

## TRDNOST (VSŠ) - 1. KOLOKVIJ (17. 12. 2003)

Pazljivo preberite besedilo vsake naloge! Naloge so točkovane enakovredno (vsaka 25%)! Pišite čitljivo!  
Uspešno reševanje!

1. Deformiranje telesa je podano s poljem pomikov  $\vec{u} = 10^{-4} [(x - z) \vec{e}_x + xy \vec{e}_y + (z^2 - y) \vec{e}_z]$ .

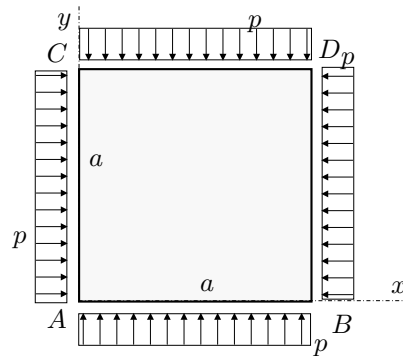
Izračunajte:

- a) tenzor majhnih deformacij;
- b) specifično spremembo dolžine vlakna v točki  $(-1, 1, 0)$  v smeri vektorja  $\vec{a} = 3\vec{e}_x - \vec{e}_y + 2\vec{e}_z$ ;
- c) spremembo pravega kota v točki  $(-1, 1, 0)$  med vektorjema  $\vec{a}$  in  $\vec{b} = \vec{e}_x + 3\vec{e}_y$ .

2. Na rob tanke kvadratne stene deluje normalna enakomerna površinska obtežba velikosti  $p$ , kot kaže slika. Privzemimo, da so napetosti po celotni prostornini stene konstantne. Določite velikost obtežbe pri kateri je specifična sprememba prostornine enaka nič ( $\varepsilon_V = 0$ ), če poznamo naslednje materialne parametre:

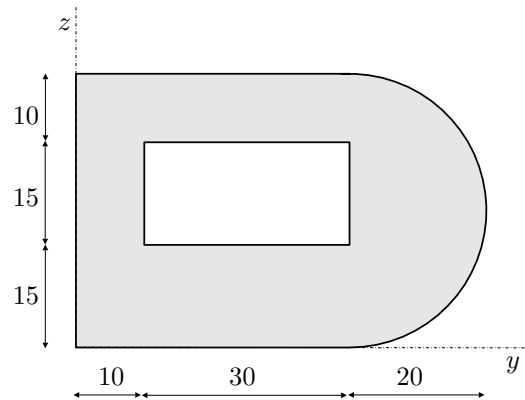
$$\nu = 0.2, E = 2.4 \cdot 10^4 \text{ kN/cm}^2,$$

$$\alpha = 10^{-5} \text{ K}^{-1}, \Delta T = 10 \text{ K}.$$



3. Izračunajte geometrijske karakteristike ( $A, y_T, z_T, I_y, I_z, I_{yz}, I_y^T, I_z^T, I_{yz}^T$ ) lika na sliki!

Podatki so v centimetrih.

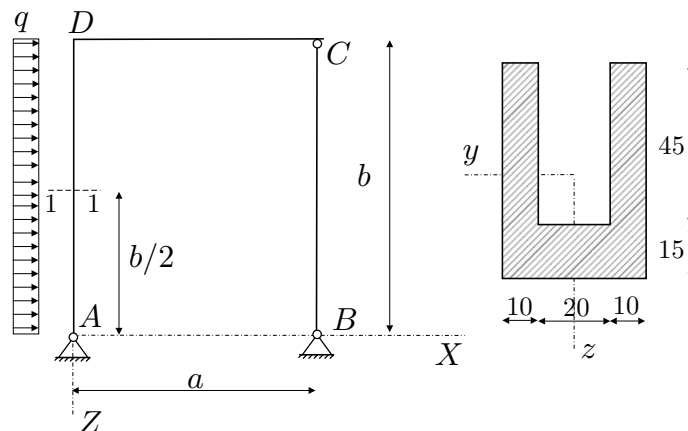


4. Za konstrukcijo na sliki izračunajte vrednosti notranjih statičnih količin ( $N_x, N_z, M_y$ ) v prerezu 1-1! Določite nekaj značilnih vrednosti in skicirajte diagrama normalne napetosti  $\sigma_{xx}$  in strižne napetosti  $\sigma_{xz}$  v tem prerezu!

Podatki:  $a = 4 \text{ m}, b = 6 \text{ m},$

$q = 2 \text{ kN/m}.$

Podatki za prerez so v centimetrih.



## REŠITVE NALOG

1. a) 
$$\begin{bmatrix} 1 & \frac{y}{2} & -\frac{1}{2} \\ \frac{y}{2} & x & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 2z \end{bmatrix};$$

b)  $D_{aa} \simeq \varepsilon_{aa} = 7.143 \cdot 10^{-6};$

c)  $D_{ab} \simeq 2\varepsilon_{aa} = 1.01419 \cdot 10^{-4};$

2.  $\sigma_{xz} = 0, \sigma_{yz} = 0, \sigma_{zz} = 0;$

$\sigma_{xx} = -p, \sigma_{xy} = 0, \sigma_{yy} = -p;$

$\varepsilon_{xx} = \varepsilon_{yy} = \left(1 - \frac{1}{3}p\right) \cdot 10^{-4}, \varepsilon_{zz} = \left(1 + \frac{1}{6}p\right) \cdot 10^{-4};$

$\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0 \Rightarrow p = 6.$

3.  $A_x = 1777.26, y_T = 28.79, z_T = 19.37;$

$I_y = 930\,594, I_z = 2\,030\,172, I_{yz} = -995\,073;$

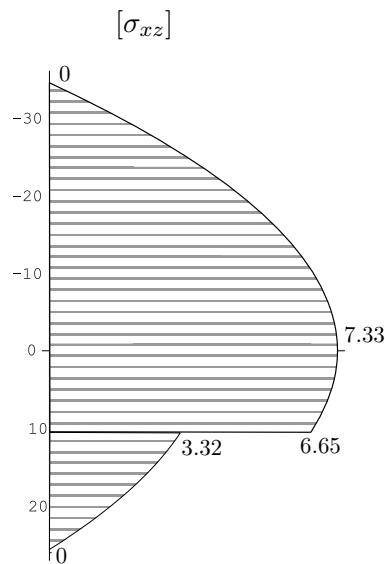
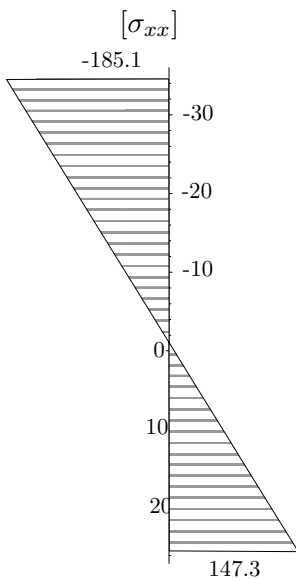
$I_y^T = 263\,988.5, I_z^T = 557\,463.5, I_{yz}^T = -4\,256.7.$

4.  $N_x = 9, N_z = 6, M_y = 27;$

$A_x = 1500, I_y^T = 487\,125;$

$\sigma_{xx} = 6 + 5.54z \text{ [N/cm}^2\text{]};$

$z^*$	$b^*$	$S_y^*$	$\sigma_{xz}$ [N/cm <sup>2</sup> ]
-34.5	20	0	0
0	20	-11902.5	7.33
10.5	20	-10800	6.65
10.5	40	-10800	3.32
25.5	40	0	0



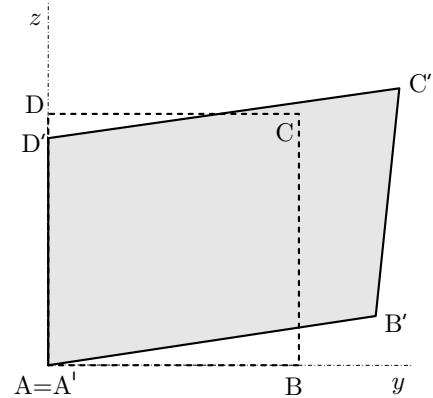
TRDNOST (VŠŠ) - 1. KOLOKVIJ (6. 12. 2004)

Pazljivo preberite besedilo vsake naloge! Pišite čitljivo! Uspešno reševanje!

1. Tanka kvadratna ploščica z robom 3 cm se deformira, kot kaže slika. Točka  $A$  se ne premakne. Nove koordinate točke  $B$  so  $B'(3.009, 0.006)$ , točka  $C$  se premakne v  $C'(3.018, 3.003)$ , točka  $D$  pa v  $D'(0, 2.997)$  Deformiranje je podano s poljem pomikov  $\vec{u} = (axy + bx) \vec{e}_x + (cx + dy) \vec{e}_y$ .

Izračunajte:

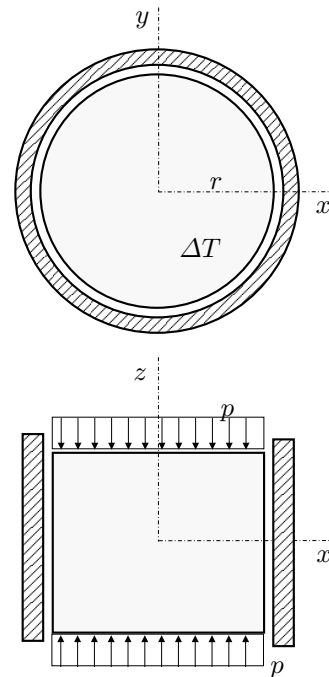
- konstante  $a, b, c$  in  $d$ ;
- tenzor majhnih deformacij;
- specifično spremembo dolžine vlakna v točki  $C$  v smeri vektorja  $\vec{AC}$ ;
- spremembo pravega kota v točki  $C$  med vektorjema  $\vec{AC}$  in  $\vec{BD}$ .



2. V togo, nerazteglivo cev je postavljen valj iz izotropnega, linearno elastičnega materiala, kot kaže slika. Polmer valja je 20 cm, razdalja med plaščem valja in cevjo pa 1 mm. Valj segrejemo za 60K in obtežimo z enakomerno površinsko obtežbo  $p$  na spodnji in zgornji ploskvi. Privzemimo, da so napetosti po celotni prostornini valja konstantne.

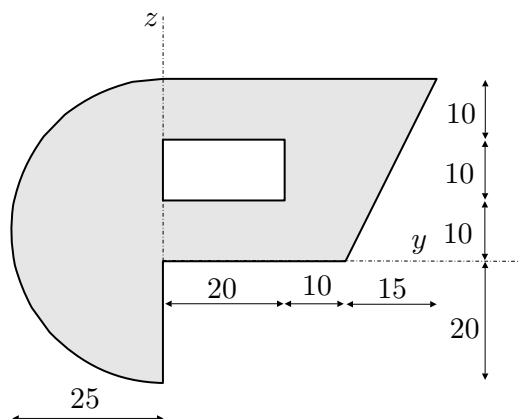
- Določite velikost obtežbe  $p$ , pri kateri se valj dotakne cevi!
- Valj obremenimo z obtežbo velikosti  $p = 500 \text{ kN/cm}^2$ . Določite napetostni tenzor! Določite tudi specifično spremembo volumna.

Podatki:  $\nu = 0.3, E = 2.1 \cdot 10^4 \text{ kN/cm}^2, \alpha = 10^{-5} \text{ K}^{-1}, \Delta T = 60\text{K}$ .



3. Izračunajte geometrijske karakteristike ( $A, y_T, z_T, I_y, I_z, I_{yz}, I_y^T, I_z^T, I_{yz}^T$ ) lika na sliki!

Podatki so v centimetrih.



## REŠITVE NALOG

1. a)  $a = 0.001, b = 0.003, c = 0.002, d = -0.001$ ;

b)  $\varepsilon = \begin{bmatrix} 3 + y & 1 + \frac{x}{2} & 0 \\ 1 + \frac{x}{2} & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ;

c)  $D_{AC} \simeq \varepsilon_{AC} = 0.005$ ;

d)  $D_{ab} \simeq 2\varepsilon_{aa} = -0.007$ ;

2. a)  $p = 308 \text{ kN/cm}^2$ ;

b)  $\sigma = \begin{bmatrix} -82.3 & 0 & 0 \\ 0 & -82.3 & 0 \\ 0 & 0 & -500 \end{bmatrix}, \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = -0.01086$ ;

3.  $A_x = 1905, y_T = 4.72, z_T = 10.45$ ;

$$I_y = 501\,966, I_z = 674\,658, I_{yz} = -280\,860;$$

$$I_y^T = 294\,076.4, I_z^T = 632\,288.6, I_{yz}^T = -187\,007.4.$$

## TRDNOST (VSŠ) - 1. KOLOKVIJ (9. 12. 2005)

Pazljivo preberite besedilo vsake naloge! Pišite čitljivo! Uspešno reševanje!

1. Deformiranje telesa je podano s poljem pomikov

$$\vec{u} = 10^{-4} (2y(x-z), xy, -x+y).$$

Izračunajte:

- tenzor velikih deformacij;
- tenzor majhnih deformacij in njegovo vrednost v točki  $T(0, 1, -1)$ ;
- specifično spremembo dolžine vlakna v točki  $T$  v smeri vektorja  $(1, 1, 0)$ ;
- spremembo pravega kota v točki  $T$  med vektorjema  $(1, 1, 0)$  in  $(1, b, 1)$ .

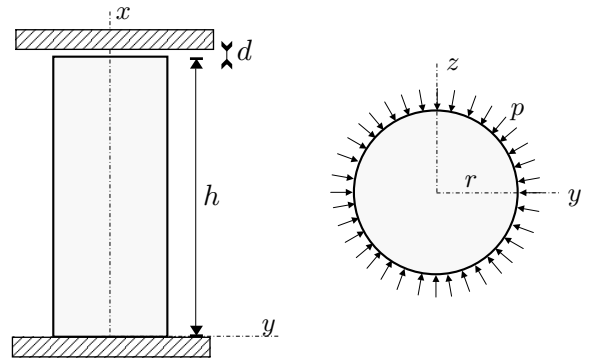
2. Valjast vzorec iz izotropnega, linearno elastičnega materiala, višine  $h = 100$  cm, je postavljen med dve togi nepomični plošči, kot kaže slika. Polmer valja je 5 cm, razdalja med valjem in zgornjo ploščo pa  $d = 1$  mm. Valj obtežimo po plašču z enakomerno normalno površinsko obtežbo  $p$ . Privzemimo, da so napetosti po celotni prostornini valja konstantne.

a) Določite velikost obtežbe  $p$ , pri kateri se valj dotakne zgornje plošče!

b) Poleg obtežbe  $p$  vzorec še segrejemo. Določite spremembo temperature, pri kateri bo specifična sprememba volumna enaka 0.

Podatki:  $\nu = 0.3$ ,  $E = 2.1 \cdot 10^4$  kN/cm<sup>2</sup>,

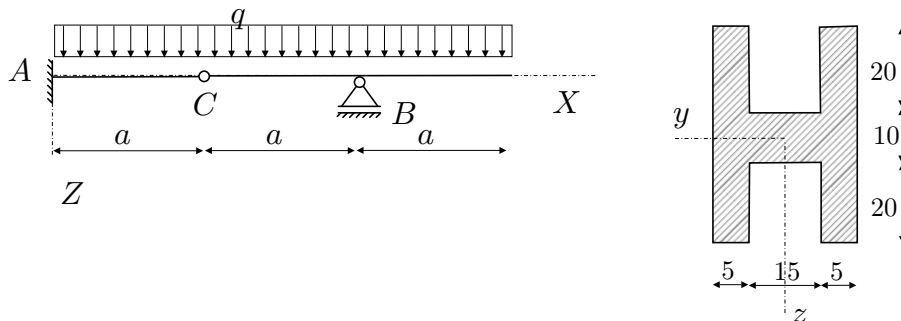
$\alpha = 1.2 \cdot 10^{-5}$  K<sup>-1</sup>.



3. Izračunajte prečne sile  $N_z$  in upogibne momente  $M_y$  za konstrukcijo na sliki. Na mestu ekstremnih upogibnih momentov izračunajte in narišite potek normalnih napetosti  $\sigma_{xx}$ . Na mestu ekstremnih prečnih sil izračunajte in narišite potek strižnih napetosti  $\sigma_{xz}$ .

Podatki:  $a = 2$  m,  $q = 10$  kN/m.

Podatki za prerez na sliki so v centimetrih.



## 1. NALOGA

$$\begin{array}{lll} \frac{\partial u_x}{\partial x} = 2y & \frac{\partial u_x}{\partial y} = 2(x-z) & \frac{\partial u_x}{\partial z} = -2y \\ \frac{\partial u_y}{\partial x} = y & \frac{\partial u_y}{\partial y} = x & \frac{\partial u_y}{\partial z} = 0 \\ \frac{\partial u_z}{\partial x} = -1 & \frac{\partial u_z}{\partial y} = 1 & \frac{\partial u_z}{\partial z} = 0 \end{array} \quad / \cdot 10^{-4}$$

a.)

$$[E] = 10^{-4} \begin{bmatrix} 2y + \frac{10^{-4}}{2}(5y^2+1) & (x-z) + \frac{y}{2} + \frac{10^{-4}}{2}(4y(x-z)+xy-1) & -y - \frac{1}{2} + \frac{10^{-4}}{2}(-4y^2) \\ x + \frac{10^{-4}}{2}(4(x-z)^2+x^2+1) & & \frac{1}{2} + \frac{10^{-4}}{2}(-4y(x-z)) \\ 0 + \frac{10^{-4}}{2}(4y^2) & & \end{bmatrix}$$

*simetrično*

b.)

$$[E] = \begin{bmatrix} 2y & x-z + \frac{y}{2} & -y - \frac{1}{2} \\ x-z + \frac{y}{2} & x & \frac{1}{2} \\ -y - \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \cdot 10^{-4} \quad [E]_T = \begin{bmatrix} 2 & \frac{3}{2} & -\frac{3}{2} \\ \frac{3}{2} & 0 & \frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} & 0 \end{bmatrix} \cdot 10^{-4}$$

c.)  $\vec{a} = (1, 1, 0)$   
 $\vec{e}_a = \frac{1}{\sqrt{2}}(1, 1, 0)$

$$\begin{aligned} \varepsilon_{aa} &= 10^{-4} \cdot \frac{1}{2} \cdot [1 \ 1 \ 0] \begin{bmatrix} 2 & \frac{3}{2} & -\frac{3}{2} \\ \frac{3}{2} & 0 & \frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{10^{-4}}{2} [1 \ 1 \ 0] \begin{bmatrix} \frac{7}{2} \\ \frac{3}{2} \\ -\frac{2}{2} \end{bmatrix} \\ &= \frac{10^{-4}}{2} \left(\frac{10}{2}\right) = \underline{\underline{2.5 \cdot 10^{-4}}} \end{aligned}$$

d.)  $\vec{b} = (1, b, 1)$   
 $\vec{a} \cdot \vec{b} = 1+b=0 \quad \boxed{b=-1}$

$$\vec{e}_b = \frac{1}{\sqrt{3}}(1, -1, 1)$$

$$\varepsilon_{ab} = 10^{-4} \cdot \frac{1}{\sqrt{6}} [1 \ -1 \ 1] \begin{bmatrix} \frac{7}{2} \\ \frac{3}{2} \\ -1 \end{bmatrix} = 10^{-4} \cdot \frac{1}{\sqrt{6}} \left(\frac{4}{2} - 1\right) = \frac{10^{-4}}{\sqrt{6}}$$

$$\text{Dab} \approx 2\varepsilon_{ab} = 8 \cdot 16 \cdot 10^{-5} \text{ radiana} \quad (4.7 \cdot 10^{-3} \text{ stopinje})$$

## 2. NALOGA

$$a.) \boxed{\varepsilon_{xx} = \frac{d}{h} = \frac{1}{1000} = 10^{-3}}$$

$$\begin{bmatrix} 0 \\ -\rho \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \boxed{\sigma_{yy} = -\rho} \quad \sigma_{xy} = 0 \quad \sigma_{yz} = 0$$

$$\begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix} = \begin{bmatrix} \sigma \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \boxed{\sigma_{zz} = -\rho} \quad \sigma_{xz} = 0 \quad \sigma_{yz} = 0$$

$$\boxed{\sigma_{xx} = 0}$$

$$\varepsilon_{xx} = \frac{1+\nu}{E} \cancel{\sigma_{xx}} - \frac{\nu}{E} (\cancel{\sigma_{xx}} + \sigma_{yy} + \sigma_{zz})$$

$$\varepsilon_{xx} = + \frac{\nu}{E} \cdot 2\rho \Rightarrow \boxed{\rho = \frac{E \cdot \varepsilon_{xx}}{2\nu} = 35 \text{ kN/cm}^2}$$

$$b.) \rho = 35 \text{ kN/cm}^2 \quad \varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0$$

$$\Delta T = ?$$

$$\sigma_{xx} = 2\mu \varepsilon_{xx} + 2\varepsilon_v - \beta_T \Delta T$$

$$\sigma_{yy} = 2\mu \varepsilon_{yy} + 2\varepsilon_v - \beta_T \Delta T$$

$$\sigma_{zz} = 2\mu \varepsilon_{zz} + 2\varepsilon_v - \beta_T \Delta T$$

$$\left. \begin{array}{l} \sigma_{xx} = 2\mu \varepsilon_{xx} + 2\varepsilon_v - \beta_T \Delta T \\ \sigma_{yy} = 2\mu \varepsilon_{yy} + 2\varepsilon_v - \beta_T \Delta T \\ \sigma_{zz} = 2\mu \varepsilon_{zz} + 2\varepsilon_v - \beta_T \Delta T \end{array} \right\} + \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = 2\mu \varepsilon_v - 3\beta_T \Delta T$$

$$\boxed{\sigma_{xx} - 2\rho = -3\beta_T \Delta T}$$

$$\sigma_{xx} = 2\mu \varepsilon_{xx} - \beta_T \Delta T$$

$$2\mu \varepsilon_{xx} - 2\rho = -2\beta_T \Delta T$$

$$\Delta T = \frac{\rho - \mu \varepsilon_{xx}}{\beta_T}$$

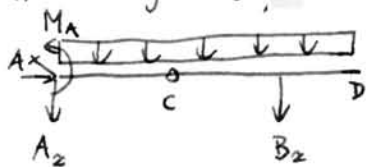
$$\mu = \frac{2 \cdot 1 \cdot 10^4}{2 \cdot 1 \cdot 3} = 0.808 \cdot 10^4 \frac{\text{kN}}{\text{cm}^2}$$

$$\beta_T = \frac{1 \cdot 2 \cdot 10^{-5} \cdot 2 \cdot 1 \cdot 10^4}{0.4} = 0.63$$

$$\boxed{\Delta T = 42.7 \text{ K}}$$

3. NALOGA

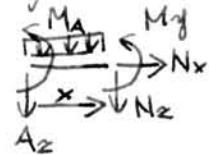
a.) Notranje sile



$$\begin{aligned} A_x &= 0 \\ A_z + B_z + g \cdot 3a &= 0 \\ M_A - B_z \cdot 2a - g \cdot 3a \cdot \frac{3a}{2} &= 0 \\ -B_z \cdot a - g \cdot 2a \cdot a &= 0 \Rightarrow \end{aligned}$$

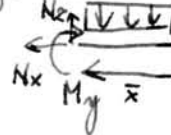
$$\begin{aligned} A_z &= -20 \text{ kN} \\ M_A &= 20 \text{ kNm} \\ B_z &= -40 \text{ kN} \end{aligned}$$

polje AB



$$\begin{aligned} N_x &= 0 \\ N_z &= +20 - gx & N_z(2) = 0, N_z(4) = -20 \\ M_y &= -20 + 20x - g \frac{x^2}{2} & M_y(2) = 0 \\ & & M_y(4) = -20 \end{aligned}$$

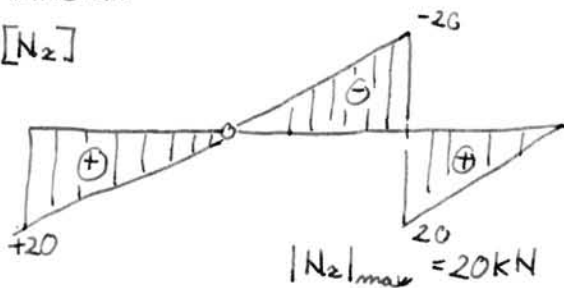
polje BD



$$\begin{aligned} N_x &= 0 \\ N_z &= gx & N_z(2) = 20 \\ M_y &= -g \frac{x^2}{2} & M_y(2) = -20 \end{aligned}$$

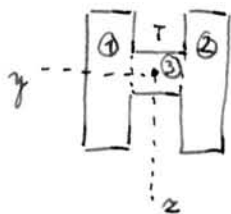
DIAGRAMI

[Nz]



$|M_y|_{max} = 20 \text{ kNm}$

b.) Karakteristike prereza



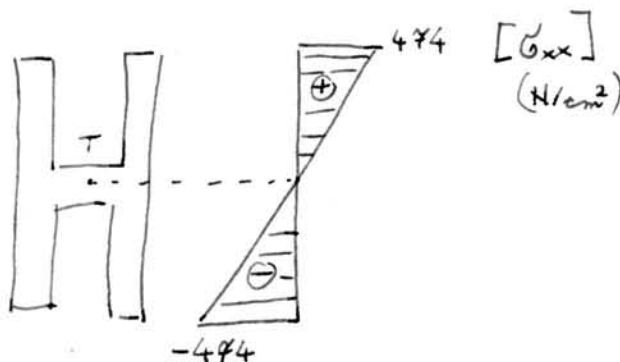
$$\begin{aligned} \textcircled{1}: A^{\textcircled{1}} &= 250 \text{ cm}^2 & \textcircled{2}: A^{\textcircled{2}} &= 250 \text{ cm}^2 & \textcircled{3}: A^{\textcircled{3}} &= 150 \text{ cm}^2 \\ y^{\textcircled{1}} &= 10 \text{ cm} & y^{\textcircled{2}} &= -10 \text{ cm} & y^{\textcircled{3}} &= 0 \\ z^{\textcircled{1}} &= 0 & z^{\textcircled{2}} &= 0 & z^{\textcircled{3}} &= 0 \\ I_y^{\textcircled{1}} &= 52083.33 & I_y^{\textcircled{2}} &= 52083.33 \text{ cm}^4 & I_y^{\textcircled{3}} &= 1250 \text{ cm}^4 \end{aligned}$$

$$I_y = I_y^{\textcircled{1}} + I_y^{\textcircled{2}} + I_y^{\textcircled{3}} = 105416.7 \text{ cm}^4$$

c.) Normalne napetosti

$$\sigma_{xx} = \frac{M_y}{I_y} \cdot z = \frac{-20 \text{ kN} \cdot 100 \text{ cm}}{105416.7 \text{ cm}^4} \cdot z = -18.97 z \text{ [N/cm}^2\text{]}$$

$$\sigma_{xx}(-25) = 474 \text{ N/cm}^2 \quad \sigma_{xx}(25) = -474 \text{ N/cm}^2$$





d.) Strižne napetosti

$$\sigma_{xz}(z^*) = -\frac{1}{b^*} \left( \frac{S_y^*}{I_y} \cdot N_z \right)$$

$$N_z = 20 \text{ kN}$$

$$I_y = 105416.7 \text{ cm}^4$$

d1)  $z^* \in [-25, -5]$

$$x \in [-25, z^*] \quad y \in [-12.5, -7.5] \quad y \in [7.5, 12.5]$$

$$S_y(z^*) = \int_{A^*} z dA^* = 2 \cdot \int_{-12.5}^{-7.5} \int_{-25}^{z^*} z dz dy = 2 \cdot \frac{z^2}{2} \Big|_{-25}^{z^*} \cdot y \Big|_{-12.5}^{-7.5} =$$

$$= 5(z^{*2} - 625)$$

$$S_y^*(-25) = 0 \quad S_y^*(-5) = -3000 \text{ cm}^3$$

$$b^* = 10 \text{ cm}$$

d2)  $z^* \in [-5, 5]$

$$S_y^* = \int_{-12.5}^{12.5} \int_{-5}^{z^*} z dz dy + S_y^*(-5) = -3000 + 25 \cdot \frac{z^{*2}}{2} \Big|_{-5}^{z^*} =$$

$$= -3000 + 12.5(z^{*2} - 25)$$

$$S_y^*(-5) = -3000 \text{ cm}^3 \quad S_y^*(5) = -3000 \text{ cm}^3$$

$$S_y^*(0) = -3312.5 \text{ cm}^3$$

$$b^* = 25 \text{ cm}$$

d3)  $z^* \in [5, 25]$

$$S_y^* = 2 \cdot \int_{-12.5}^{7.5} \int_5^{z^*} z dz dy - 3000$$

$$= -3000 + 5 \cdot (z^{*2} - 25)$$

$$S_y^*(5) = -3000 \text{ cm}^3$$

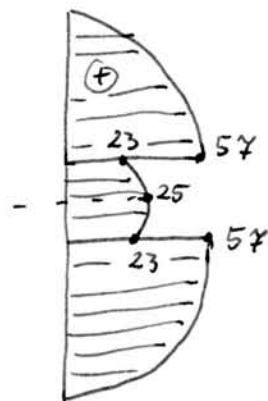
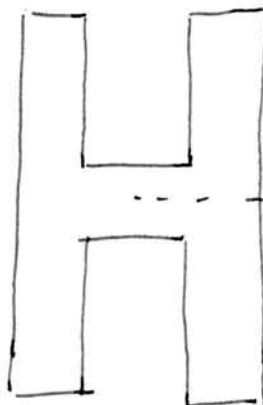
$$S_y^*(25) = 0$$

$$b^* = 10 \text{ cm}$$

TABELA

$z^*$	$S_y^*$	$b^*$	$\sigma_{xz}^* \text{ [N/cm}^2\text{]}$
-25	0	10	0
-5	-3000	10	56.9
-5	-3000	25	22.77
0	-3312	25	25.13
5	-3000	25	22.77
5	-3000	10	56.9
25	0	10	0

$[\sigma_{xz}]$



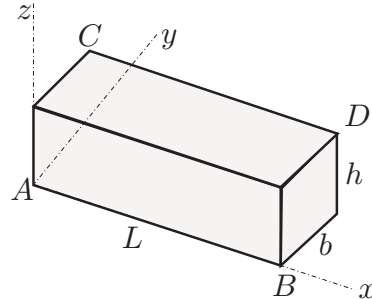
## TRDNOST (VŠŠ) - 1. KOLOKVIJ (15. 12. 2006)

Pazljivo preberite besedilo vsake naloge! Naloge so točkovane enakovredno (vsaka 25%).  
Uspešno reševanje!

1. Kvader na sliki ima stranice dolžin  $L = 20$  cm,  $b = h = 5$  cm. Izmerjene spremembe dolžin stranic znašajo  $\Delta L = 0.2$  mm,  $\Delta b = \Delta h = 0.1$  mm. Telesna diagonala  $AD$  se je podaljšala za 0.8 mm, kot med osema  $y$  in  $z$  pa se ni spremenil. Spremembi pravih kotov med osema  $x$  in  $y$  ter  $x$  in  $z$  sta enaki.

Izračunajte:

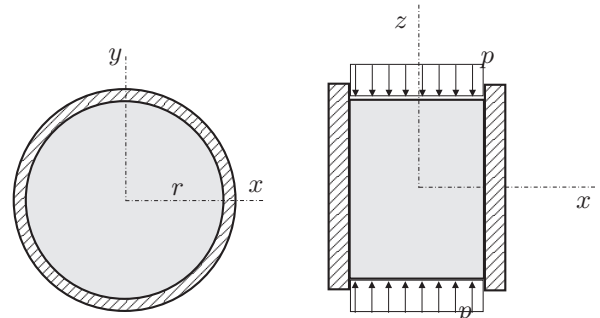
- tenzor majhnih deformacij;
- specifično spremembo volumna;
- spremembo dolžine diagonale  $BC$ .



2. V togo, nerazteglivo cev je postavljen valj iz izotropnega, linearno elastičnega materiala. Polmer valja je 5 cm, višina pa 20 cm. Valj obremenimo z enakomerno površinsko obtežbo  $p$  na spodnji in zgornji ploskvi. Privzemimo, da so napetosti po celotni prostornini valja konstantne. Trenje med valjem in cevjo zanemarimo.

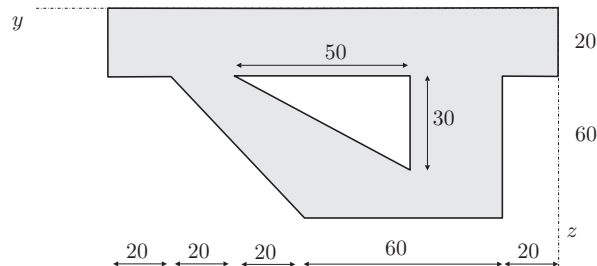
- Določite normalne napetosti med valjem in cevjo!
- Določite tudi spremembo višine valja.

Podatki:  $\nu = 0.2$ ,  $E = 2 \cdot 10^4$  kN/cm<sup>2</sup>,  
 $p = 3$  kN/cm<sup>2</sup>.



3. Izračunajte geometrijske karakteristike ( $A$ ,  $y_T$ ,  $z_T$ ,  $I_y$ ,  $I_z$ ,  $I_{yz}$ ,  $I_y^T$ ,  $I_z^T$ ,  $I_{yz}^T$  ter glavna vztrajnostna momenta) lika na sliki!

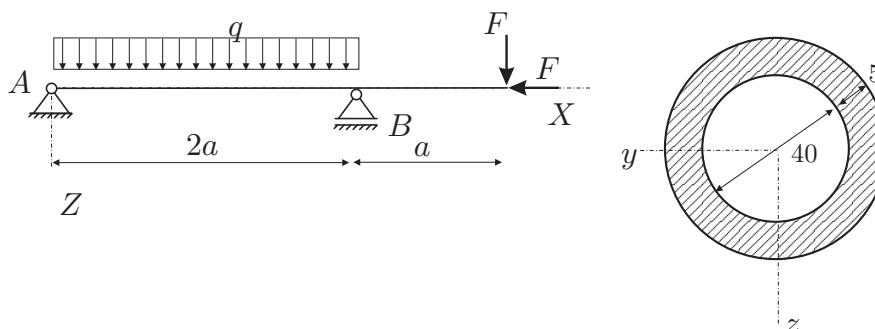
Podatki so v centimetrih.



4. Na mestu največjih in najmanjših upogibnih momentov konstrukcije na sliki izračunajte in narišite potek normalnih napetosti  $\sigma_{xx}$ .

Podatki:  $a = 2$  m,  $q = 5$  kN/m,  $F = 10$  kN.

Podatki za prerez na sliki so v centimetrih.



## 1. NALOGA

$$\epsilon_{xx} = \frac{\Delta L}{L} = \frac{2}{2000} = 0.001 = 1 \cdot 10^{-3}$$

$$\epsilon_{yy} = \epsilon_{zz} = \frac{1}{500} = 0.002 = 2 \cdot 10^{-3}$$

$$\epsilon_{yz} = 0$$

$$\epsilon_{xy} = \epsilon_{xz}$$

$$\epsilon_{dd} = \frac{\Delta d}{d} = \frac{8}{\sqrt{450} \cdot 100} = 3771 \cdot 10^{-3}$$

$$d = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{450} \text{ cm}$$

$$[\epsilon] = \begin{bmatrix} 1 & \epsilon_{xy} & \epsilon_{xy} \\ \epsilon_{xy} & 2 & 0 \\ \epsilon_{xy} & 0 & 2 \end{bmatrix}$$

$$\vec{d} = (20, 5, 5)$$

$$\vec{e}_d = \frac{1}{\sqrt{450}} (20, 5, 5) = \frac{5}{\sqrt{450}} (4, 1, 1)$$

$$\epsilon_{dd} = \frac{25}{450} [4 \ 1 \ 1] \begin{bmatrix} 1 & \epsilon_{xy} & \epsilon_{xy} \\ \epsilon_{xy} & 2 & 0 \\ \epsilon_{xy} & 0 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} \cdot 10^{-3} =$$

$$= \frac{10^{-3}}{18} [4 \ 1 \ 1] \begin{bmatrix} 4 + 2\epsilon_{xy} \\ 4\epsilon_{xy} + 2 \\ 4\epsilon_{xy} + 2 \end{bmatrix} = \frac{1}{18} (16\epsilon_{xy} + 20) \cdot 10^{-3}$$

$$\epsilon_{xy} = \frac{18\epsilon_{dd} - 20}{16} = 2.99 \cdot 10^{-3} \quad \Rightarrow [\epsilon] = 10^{-3} \begin{bmatrix} 1 & 3 & 3 \\ 3 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

$$b.) \epsilon_V = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = 5 \cdot 10^{-3}$$

$$c.) \vec{BC} = (-20, 4, 4) \Rightarrow e_{BC} = \frac{5}{\sqrt{450}} (-4, 1, 1)$$

$$\epsilon_{BC} = \frac{25}{450} [-4 \ 1 \ 1] \cdot 10^{-3} \begin{bmatrix} 1 & 3 & 3 \\ 3 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix} = -1549 \cdot 10^{-3}$$

## 2. NALOGA

a.) deformacija  $\epsilon_{xx} = \epsilon_{yy} = 0$

b.) napetosti

NORMALA  $\lambda$

$$[\mu_s] = \begin{bmatrix} 0 \\ 0 \\ -\mu \end{bmatrix} \quad [\epsilon_s] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[G][\epsilon_s] = [\mu_s] \Rightarrow \sigma_{xz} = 0$$

$$\sigma_{yz} = 0$$

$$\sigma_{zz} = -\mu = -3 \text{ kN/cm}^2$$

c.) Hookov zakon

$$\epsilon_v = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$$

$$\sigma_{zz} = 2G \epsilon_{zz} + \lambda \epsilon_v = 2G \epsilon_{zz} + \lambda \epsilon_{zz}$$

$$\epsilon_{zz} = \frac{\sigma_{zz}}{2G + \lambda}$$

$$G = \frac{E}{2(1+\nu)} = \frac{2 \cdot 10^4}{2 \cdot 1 \cdot 2} = \frac{5}{6} \cdot 10^4 \text{ kN/cm}^2$$

$$\lambda = \frac{0.2E}{12 \cdot 0.6} = \frac{10}{18} \cdot 10^4 \text{ kN/cm}^2$$

$$\epsilon_{zz} = - \frac{3}{\frac{25}{18}} \cdot 10^{-4}$$

$$\epsilon_{zz} = -1.35 \cdot 10^{-4}$$

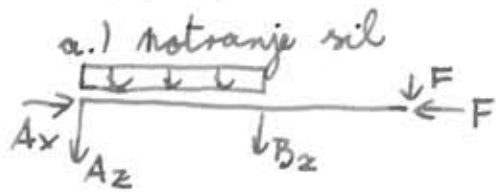
$$\sigma_{xx} = 2G \epsilon_{xx} + \lambda \epsilon_{zz}$$

$$\sigma_{xx} = \frac{10}{18} \cdot 10^4 \cdot (-1.35) \cdot 10^{-4} = -\frac{15}{20} = -0.75 \text{ kN/cm}^2$$

$$\sigma_{yy} = -0.75 \text{ kN/cm}^2$$

$$\Delta h = \epsilon_{zz} \cdot h = -1.35 \cdot 10^{-4} \cdot 200 \text{ mm} = -0.027 \text{ mm}$$

#### 4. NALOGA



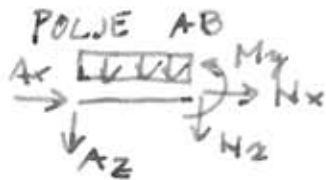
$$A_x = F$$

$$A_z + B_z = q \cdot 2a + F$$

$$-B_z \cdot 2a - q \cdot 2a \cdot a - F \cdot 3a = 0$$

$$A_z = -5 \text{ kN}$$

$$B_z = -\frac{1}{2}(2qa + F)$$



$$N_x = -A_x \quad N_x = -10 \text{ kN}$$

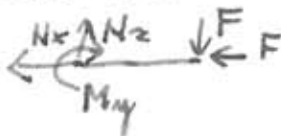
$$N_z = -A_z - qx \quad N_z = 5 - 5x$$

$$B_z = -25 \text{ kN}$$

$$M_y = -A_z x - q \frac{x^2}{2} \quad M_y = 5x - 2.5x^2$$

$$M_y(1) = 2.5 \text{ kNm} \quad (\text{ekstrem!})$$

POLJE BC



$$N_x = -F$$

$$N_x = -10 \text{ kN}$$

$$M_y(4) = -20 \text{ kNm}$$

$$N_z = F$$

$$N_z = 10 \text{ kN}$$

$$M_y = -Fx$$

$$M_y = -20 \text{ kNm}$$

$$N_x = -10$$

[My]



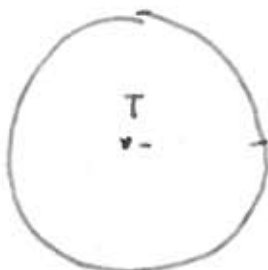
b.) površina:  $A = \pi(25^2 - 20^2) = 706.8 \text{ cm}^2$

$$I_y = \frac{\pi \cdot 450^4}{64} - \frac{\pi \cdot 40^4}{64} = 1.811 \cdot 10^5 \text{ cm}^4$$

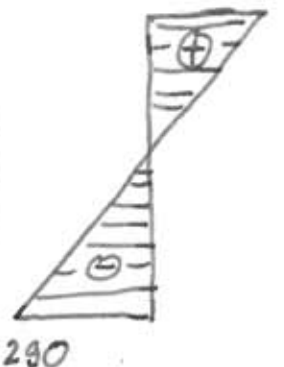
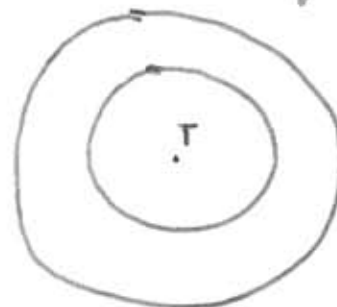
$$\begin{aligned} \sigma_{xx}^{(1)} &= \frac{N_x}{A_x} + \frac{M_y}{I_y} \cdot z = -\frac{10 \cdot 1000 \text{ N}}{706.8 \text{ cm}^2} + \frac{2.5 \cdot 1000 \text{ N} \cdot 100 \text{ cm}}{1.811 \cdot 10^5 \text{ cm}^4} \cdot z \\ &= -14.15 + 1.38 z \quad [\text{N/cm}^2] \end{aligned}$$

$$\sigma_{xx}^{(2)} = -14.15 - \frac{20}{1.811} z = -14.15 - 11.04 z \quad [\text{N/cm}^2]$$

①  $M_y = 2.5$



$M_y = -20$



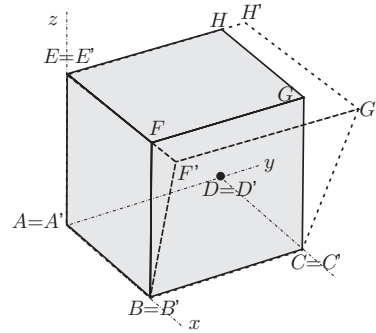
## TRDNOST (VSŠ) - 1. KOLOKVIJ (7. 12. 2007)

Pazljivo preberite besedilo vsake naloge! Prva in četrta naloga sta vredni 30%, druga in tretja pa 20%.  
Uspešno reševanje!

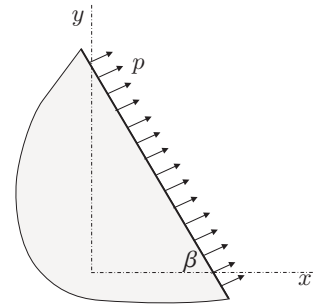
1. Kocka s stranico dolžine  $a = 1$  cm se deformira, kot kaže slika. Tako deformiranje opišemo s pomiki oblike  $\vec{u} = (axz, byz, 0)$ . Lege točk  $A, B, C, D$  in  $E$  se ne spremenijo. Nove koordinate točke  $F$  so  $F'(1.001, 0, 1)$ , točka  $H$  pa se premakne v  $H'(0, 1.002, 1)$ .

Izračunajte:

- konstanti  $a$  in  $b$ ;
- ново lego točke  $G$ ;
- tenzor majhnih deformacij;
- spremembo dolžine diagonale  $AF$ ;
- spremembo pravega kota med  $AF$  in  $AD$ .

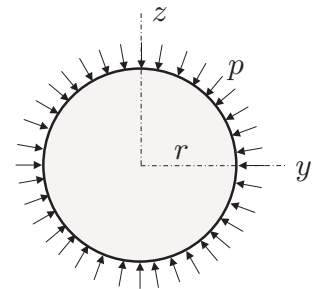


2. Na rob tanke stene, ki leži pod kotom  $\beta = 60^\circ$  glede na os  $x$ , deluje enakomerna površinska obtežba velikosti  $p = 10$  kN/cm<sup>2</sup> pravokotno na rob, kot kaže slika. Privzemimo, da so napetosti po celotni prostornini stene konstantne. Specifična sprememba dolžine v smeri osi  $x$  ( $\varepsilon_{xx}$ ) pa je enaka nič. Določite napetostni tenzor!  
Podatki:  $\nu = 0.3, E = 2 \cdot 10^4$  kN/cm<sup>2</sup>.



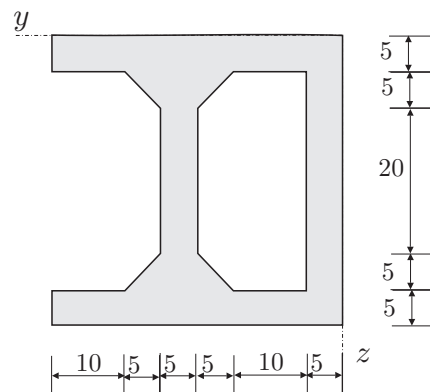
3. Kroglico iz izotropnega linearno elastičnega materiala obtežimo po površini z enakomerno normalno površinsko obtežbo  $p$ . Privzemimo, da so napetosti po celotni prostornini kroglice konstantne. Določite spremembo temperature, pri kateri bo specifična sprememba volumna enaka 0.

Podatki:  $\nu = 0.2, E = 2.1 \cdot 10^4$  kN/cm<sup>2</sup>,  
 $\alpha = 1.2 \cdot 10^{-5}$  K<sup>-1</sup>.



4. Izračunajte geometrijske karakteristike ( $A, y_T, z_T, I_y, I_z, I_{yz}, I_y^T, I_z^T, I_{yz}^T$  ter glavna vztrajnostna momenta) lika na sliki!

Podatki so v centimetrih.

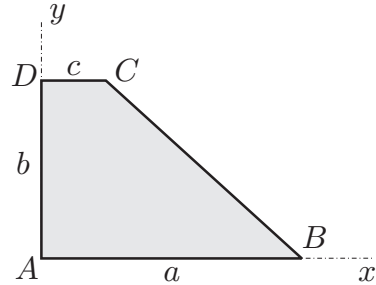


## TRDNOST (VŠŠ) - 1. KOLOKVIJ (12. 12. 2008)

Pazljivo preberite besedilo vsake naloge! Naloge so točkovane enakovredno (vsaka 25%).  
Uspešno reševanje!

1. V telesu na sliki vlada homogeno ravninsko deformacijsko stanje. Poznamo tri spremembe dolžin. Stranica  $AB$  se je podaljšala za 0.3 mm, stranica  $AD$  pa skrajšala za 0.1mm. Daljica  $AC$  se je podaljšala za 0.2 mm. Določite tenzor majhnih deformacij!

Podatki:  $a = 30$  cm,  $b = 20$  cm,  $c = 10$  cm.

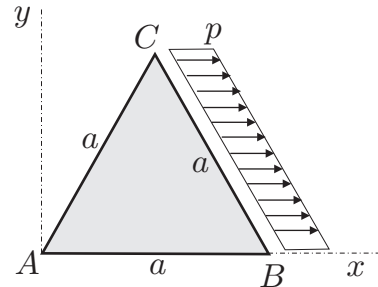


2. V tanki enakostranični trikotni prizmi iz izotropnega, linearno elastičnega materiala vlada homogeno ravninsko napetostno stanje. Poznamo obtežbo vzdolž robu  $BC$  in specifično spremembo volumna  $\epsilon_V$ .

a) Določite tenzor napetosti!

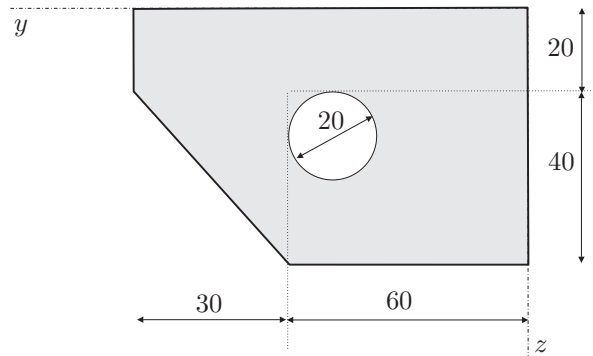
b) Določite obtežbo na ostalih robovih!

Podatki:  $\nu = 0.3$ ,  $E = 2 \cdot 10^4$  kN/cm<sup>2</sup>,  
 $p = 3$  kN/cm<sup>2</sup>,  $\epsilon_V = 10^{-4}$ .



3. Izračunajte geometrijske karakteristike ( $A$ ,  $y_T$ ,  $z_T$ ,  $I_y$ ,  $I_z$ ,  $I_{yz}$ ,  $I_y^T$ ,  $I_z^T$ ,  $I_{yz}^T$  ter glavna vztrajnostna momenta) lika na sliki!

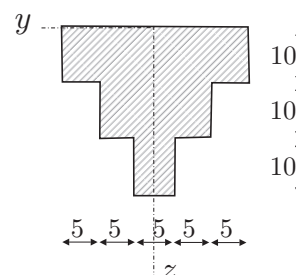
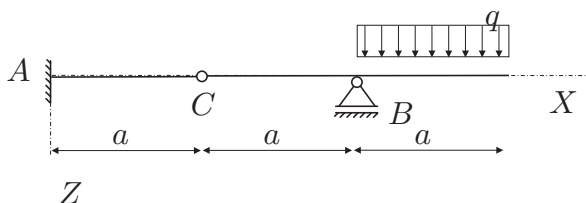
Podatki so v centimetrih.



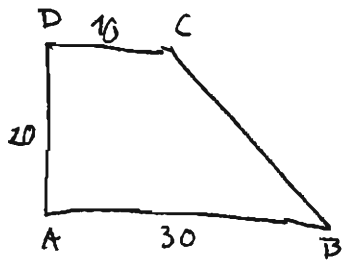
4. Izračunajte prečne sile  $N_z$  in upogibne momente  $M_y$  za konstrukcijo na sliki. Na mestu ekstremnih upogibnih momentov izračunajte in narišite potek normalnih napetosti  $\sigma_{xx}$ . Na mestu ekstremnih prečnih sil izračunajte in narišite potek strižnih napetosti  $\sigma_{xz}$ .

Podatki:  $a = 2$  m,  $q = 10$  kN/m.

Podatki za prerez na sliki so v centimetrih.



1. NALOGA



$$\Delta AB = 0.3 \text{ mm} \Rightarrow \epsilon_{xx} = \frac{0.3}{300} = 10^{-3}$$

$$\Delta AD = 0.1 \text{ mm} \Rightarrow \epsilon_{yy} = -\frac{0.1}{200} = -0.5 \cdot 10^{-3}$$

$$\Delta AC = 0.2 \text{ mm}$$

$$\Rightarrow \epsilon_{AC} = \frac{0.2}{\sqrt{200^2 + 100^2}} = 8.94 \cdot 10^{-4}$$

$$\vec{AC} = (10, 20)$$

$$|\vec{AC}| = \sqrt{500}$$

$$\epsilon_{AC} = \frac{1}{\sqrt{500}} \cdot \frac{1}{\sqrt{500}} \begin{bmatrix} 10 & 20 \end{bmatrix} \begin{bmatrix} 10^{-3} & \epsilon_{xy} \\ \epsilon_{xy} & -5 \cdot 10^{-4} \end{bmatrix} \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

$$\epsilon_{AC} = \frac{1}{5} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 10^{-3} + 2\epsilon_{xy} \\ \epsilon_{xy} - 10^{-3} \end{bmatrix} = \frac{1}{5} (10^{-3} + 4\epsilon_{xy} - 2 \cdot 10^{-3})$$

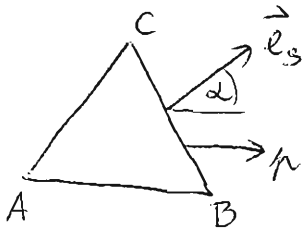
$$4\epsilon_{xy} = 5 \cdot 8.94 \cdot 10^{-4} + 10^{-3}$$

$$\epsilon_{xy} = 1.368 \cdot 10^{-3}$$

$$[\epsilon] = 10^{-4} \begin{bmatrix} 10 & 13.4 & 0 \\ 13.4 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



## 2. NALOGA



$$\epsilon_v = 10^{-4}$$

$$\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$$

$$\vec{e}_s = \cos 30^\circ \vec{e}_x + \sin 30^\circ \vec{e}_y$$

$$\vec{n} = n \vec{e}_x + 0 \vec{e}_y$$

$$[\sigma] \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} n \\ 0 \end{bmatrix}$$

$$\frac{\sqrt{3}}{2} \sigma_{xx} + \frac{1}{2} \sigma_{xy} = 3 \quad / \sqrt{3}$$

$$\frac{\sqrt{3}}{2} \sigma_{xy} + \frac{1}{2} \sigma_{yy} = 0 \quad \leftarrow$$

$$3\sigma_{xx} - \sigma_{yy} = 3\sqrt{3} \cdot 2$$

$$\sigma_{xx} = 2G \epsilon_{xx} + \lambda \epsilon_v$$

$$\sigma_{yy} = 2G \epsilon_{yy} + \lambda \epsilon_v$$

$$0 = \sigma_{zz} = 2G \epsilon_{zz} + \lambda \epsilon_v$$

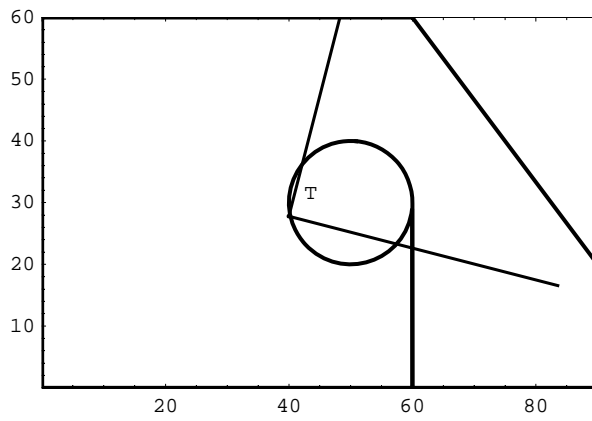
$$\sigma_{xx} + \sigma_{yy} = (2G + 3\lambda) \epsilon_v = 5 \cdot 25$$

$$[\sigma] = \begin{bmatrix} 3.91 & -0.77 \\ -0.77 & 1.34 \end{bmatrix}$$

b.)

$$\sigma_{AC} = [\sigma] \begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -3.074 \\ 1.034 \end{bmatrix}$$

$$\sigma_{AB} = [\sigma] \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.77 \\ -1.34 \end{bmatrix}$$



$$Ax = 4485.64$$

$$Sy = 124577.$$

$$Sz = 179275.$$

$$y_T = 39.9664$$

$$z_T = 27.7723$$

$$Iy = 4.82958 \times 10^6$$

$$Iz = 9.91547 \times 10^6$$

$$Iyz = -4.59869 \times 10^6$$

$$Iy^T = 1.3698 \times 10^6$$

$$Iz^T = 2.75051 \times 10^6$$

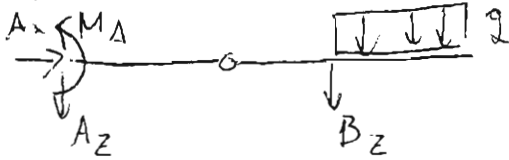
$$Iyz^T = 380186.$$

$$\alpha_G = -14.421$$

$$I1 = 2.48148 \times 10^6$$

$$I2 = 1.63884 \times 10^6$$

#### 4. NALOBA



$$A_x = 0$$

$$-B_z \cdot a - q \cdot \frac{3a}{2} = 0$$

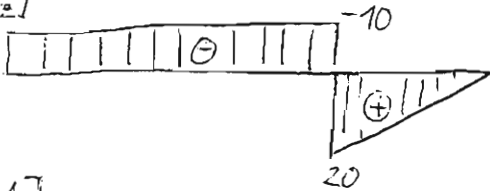
$$\begin{cases} B_z = -30 \text{ kN} \\ A_z = 10 \text{ kN} \end{cases}$$

DIAGRAMI

$[N_x]$

0

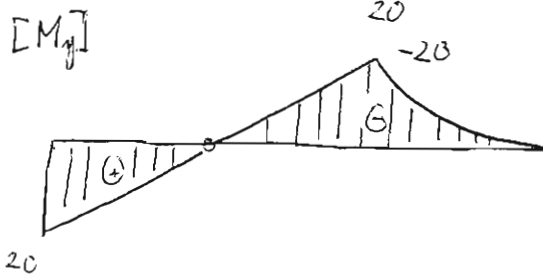
$[N_z]$



$$N_{z, \text{act}} = 20 \text{ kN}$$

$$M_{y, \text{act}} = \pm 20 \text{ kNm}$$

$[M_y]$



KARAKTERISTIKE

$$A = 25 \cdot 10 + 15 \cdot 10 + 5 \cdot 10 = 450$$

$$e_T = \frac{5 \cdot 25 \cdot 10 + 15 \cdot 15 \cdot 10 + 25 \cdot 5 \cdot 10}{450} = 10.55 \text{ cm}$$

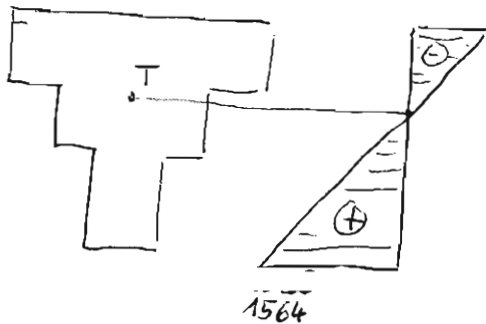
$$I_y = \frac{25 \cdot 10^3}{12} + \frac{15 \cdot 10^3}{12} + \frac{5 \cdot 10^3}{12} + 5^2 \cdot 25 \cdot 10 + 15^2 \cdot 15 \cdot 10 + 25^2 \cdot 5 \cdot 10 = 75000 \text{ cm}^4$$

$$I_y^T = 75000 - e_T^2 \cdot A = 24861 \text{ cm}^4$$

$\sigma_{xx}$

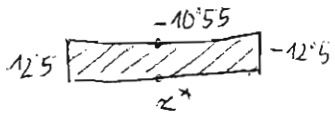
$$\sigma_{xx} = \frac{N_x}{A_x} + \frac{M_y}{I_y} \cdot z = \frac{20 \text{ kN} \cdot 100 \text{ cm}}{24861 \text{ cm}^4} \cdot z = 80 z \text{ [N/cm}^2]$$

$[\sigma_{xx}]$  -349



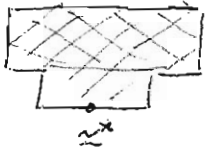
$\sigma_{xz}$

STATIČNI MOMENTI



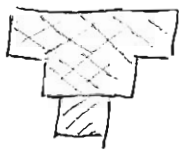
$$S_{yz}^* = \int_{-10.55}^{12.5} z dz \int_{-12.5}^{12.5} dy = 12.5 (z^{*2} - 10.55^2)$$

$$S_{yz}^* (-10.55) = 0 \quad S_{yz}^* (-0.55) = -1389 \text{ cm}^3$$



$$S_{yz}^* = -1389 + \int_{-0.55}^{z^*} z dz \int_{-7.5}^{7.5} dy = -1389 + 7.5(z^{*2} - 0.55^2)$$

$$S_{yz}^* (9.45) = -722 \text{ cm}^3$$



$$S_{yz}^* = -722 + \int_{9.45}^{z^*} z dz \int_{-2.5}^{2.5} dy = -722 + 2.5(z^{*2} - 9.45^2)$$

$$S_{yz}^* (19.45) = 0 \quad \checkmark$$

$$\sigma_{xz} = -\frac{1}{b^*} \frac{N_z}{I_y} S_{yz}^*$$

$$\frac{N_z}{I_y} = 0.8045 \text{ N/cm}^4$$

$z^*$	$b^*$	$S_{yz}^*$	$\sigma_{xz} \text{ [N/cm}^2\text{]}$
-10.55	25	0	0
-0.55	25	-1389	44.7
-0.55	15	-1389	74.5
0	15	-1393	74.7 (ekstrem)
9.45	15	-722	38.7
9.45	5	-722	116.2
19.45	5	0	0

