikov $\vec{u} = 10^{-4}\left[(x-z)\,\vec{e}_x + x\,y\,\vec{e}_y + \left(z^2-y\right)\vec{e}_z\right]$.
Izračunajte:
a) tenzor majhnih deformacij;
b) specifično spremembo dolžine vlakna v točki $(-1,1,0)$ v smeri vektorja $\vec{a} = 3\vec{e}_x - \vec{e}_y + 2\vec{e}_z$;
c) spremembo pravega kota v točki $(-1,1,0)$ med vektorjema $\vec{a}$ in $\vec{b} = \vec{e}_x + 3\vec{e}_y$.

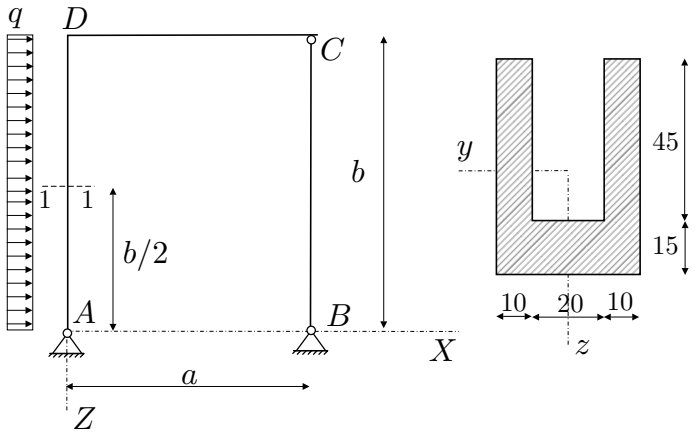
2. Na rob tanke kvadratne stene deluje normalna enakomerna površinska obtežba velikosti p, kot kaže slika. Privzemimo, da so napetosti po celotni prostornini stene konstantne. Določite velikost obtežbe pri kateri je specifična sprememba prostornine enaka nič ($\varepsilon_V = 0$), če poznamo naslednje materialne parametre:
$\nu = 0.2$, $E = 2.4 \cdot 10^4\,\mathrm{kN/cm^2}$,
$\alpha = 10^{-5}\,\mathrm{K}^{-1}$, $\Delta T = 10\mathrm{K}$.



3. Izračunajte geometrijske karakteristike (A, y_T, z_T, I_y, I_z, I_{yz}, I_y^T, I_z^T, I_{yz}^T) lika na sliki!
Podatki so v centimetrih.



4. Za konstrukcijo na sliki izračunajte vrednosti notranjih statičnih količin (N_x, N_z, M_y) v prerezu $1-1$! Določite nekaj značilnih vrednosti in skicirajte diagrama normalne napetosti σ_{xx} in strižne napetosti σ_{xz} v tem prerezu!
Podatki: $a = 4\,\mathrm{m}$, $b = 6\,\mathrm{m}$,
$q = 2\,\mathrm{kN/m}$.
Podatki za prerez so v centimetrih.



REŠITVE NALOG

1. a) $\begin{bmatrix} 1 & \frac{y}{2} & -\frac{1}{2} \\ \frac{y}{2} & x & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 2\,z \end{bmatrix}$;

   b) $D_{aa} \simeq \varepsilon_{aa} = 7.143 \cdot 10^{-6}$;

   c) $D_{ab} \simeq 2\varepsilon_{aa} = 1.01419 \cdot 10^{-4}$;

2. $\sigma_{xz} = 0$, $\sigma_{yz} = 0$, $\sigma_{zz} = 0$;

   $\sigma_{xx} = -p$, $\sigma_{xy} = 0$, $\sigma_{yy} = -p$;

   $\varepsilon_{xx} = \varepsilon_{yy} = \left(1 - \frac{1}{3}p\right) \cdot 10^{-4}$, $\varepsilon_{zz} = \left(1 + \frac{1}{6}p\right) \cdot 10^{-4}$;

   $\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0 \quad \Rightarrow \quad p = 6.$

3. $A_x = 1777.26$, $y_T = 28.79$, $z_T = 19.37$;

   $I_y = 930\,594$, $I_z = 2\,030\,172$, $I_{yz} = -995\,073$;

   $I_y^T = 263\,988.5$, $I_z^T = 557\,463.5$, $I_{yz}^T = -4\,256.7$.

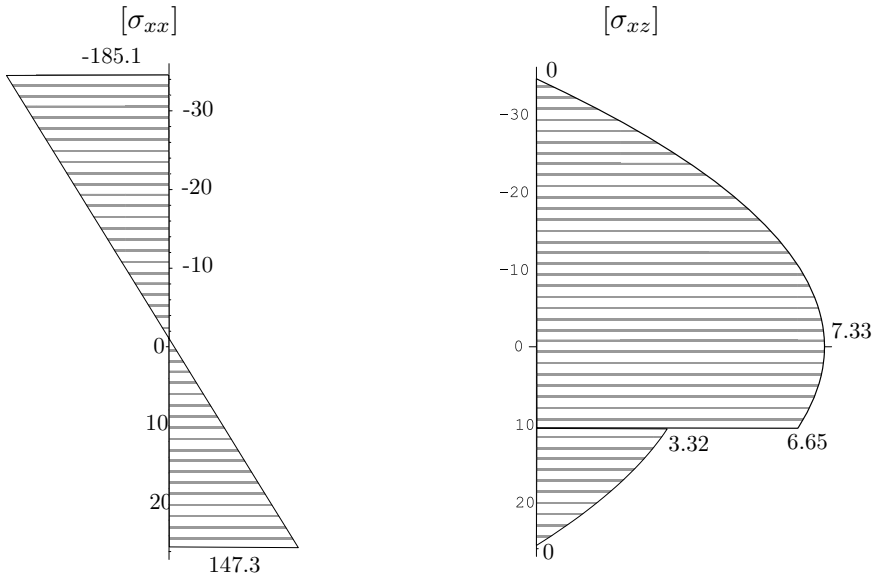
4. $N_x = 9$, $N_z = 6$, $M_y = 27$;

   $A_x = 1500$, $I_y^T = 487\,125$;

   $\sigma_{xx} = 6 + 5.54z \ \left[\mathrm{N/cm^2}\right]$;

z^*	b^*	S_y^*	$\sigma_{xz} \quad \left[\mathrm{N/cm^2}\right]$
-34.5	20	0	0
0	20	-11902.5	7.33
10.5	20	-10800	6.65
10.5	40	-10800	3.32
25.5	40	0	0

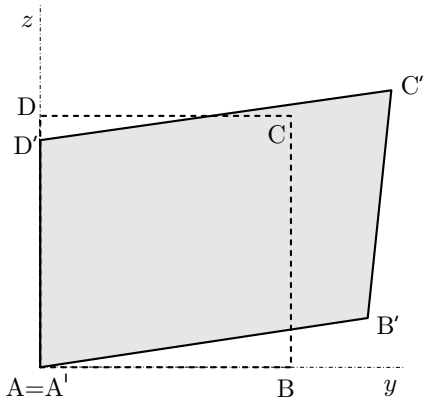
.



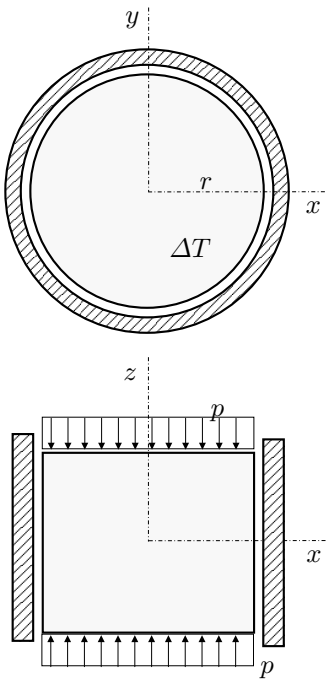
TRDNOST (VSŠ) - 1. KOLOKVIJ (6. 12. 2004)

Pazljivo preberite besedilo vsake naloge! Pišite čitljivo! Uspešno reševanje!

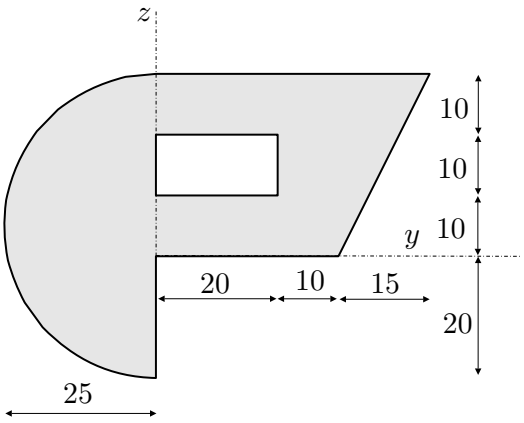
1. Tanka kvadratna ploščica z robom 3 cm se deformira, kot kaže slika. Točka A se ne premakne. Nove koordinate točke B so $B'(3.009, 0.006)$, točka C se premakne v $C'(3.018, 3.003)$, točka D pa v $D'(0, 2.997)$ Deformiranje je podano s poljem pomikov
$\vec{u} = (axy + bx)\,\vec{e}_x + (cx + dy)\,\vec{e}_y$.
Izračunajte:
a) konstante a, b, c in d;
b) tenzor majhnih deformacij;
c) specifično spremembo dolžine vlakna v točki C v smeri vektorja $\vec{AC}$;
d) spremembo pravega kota v točki C med vektorjema $\vec{AC}$ in $\vec{BD}$.



2. V togo, nerazteglivo cev je postavljen valj iz izotropnega, linearno elastičnega materiala, kot kaže slika. Polmer valja je 20 cm, razdalja med plaščem valja in cevjo pa 1 mm. Valj segrejemo za 60K in obtežimo z enakomerno površinsko obtežbo p na spodnji in zgornji ploskvi. Privzemimo, da so napetosti po celotni prostornini valja konstantne.
a) Določite velikost obtežbe p, pri kateri se valj dotakne cevi!
b) Valj obremenimo z obtežbo velikosti $p = 500\ \mathrm{kN/cm^2}$. Določite napetostni tenzor! Določite tudi specifično spremembo volumna.
Podatki: $\nu = 0.3$, $E = 2.1 \cdot 10^4\ \mathrm{kN/cm^2}$,
$\alpha = 10^{-5}\ \mathrm{K^{-1}}$, $\Delta T = 60\mathrm{K}$.



3. Izračunajte geometrijske karakteristike (A, y_T, z_T, I_y, I_z, I_{yz}, I_y^T, I_z^T, I_{yz}^T) lika na sliki!
Podatki so v centimetrih.



REŠITVE NALOG

1. a) $a = 0.001, b = 0.003, c = 0.002, d = -0.001$;

   b) $\varepsilon = \begin{bmatrix} 3+y & 1+\frac{x}{2} & 0 \\ 1+\frac{x}{2} & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$;

   c) $D_{AC} \simeq \varepsilon_{AC} = 0.005$;

   d) $D_{ab} \simeq 2\varepsilon_{aa} = -0.007$;

2. a) $p = 308\ \mathrm{kN/cm^2}$;

   b) $\sigma = \begin{bmatrix} -82.3 & 0 & 0 \\ 0 & -82.3 & 0 \\ 0 & 0 & -500 \end{bmatrix}, \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = -0.01086$;

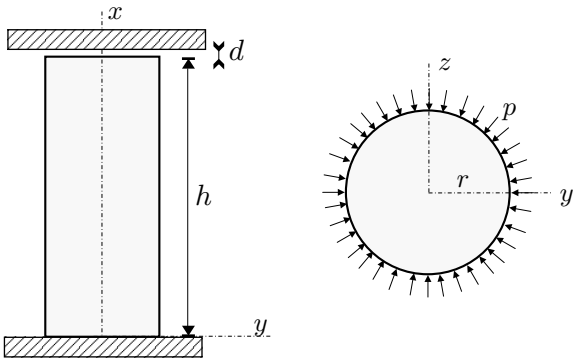
3. $A_x = 1905$, $y_T = 4.72$, $z_T = 10.45$;
   $I_y = 501\,966$, $I_z = 674\,658$, $I_{yz} = -280\,860$;
   $I_y^T = 294\,076.4$, $I_z^T = 632\,288.6$, $I_{yz}^T = -187\,007.4$.

TRDNOST (VSŠ) - 1. KOLOKVIJ (9. 12. 2005)

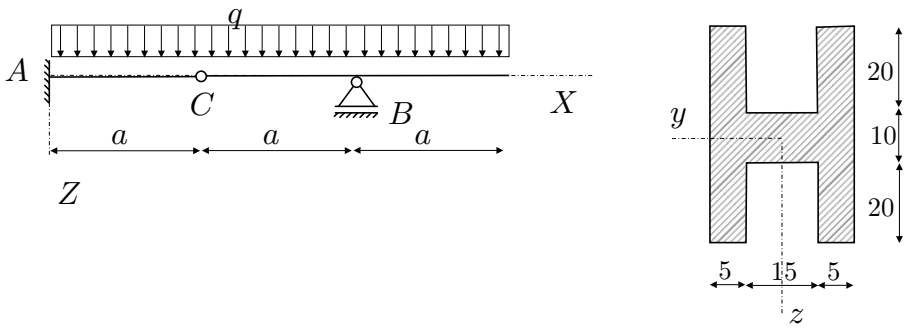
Pazljivo preberite besedilo vsake naloge! Pišite čitljivo! Uspešno reševanje!

1. Deformiranje telesa je podano s poljem pomikov
$\overrightarrow{u} = 10^{-4}\,(\,2y\,(x-z)\,,\,xy,\,-x+y\,)$.
Izračunajte:
a) tenzor velikih deformacij;
b) tenzor majhnih deformacij in njegovo vrednost v točki $T\,(0,1,-1)$;
c) specifično spremembo dolžine vlakna v točki T v smeri vektorja $(1,1,0)$;
d) spremembo pravega kota v točki T med vektorjema $(1,1,0)$ in $(1,b,1)$.

2. Valjast vzorec iz izotropnega, linearno elastičnega materiala, višine $h = 100$ cm, je postavljen med dve togi nepomični plošči, kot kaže slika. Polmer valja je 5 cm, razdalja med valjem in zgornjo ploščo pa $d = 1$ mm. Valj obtežimo po plašču z enakomerno normalno površinsko obtežbo p. Privzemimo, da so napetosti po celotni prostornini valja konstantne.
a) Določite velikost obtežbe p, pri kateri se valj dotakne zgornje plošče!
b) Poleg obtežbe p vzorec še segrejemo. Določite spremembo temperature, pri kateri bo specifična sprememba volumna enaka 0.
Podatki: $\nu = 0.3$, $E = 2.1 \cdot 10^4\,\mathrm{kN/cm^2}$,
$\alpha = 1.2 \cdot 10^{-5}\,\mathrm{K}^{-1}$.



3. Izračunajte prečne sile N_z in upogibne momente M_y za konstrukcijo na sliki. Na mestu ekstremnih upogibnih momentov izračunajte in narišite potek normalnih napetosti σ_{xx}. Na mestu ekstremnih prečnih sil izračunajte in narišite potek strižnih napetosti σ_{xz}.
Podatki: $a = 2\,\mathrm{m}$, $q = 10\,\mathrm{kN/m}$.
Podatki za prerez na sliki so v centimetrih.



TRDNOST - VSŠ 9. 12. 05

1. NALOGA

$$\frac{\partial u_x}{\partial x} = 2y \quad \frac{\partial u_x}{\partial y} = 2(x-z) \quad \frac{\partial u_x}{\partial z} = -2y$$

$$\frac{\partial u_y}{\partial x} = y \quad \frac{\partial u_y}{\partial y} = x \quad \frac{\partial u_y}{\partial z} = 0$$

$$\frac{\partial u_z}{\partial x} = -1 \quad \frac{\partial u_z}{\partial y} = 1 \quad \frac{\partial u_z}{\partial z} = 0 \qquad / \cdot 10^{-4}$$

a.)

$$[E] = 10^{-4}\begin{bmatrix} 2y + \frac{10^{-4}}{2}(5y^2+1) & (x-z)+\frac{y}{2}+\frac{10^{-4}}{2}(4y(x-z)+xy-1) & -y-\frac{1}{2}+\frac{10^{-4}}{2}(-4y^2) \\ & x+\frac{10^{-4}}{2}(4(x-z)^2+x^2+1) & \frac{1}{2}+\frac{10^{-4}}{2}(-4y(x-z)) \\ \text{simetrično} & & 0+\frac{10^{-4}}{2}(4y^2 \end{bmatrix}$$

b.)

$$[\varepsilon] = \begin{bmatrix} 2y & x-z+\frac{y}{2} & -y-\frac{1}{2} \\ x-z+\frac{y}{2} & x & \frac{1}{2} \\ -y-\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \cdot 10^{-4} \qquad [\varepsilon]_T = \begin{bmatrix} 2 & \frac{3}{2} & -\frac{3}{2} \\ \frac{3}{2} & 0 & \frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} & 0 \end{bmatrix} \cdot 10^{-4}$$

c.) $\vec{a} = (1,1,0)$

$\vec{e}_a = \frac{1}{\sqrt{2}}(1,1,0)$

$$\varepsilon_{aa} = 10^{-4} \cdot \frac{1}{2} \cdot [1\ 1\ 0] \begin{bmatrix} 2 & \frac{3}{2} & -\frac{3}{2} \\ \frac{3}{2} & 0 & \frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{10^{-4}}{2} [1\ 1\ 0] \begin{bmatrix} \frac{7}{2} \\ \frac{3}{2} \\ -\frac{2}{2} \end{bmatrix}$$

$$= \frac{10^{-4}}{2}\left(\frac{10}{2}\right) = \underline{\underline{2 \cdot 5 \cdot 10^{-4}}}$$

d.) $\vec{b} = (1, b, 1)$

$\vec{a} \cdot \vec{b} = 1 + b = 0 \quad \boxed{b = -1}$

$\vec{e}_b = \frac{1}{\sqrt{3}}(1,-1,1)$

$$\varepsilon_{ab} = 10^{-4} \cdot \frac{1}{\sqrt{6}} [1\ -1\ 1] \begin{bmatrix} \frac{7}{2} \\ \frac{3}{2} \\ -1 \end{bmatrix} = 10^{-4} \cdot \frac{1}{\sqrt{6}} \left(\frac{4}{2} - 1\right) = \frac{10^{-4}}{\sqrt{6}}$$

$D_{ab} \approx 2\varepsilon_{ab} = 8 \cdot 16 \cdot 10^{-5}$ radiana $(4 \cdot 7 \cdot 10^{-3}$ stopinje$)$

TRDNOST - VSŠ 9. 12. 2005

2. NALOGA

a.) $\boxed{\varepsilon_{xx} = \frac{d}{h} = \frac{1}{1000} = 10^{-3}}$

$$\begin{bmatrix} 0 \\ -p \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \boxed{\sigma_{yy} = -p} \quad \sigma_{xy} = 0 \quad \sigma_{yz} = 0$$

$$\begin{bmatrix} 0 \\ 0 \\ -p \end{bmatrix} = \begin{bmatrix} \sigma \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \boxed{\sigma_{zz} = -p} \quad \sigma_{xz} = 0 \quad \sigma_{yz} = 0$$

$$\boxed{\sigma_{xx} = 0}$$

$$\varepsilon_{xx} = \frac{1+\nu}{E}\sigma_{xx} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

$$\varepsilon_{xx} = +\frac{\nu}{E} \cdot 2p \Rightarrow \boxed{p = \frac{E \cdot \varepsilon_{xx}}{2\nu} = 35\ kN/cm^2}$$

b.) $p = 35\ kN/cm^2 \quad \varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0$

$\Delta T = ?$

$$\left.\begin{aligned} \sigma_{xx} &= 2\mu\varepsilon_{xx} + \lambda\varepsilon_v - \beta_T\Delta T \\ \sigma_{yy} &= 2\mu\varepsilon_{yy} + \lambda\varepsilon_v - \beta_T\Delta T \\ \sigma_{zz} &= 2\mu\varepsilon_{zz} + \lambda\varepsilon_v - \beta_T\Delta T \end{aligned}\right\} + \quad \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = 2\mu\underbrace{\varepsilon_v}_{0} - 3\beta_T\Delta T$$

$$\boxed{\sigma_{xx} - 2p = -3\beta_T\Delta T}$$

$$\sigma_{xx} = 2\mu\varepsilon_{xx} - \beta_T\Delta T$$

$$2\mu\varepsilon_{xx} - 2p = -2\beta_T\Delta T$$

$$\Delta T = \frac{p - \mu\varepsilon_{xx}}{\beta_T}$$

$$\mu = \frac{2 \cdot 1 \cdot 10^4}{2 \cdot 1 \cdot 3} = 0{\cdot}808 \cdot 10^4 \frac{kN}{cm^2}$$

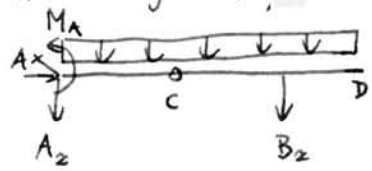
$$\beta_T = \frac{1{\cdot}2 \cdot 10^{-5} \cdot 2{\cdot}1 \cdot 10^4}{0{\cdot}4} = 0{\cdot}63$$

$$\boxed{\Delta T = 42{\cdot}7\ K}$$

TRDNOST - VSŠ 9. 12. 2005

3. NALOGA

a.) Notranje sile



$A_x = 0$

$A_z + B_z + g \cdot 3a = 0$

$M_A - B_z \cdot 2a - g\,3a\,\frac{3a}{2} = 0$

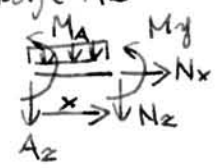
$-B_z \cdot a - g \cdot 2a \cdot a = 0 \Rightarrow$

$A_z = -20\,kN$

$M_A = 20\,kN$

$B_z = -40\,kN$

polje AB



$N_x = 0$

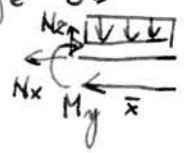
$N_z = +20 - gx$ $\quad N_z(2) = 0,\ N_z(4) = -20$

$M_y = -20 + 20x - g\frac{x^2}{2}$

$M_y(2) = 0$

$M_y(4) = -20$

polje BD



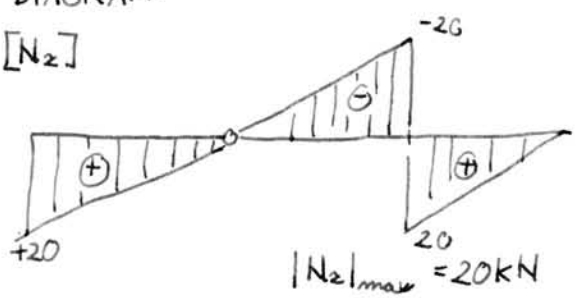
$N_x = 0$

$N_z = g\bar{x}$ $\quad N_z(2) = 20$

$M_y = -g\frac{\bar{x}^2}{2}$ $\quad M_y(2) = -20$

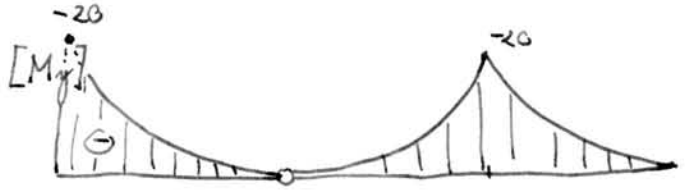
DIAGRAMI

$[N_z]$



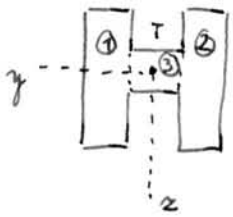
$|N_z|_{max} = 20kN$

$[M_y]$



$|M_y|_{max} = 20kNm$

b.) Karakteristike prereza



① $A^{①} = 250\,cm^2$ $\quad y^{①} = 10\,cm$ $\quad z^{①} = 0$

② $A^{②} = 250\,cm^2$ $\quad y^{②} = -10\,cm$ $\quad z^{②} = 0$

③ $A^{③} = 150\,cm^2$ $\quad y^{③} = 0$ $\quad z^{③} = 0$

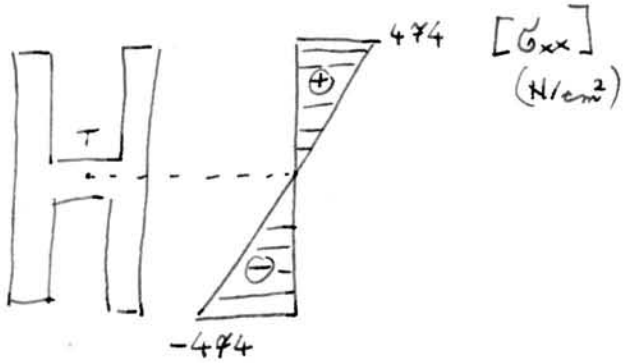
$I_y^{①} = 52083.\overline{33}$ $\quad I_y^{②} = 52083.\overline{33}\,cm^4$ $\quad I_y^{③} = 1250\,cm^4$

$$I_y = I_y^{①} + I_y^{②} + I_y^{③} = 105\,416.7\,cm^4$$

c.) Normalne napetosti

$$\sigma_{xx} = \frac{M_y}{I_y} \cdot z = \frac{-20\,kN \cdot 100\,cm}{105\,416.7\,cm^4} \cdot z = -18.97\,z\ [N/cm^2]$$

$\sigma_{xx}(-25) = 474\ N/cm^2$ $\quad \sigma_{xx}(25) = -474\ N/cm^2$



d.) Strižne napetosti

$$\sigma_{xz}(z^*) = -\frac{1}{b^*}\left(\frac{S_y^*}{I_y}\cdot N_z\right) \qquad N_z = 20\ kN$$
$$I_y = 10541 6.7\ cm^4$$

d1.) $z^* \in [-25, -5]$

$z \in [-25, z^*] \quad y \in [-12.5, -7.5] \quad y \in [7.5, 12.5]$

$$S_y(z^*) = \int_{A^*} z\,dA^* = 2\cdot\int_{-12.5}^{-7.5}\int_{-25}^{z^*} z\,dz\,dy = 2\cdot\frac{z^2}{2}\Big|_{-25}^{z^*}\cdot y\Big|_{-12.5}^{-7.5} =$$

$$= 5(z^{*2} - 625)$$

$S_y^*(-25) = 0$ | $S_y^*(-5) = -3000\ cm^3$ | $b^* = 10\ cm$

d2) $z^* \in [-5, 5]$

$$S_y^* = \int_{-12.5}^{12.5}\int_{-5}^{z^*} z\,dz\,dy + S_y^*(-5) = -3000 + 25\cdot\frac{z^{*2}}{2}\Big|_{-5}^{z^*} =$$

$$= -3000 + 12.5(z^{*2} - 25)$$

$S_y^*(-5) = -3000\ cm^3$ | $S_y^*(5) = -3000\ cm^3$ | $b^* = 25\ cm$

$S_y^*(0) = -3312.5\ cm^3$

d3) $z^* \in [5, 25]$

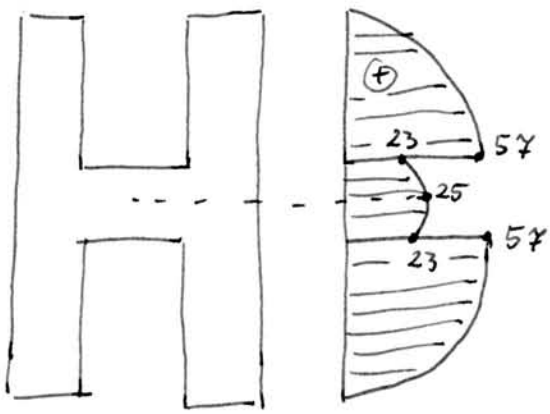
$$S_y^* = 2\cdot\int_{-12.5}^{7.5}\int_{5}^{z^*} z\,dz\,dy - 3000$$

$$= -3000 + 5\cdot(z^{*2} - 25)$$

$S_y^*(5) = -3000\ cm^3$ | $S_y^*(25) = 0$ | $b^* = 10\ cm$

TABELA

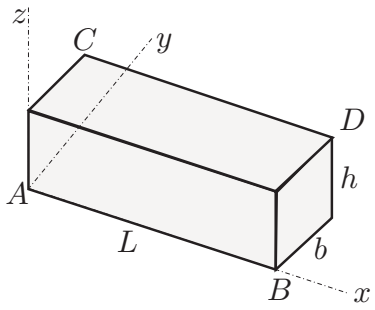
z^*	S_y^*	b^*	σ_{xz}^* [N/cm²]
-25	0	10	0
-5	-3000	10	56.9
-5	-3000	25	22.77
0	-3312	25	25.13
5	-3000	25	22.77
5	-3000	10	56.9
25	0	10	0



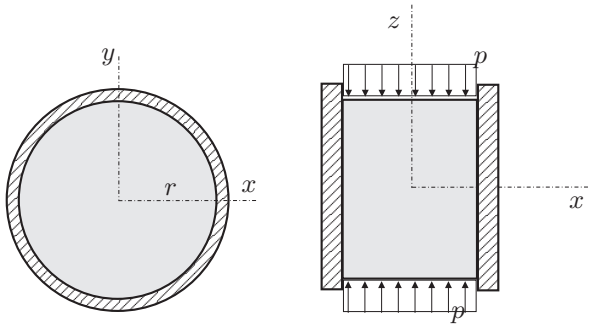
TRDNOST (VSŠ) - 1. KOLOKVIJ (15. 12. 2006)

Pazljivo preberite besedilo vsake naloge! Naloge so točkovane enakovredno (vsaka 25%).
Uspešno reševanje!

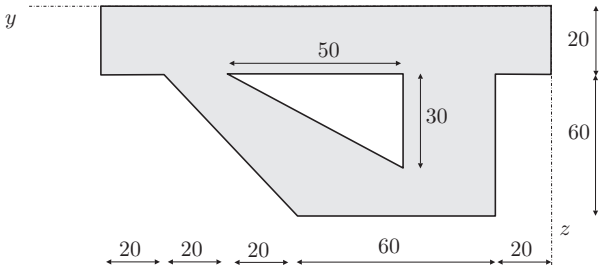
1. Kvader na sliki ima stranice dolžin $L = 20$ cm, $b = h = 5$ cm. Izmerjene spremembe dolžin stranic znašajo $\Delta L = 0.2$ mm, $\Delta b = \Delta h = 0.1$ mm. Telesna diagonala AD se je podaljšala za 0.8 mm, kot med osema y in z pa se ni spremenil. Spremembi pravih kotov med osema x in y ter x in z sta enaki.
Izračunajte:
a) tenzor majhnih deformacij;
b) specifično spremembo volumna;
c) spremembo dolžine diagonale BC.



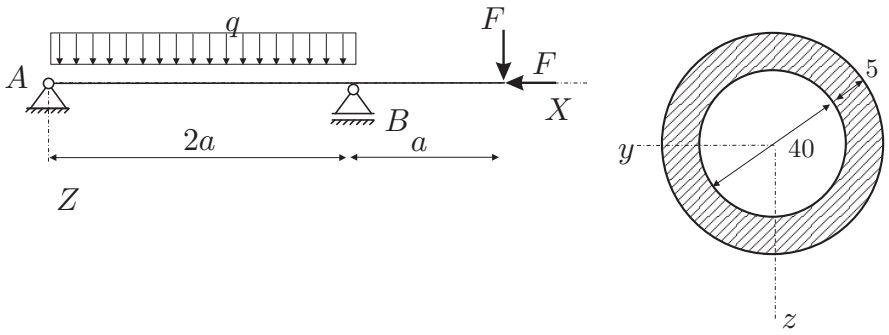
2. V togo, nerazteglivo cev je postavljen valj iz izotropnega, linearno elastičnega materiala. Polmer valja je 5 cm, višina pa 20 cm. Valj obremenimo z enakomerno površinsko obtežbo p na spodnji in zgornji ploskvi. Privzemimo, da so napetosti po celotni prostornini valja konstantne. Trenje med valjem in cevjo zanemarimo.
a) Določite normalne napetosti med valjem in cevjo!
b) Določite tudi spremembo višine valja.
Podatki: $\nu = 0.2$, $E = 2 \cdot 10^4\,\mathrm{kN/cm^2}$,
$p = 3\,\mathrm{kN/cm^2}$.



3. Izračunajte geometrijske karakteristike (A, y_T, z_T, I_y, I_z, I_{yz}, I_y^T, I_z^T, I_{yz}^T ter glavna vztrajnostna momenta) lika na sliki!
Podatki so v centimetrih.



4. Na mestu največjih in najmanjših upogibnih momentov konstrukcije na sliki izračunajte in narišite potek normalnih napetosti σ_{xx}.
Podatki: $a = 2\,\mathrm{m}$, $q = 5\,\mathrm{kN/m}$, $F = 10\,\mathrm{kN}$.
Podatki za prerez na sliki so v centimetrih.



TRDNOST VSŠ 1. KOLOKVIJ 15. 12. 06

1. NALOGA

$$\varepsilon_{xx} = \frac{\Delta L}{L} = \frac{2}{2000} = 0.001 = 1 \cdot 10^{-3}$$

$$\varepsilon_{yy} = \varepsilon_{zz} = \frac{1}{500} = 0.002 = 2 \cdot 10^{-3}$$

$$\varepsilon_{yz} = 0$$

$$\varepsilon_{xy} = \varepsilon_{xz}$$

$$\varepsilon_{dd} = \frac{\Delta d}{d} = \frac{8}{\sqrt{450} \cdot 100} = 3.771 \cdot 10^{-3}$$

$$d = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{450}\ cm$$

$$[\varepsilon] = \begin{bmatrix} 1 & \varepsilon_{xy} & \varepsilon_{xy} \\ \varepsilon_{xy} & 2 & 0 \\ \varepsilon_{xy} & 0 & 2 \end{bmatrix} \qquad \vec{d} = (20, 5, 5)$$

$$\vec{e}_d = \frac{1}{\sqrt{450}} (20, 5, 5) = \frac{5}{\sqrt{450}} (4, 1, 1)$$

$$\varepsilon_{dd} = \frac{25}{450} \begin{bmatrix} 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & \varepsilon_{xy} & \varepsilon_{xy} \\ \varepsilon_{xy} & 2 & 0 \\ \varepsilon_{xy} & 0 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} \cdot 10^{-3} =$$

$$= \frac{10^{-3}}{18} \begin{bmatrix} 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 + 2\varepsilon_{xy} \\ 4\varepsilon_{xy} + 2 \\ 4\varepsilon_{xy} + 2 \end{bmatrix} = \frac{1}{18} (16\varepsilon_{xy} + 20) \cdot 10^{-3}$$

$$\varepsilon_{xy} = \frac{18\varepsilon_{dd} - 20}{16} = 2.99 \cdot 10^{-3} \qquad \Rightarrow [\varepsilon] = 10^{-3} \begin{bmatrix} 1 & 3 & 3 \\ 3 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

b.) $\varepsilon_V = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 5 \cdot 10^{-3}$

c.) $\vec{BC} = (-20, 4, 4) \Rightarrow e_{BC} = \frac{5}{\sqrt{450}} (-4, 1, 1)$

$$\varepsilon_{BC} = \frac{25}{450} \begin{bmatrix} -4 & 1 & 1 \end{bmatrix} 10^{-3} \begin{bmatrix} 1 & 3 & 3 \\ 3 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix} = -1.549 \cdot 10^{-3}$$

2. NALOGA

a.) deformacije $\varepsilon_{xx} = \varepsilon_{yy} = 0$

b.) napetosti

NORMALA z

$$[p_s] = \begin{bmatrix} 0 \\ 0 \\ -p \end{bmatrix} \qquad [e_s] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[\sigma][e_s] = [p_s] \Rightarrow \sigma_{xz} = 0$$

$$\sigma_{yz} = 0$$

$$\boxed{\sigma_{zz} = -p = -3\ kN/cm^2}$$

c.) Hookov zakon

$$\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

$$\sigma_{zz} = 2G\varepsilon_{zz} + \lambda\varepsilon_v = 2G\varepsilon_{zz} + \lambda\varepsilon_{zz}$$

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{2G+\lambda}$$

$$G = \frac{E}{2(1+\nu)} = \frac{2\cdot 10^4}{2\cdot 1{,}2} = \frac{5}{6}\cdot 10^4\ kN/cm^2$$

$$\lambda = \frac{0{,}2E}{1{,}2\cdot 0{,}6} = \frac{10}{18}\ 10^4\ kN/cm^2$$

$$\varepsilon_{zz} = -\frac{3}{\frac{25}{18}}\cdot 10^{-4}$$

$$\varepsilon_{zz} = -1{,}35\ 10^{-4}$$

$$\sigma_{xx} = 2G\overset{0}{\varepsilon_{xx}} + \lambda\varepsilon_{zz}$$

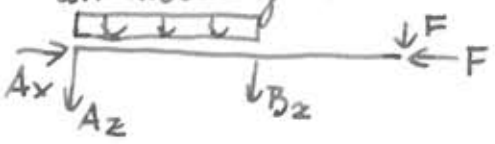
$$\boxed{\sigma_{xx} = \frac{10}{18}\cdot 10^4(-1{,}35)10^{-4} = -\frac{15}{20} = -0{,}75\ kN/cm^2}$$

$$\boxed{\sigma_{yy} = -0{,}75\ kN/cm^2}$$

$$\boxed{\Delta h = \varepsilon_{zz}\cdot h = -1{,}35\ 10^{-4}\cdot 200\ mm = -0{,}027\ mm}$$

4. NALOGA

a.) notranje sil



$A_x = F$

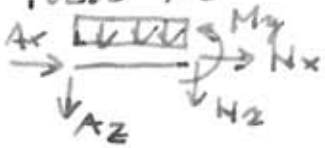
$A_z + B_z = g \cdot 2a + F$

$-B_z \cdot 2a + g \cdot 2a \cdot a - F \cdot 3a = 0$

$B_z = -\frac{1}{2}(2ga + 3F)$

$A_z = -5\ kN$

POLJE AB



$N_x = -A_x$ $\quad N_x = -10\ kN$

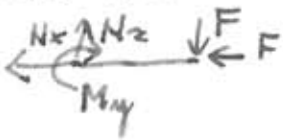
$N_z = -A_z - gx$ $\quad N_z = 5 - 5x$

$M_y = -A_z x - g\frac{x^2}{2}$ $\quad M_y = 5x - 2{,}5x^2$

$B_z = -25\ kN$

$M_y(1) = 2{,}5\ kNm$ (ekstrem!)

POLJE BC



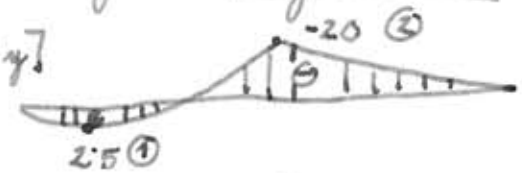
$N_x = -F$ $\quad N_x = -10\ kN$ $\quad M_y(4) = -20\ kNm$

$N_z = F$ $\quad N_z = 10\ kN$

$M_y = -Fx$ $\quad M_y = -20\ kNm$

$N_x = -10$

[My]



b.) Prerez: $A = \pi(25^2 - 20^2) = 706{,}8\ cm^2$

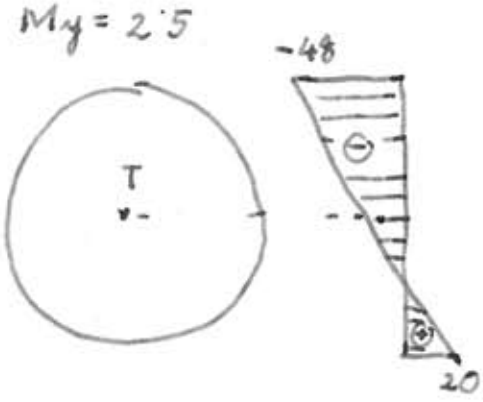
$I_y = \frac{\pi \cdot 45^4}{64} - \frac{\pi \cdot 40^4}{64} = 1{,}811 \cdot 10^5\ cm^4$

c.) $\sigma_{xx}^{(4)} = \frac{N_x}{A_x} + \frac{M_y}{I_y} \cdot z = -\frac{10 \cdot 1000\ N}{706{,}8\ cm^2} + \frac{2{,}5 \cdot 1000\ N \cdot 100\ cm}{1{,}811 \cdot 10^5\ cm^4} \cdot z$

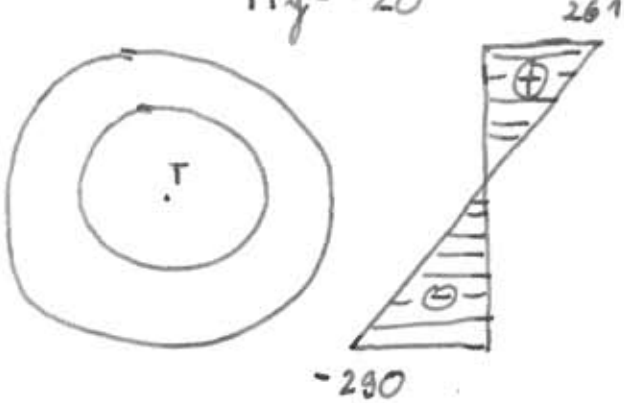
$= -14{,}15 + 1{,}38\ z\ [N/cm^2]$

$\sigma_{xx}^{(2)} = -14{,}15 - \frac{20}{1{,}811} z = -14{,}15 - 11{,}04\ z\ [N/cm^2]$

(1) $M_y = 2{,}5$



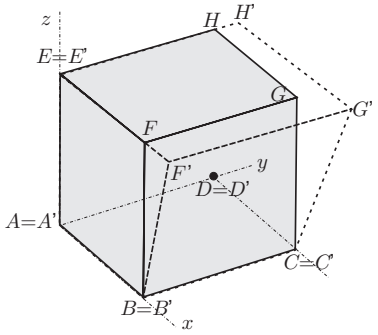
$M_y = -20$



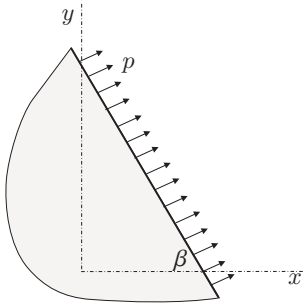
TRDNOST (VSŠ) - 1. KOLOKVIJ (7. 12. 2007)

Pazljivo preberite besedilo vsake naloge! Prva in četrta naloga sta vredni 30%, druga in tretja pa 20%. Uspešno reševanje!

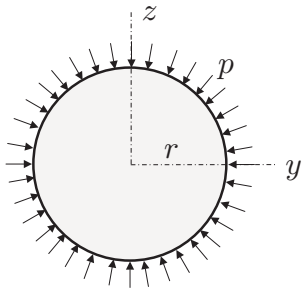
1. Kocka s stranico dolžine $a = 1$ cm se deformira, kot kaže slika. Tako deformiranje opišemo s pomiki oblike $\vec{u} = (axz, byz, 0)$. Lege točk A, B, C, D in E se ne spremenijo. Nove koordinate točke F so $F'(1.001, 0, 1)$, točka H pa se premakne v $H'(0, 1.002, 1)$.
Izračunajte:
a) konstanti a in b;
b) novo lego točke G;
c) tenzor majhnih deformacij;
d) spremembo dolžine diagonale AF;
e) spremembo pravega kota med AF in AD.



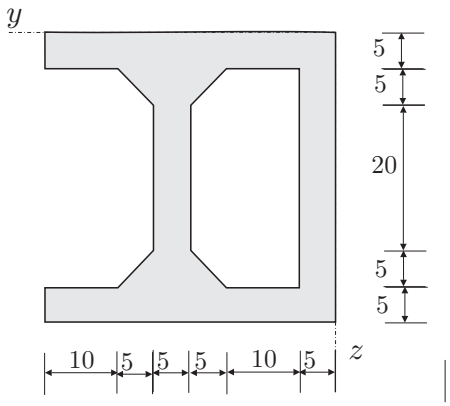
2. Na rob tanke stene, ki leži pod kotom $\beta = 60°$ glede na os x, deluje enakomerna površinska obtežba velikosti $p = 10\,\mathrm{kN/cm^2}$ pravokotno na rob, kot kaže slika. Privzemimo, da so napetosti po celotni prostornini stene konstantne. Specifična sprememba dolžine v smeri osi x (ε_{xx}) pa je enaka nič. Določite napetostni tenzor!
Podatki: $\nu = 0.3$, $E = 2 \cdot 10^4\,\mathrm{kN/cm^2}$.



3. Kroglico iz izotropnega linearno elastičnega materiala obtežimo po površini z enakomerno normalno površinsko obtežbo p. Privzemimo, da so napetosti po celotni prostornini kroglice konstantne. Določite spremembo temperature, pri kateri bo specifična sprememba volumna enaka 0.
Podatki: $\nu = 0.2$, $E = 2.1 \cdot 10^4\,\mathrm{kN/cm^2}$,
$\alpha = 1.2 \cdot 10^{-5}\,\mathrm{K^{-1}}$.



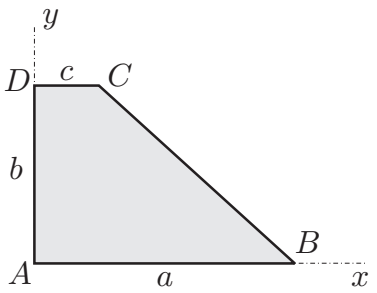
4. Izračunajte geometrijske karakteristike (A, y_T, z_T, I_y, I_z, I_{yz}, I_y^T, I_z^T, I_{yz}^T ter glavna vztrajnostna momenta) lika na sliki!
Podatki so v centimetrih.



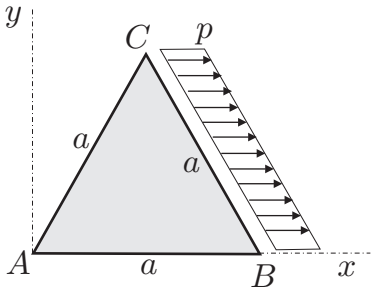
TRDNOST (VSŠ) - 1. KOLOKVIJ (12. 12. 2008)

Pazljivo preberite besedilo vsake naloge! Naloge so točkovane enakovredno (vsaka 25%).
Uspešno reševanje!

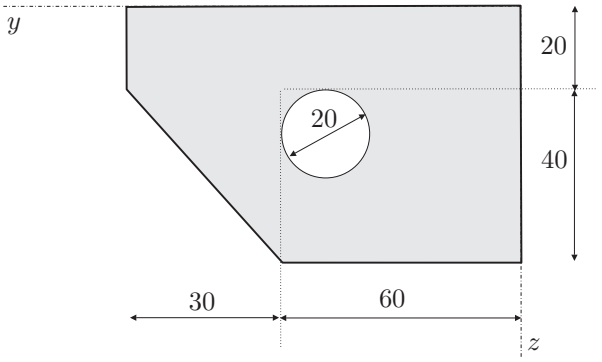
1. V telesu na sliki vlada homogeno ravninsko deformacijsko stanje. Poznamo tri spremembe dolžin. Stranica AB se je podaljšala za 0.3 mm, stranica AD pa skrajšala za 0.1mm. Daljica AC se je podaljšala za 0.2 mm. Določite tenzor majhnih deformacij!
Podatki: $a = 30\,\mathrm{cm}$, $b = 20\,\mathrm{cm}$, $c = 10\,\mathrm{cm}$.



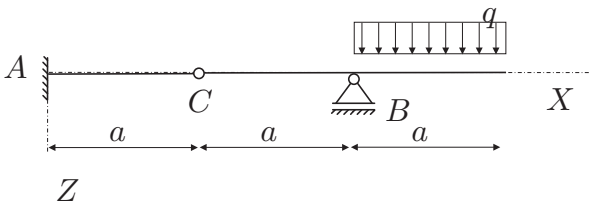
2. V tanki enakostranični trikotni prizmi iz izotropnega, linearno elastičnega materiala vlada homogeno ravninsko napetostno stanje. Poznamo obtežbo vzdolž robu BC in specifično spremembo volumna ϵ_V .
a) Določite tenzor napetosti!
b) Določite obtežbo na ostalih robovih!
Podatki: $\nu = 0.3$, $E = 2 \cdot 10^4\,\mathrm{kN/cm^2}$,
$p = 3\,\mathrm{kN/cm^2}$, $\epsilon_V = 10^{-4}$.

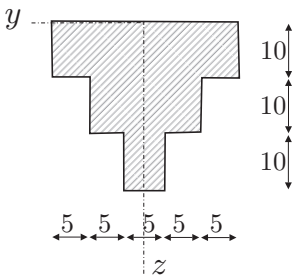


3. Izračunajte geometrijske karakteristike (A, y_T, z_T, I_y, I_z, I_{yz}, I_y^T, I_z^T, I_{yz}^T ter glavna vztrajnostna momenta) lika na sliki!
Podatki so v centimetrih.



4. Izračunajte prečne sile N_z in upogibne momente M_y za konstrukcijo na sliki. Na mestu ekstremnih upogibnih momentov izračunajte in narišite potek normalnih napetosti σ_{xx}. Na mestu ekstremnih prečnih sil izračunajte in narišite potek strižnih napetosti σ_{xz}.
Podatki: $a = 2\,\mathrm{m}$, $q = 10\,\mathrm{kN/m}$.
Podatki za prerez na sliki so v centimetrih.

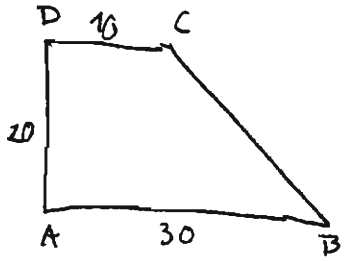




TRDNOST - VSŠ 12. 12. 2008

1. KOLOKVIJ

1. NALOGA



$\Delta AB = 0{\cdot}3\ mm \Rightarrow \varepsilon_{xx} = \frac{0{\cdot}3}{300} = 10^{-3}$

$\Delta AD = 0{\cdot}1\ mm \Rightarrow \varepsilon_{yy} = -\frac{0{\cdot}1}{200} = -0{\cdot}5 \cdot 10^{-3}$

$\Delta AC = 0{\cdot}2\ mm$

$$\Rightarrow \varepsilon_{AC} = \frac{0{\cdot}2}{\sqrt{200^2 + 100^2}} = 8{\cdot}94 \cdot 10^{-4}$$

$\vec{AC} = (10, 20)$

$|\vec{AC}| = \sqrt{500}$

$$\varepsilon_{AC} = \frac{1}{\sqrt{500}} \cdot \frac{1}{\sqrt{500}} \begin{bmatrix} 10 & 20 \end{bmatrix} \begin{bmatrix} 10^{-3} & \varepsilon_{xy} \\ \varepsilon_{xy} & -5 \cdot 10^{-4} \end{bmatrix} \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

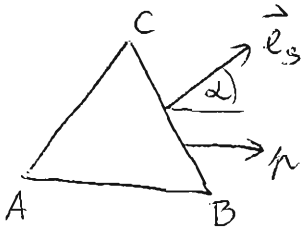
$$\varepsilon_{AC} = \frac{1}{5} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 10^{-3} + 2\varepsilon_{xy} \\ \varepsilon_{xy} - 10^{-3} \end{bmatrix} = \frac{1}{5} \left(10^{-3} + 4\varepsilon_{xy} - 2 \cdot 10^{-3} \right)$$

$$4\varepsilon_{xy} = 5 \cdot 8{\cdot}94 \cdot 10^{-4} + 10^{-3}$$

$$\varepsilon_{xy} = 1{\cdot}368 \cdot 10^{-3}$$

$$[\varepsilon] = 10^{-4} \begin{bmatrix} 10 & 13{\cdot}4 & 0 \\ 13{\cdot}4 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2. NALOGA



$$\varepsilon_v = 10^{-4}$$

$$\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$$

$$\vec{e}_s = \cos 30^\circ \vec{e}_x + \sin 30^\circ \vec{e}_y$$

$$\vec{p}_s = p \vec{e}_x + 0 \vec{e}_y$$

$$[\sigma]\begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} p \\ 0 \end{bmatrix}$$

$$\frac{\sqrt{3}}{2}\sigma_{xx} + \frac{1}{2}\sigma_{xy} = 3 \quad /\sqrt{3}$$

$$\frac{\sqrt{3}}{2}\sigma_{xy} + \frac{1}{2}\sigma_{yy} = 0 \quad /\cdot(-)$$

$$3\sigma_{xx} - \sigma_{yy} = 3\sqrt{3} \cdot 2$$

$$\sigma_{xx} = 2G\varepsilon_{xx} + \lambda\varepsilon_v$$

$$\sigma_{yy} = 2G\varepsilon_{yy} + \lambda\varepsilon_v$$

$$0 = \sigma_{zz} = 2G\varepsilon_{zz} + \lambda\varepsilon_v$$

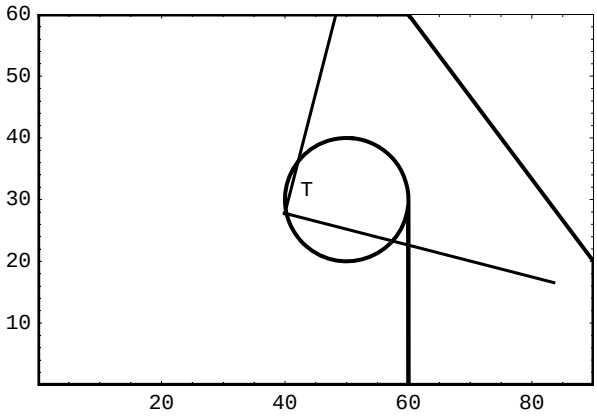
$$\sigma_{xx} + \sigma_{yy} = (2G + 3\lambda)\varepsilon_v = 5{,}25$$

$$[\sigma] = \begin{bmatrix} 3{,}91 & -0{,}77 \\ -0{,}77 & 1{,}34 \end{bmatrix}$$

b.)

$$p_{AC} = [\sigma]\begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -3{,}77 \\ 1{,}34 \end{bmatrix}$$

$$p_{AB} = [\sigma]\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0{,}77 \\ -1{,}34 \end{bmatrix}$$



Ax = 4485.64

Sy = 124577.

Sz = 179275.

y_T = 39.9664

z_T = 27.7723

Iy = 4.82958×10^6

Iz = 9.91547×10^6

Iyz = -4.59869×10^6

Iy^T = 1.3698×10^6

Iz^T = 2.75051×10^6

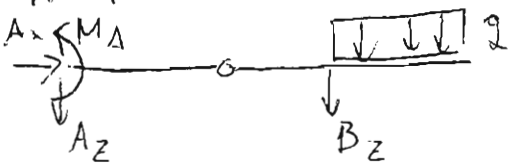
Iyz^T = 380186.

α_G = −14.421

I1 = 2.48148×10^6

I2 = 1.63884×10^6

4. NALOGA



$$\boxed{A_x = 0}$$

$$-B_z \cdot a - q \cdot a \cdot \frac{3a}{2} = 0$$

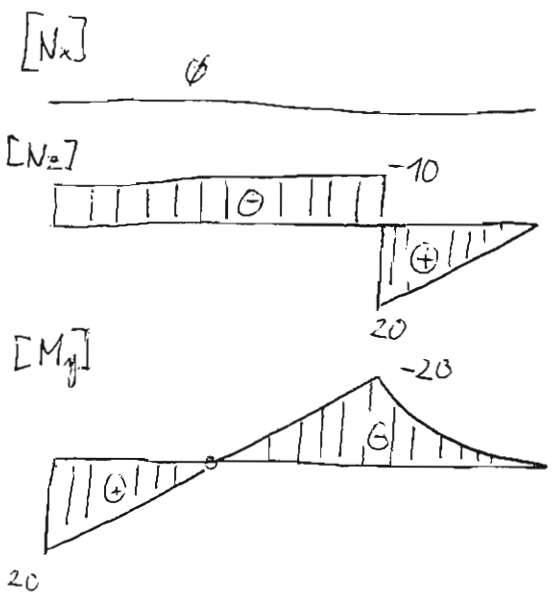
$$\boxed{B_z = -30\ kN}$$

$$\boxed{A_z = 10\ kN}$$

$$A_z \cdot a + M_A = 0$$

$$\boxed{M_A = -20\ kNm}$$

DIAGRAMI



$N_{z,ect} = 20\ kN$

$M_{y,ect} = \pm 20\ kNm$

KARAKTERISTIKE

$$A = 25 \cdot 10 + 15 \cdot 10 + 5 \cdot 10 = 450$$

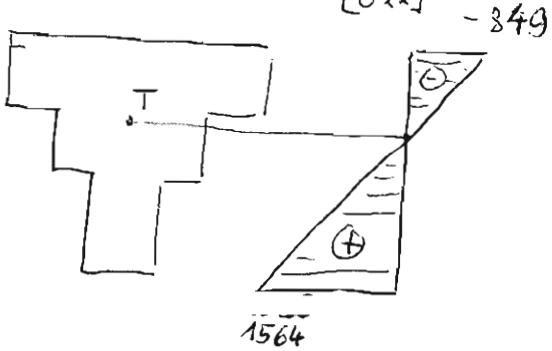
$$z_T = \frac{5 \cdot 25 \cdot 10 + 15 \cdot 15 \cdot 10 + 25 \cdot 5 \cdot 10}{450} = 10{,}\overline{55}\ cm$$

$$I_y = \frac{25 \cdot 10^3}{12} + \frac{15 \cdot 10^3}{12} + \frac{5 \cdot 10^3}{12} + 5^2 \cdot 25 \cdot 10 + 15^2 \cdot 15 \cdot 10 + 25^2 \cdot 5 \cdot 10 = 75000\ cm^4$$

$$I_y^T = 75000 - z_T^2 \cdot A = 24861\ cm^4$$

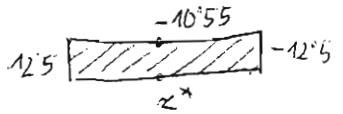
σ_{xx}

$$\sigma_{xx} = \frac{N_x}{A_x} + \frac{M_y}{I_y} \cdot z = \frac{20\ kN \cdot 100\ cm}{2486\ cm^4} \cdot z = 80z\ \left[N/cm^2\right]$$



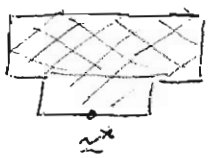
σ_{xz}

STATIČNI MOMENTI



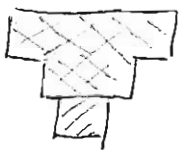
$$S_y^* = \int_{-10'55}^{z^*} z\,dz \int_{-12'5}^{12'5} dy = 12'5\,(z^{*2} - 10'55^2)$$

$$S_y^*(-10'55) = 0 \qquad S_y^*(-0'55) = -1389\ cm^3$$



$$S_y^* = -1389 + \int_{-0'55}^{z^*} z\,dz \int_{-7'5}^{7'5} dy = -1389 + 7'5(z^{*2} - 0'55^2)$$

$$S_y(9'45) = -722\ cm^3$$



$$S_y^* = -722 + \int_{9'45}^{z^*} z\,dz \int_{-2'5}^{2'5} = -722 + 2'5\,(z^{*2} - 9'45^2)$$

$$S_y(19'45) = 0 \checkmark$$

$$\sigma_{xz} = -\frac{1}{b^*}\frac{N_z}{I_y} S_y^* \qquad \frac{N_z}{I_y} = 0'8045 \quad N/cm^4$$

z^*	b^*	S_y^*	σ_{xz} [N/cm^2]
-10'55	25	0	0
-0'55	25	-1389	44'7
-0'55	15	-1389	74'5
0	15	-1393	74'7 (ekstrem)
9'45	15	-722	38'7
9'45	5	-722	116'2
19'45	5	0	0