Vpisna številka: 261 _ _ _ _ _

naloga	točk
1	
2	
3	
4	

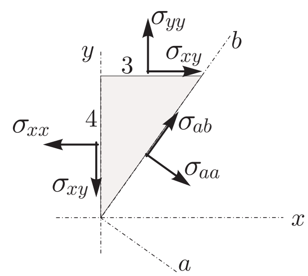
TRDNOST (VSŠ) - 1. KOLOKVIJ (18. 11. 2014)

Pazljivo preberite besedilo vsake naloge! Pišite čitljivo! Uspešno reševanje!

1. Deformiranje nekega telesa lahko opišemo s pomiki oblike $\vec{u} = 10^{-3}\left(xy, z+2x, y^2-x\right)$. Določite:
i) tenzor majhnih deformacij v točki $T(1,-1,0)$;
ii) specifično spremembo dolžine vlakna v točki $T(1,-1,0)$ v smeri $\vec{a} = 2\vec{e}_x + 1\vec{e}_y$! (20%)

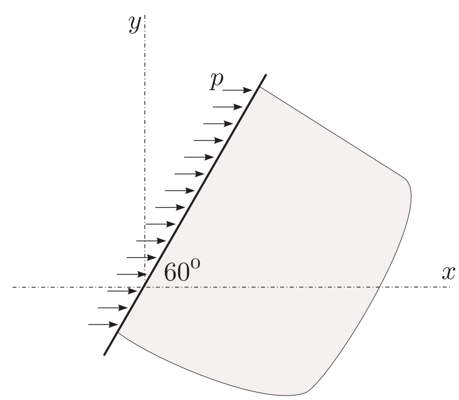
2. V tanki trikotni ploščici vlada homogeno ravninsko napetostno stanje. Znane so napetosti σ_{xx}, σ_{aa} in σ_{ab}. Določite napetostni tenzor v koordinatah x, y, z! (20%)

Podatki: $\sigma_{xx} = 20$MPa, $\sigma_{aa} = 60\,$MPa, $\sigma_{ab} = -30\,$MPa.



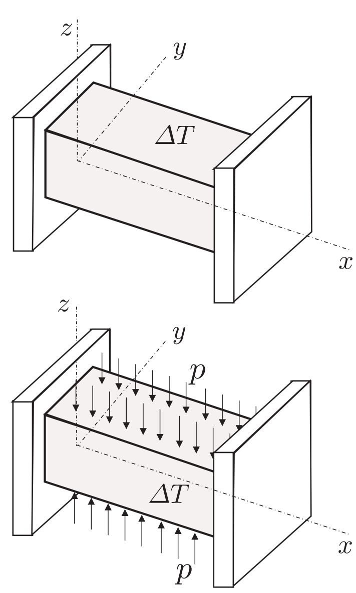
3. Na rob tanke stene, ki leži pod kotom 60° glede na os x, deluje enakomerna površinska obtežba velikosti $p = 20\,\text{kN/cm}^2$, kot kaže slika. Napetosti so konstantne po celotni prostornini stene. Normalna deformacija v navpični smeri znaša $\epsilon_{yy} = 1\cdot 10^{-3}$.
Določite napetostni tenzor v koordinatah x, y, z! (30%)

Podatki: $\nu = 0.25$, $E = 2\cdot 10^5\,$MPa.



4. Kvader iz izotropnega, linearno elastičnega materiala postavimo med dve togi plošči in segrejemo za ΔT. Določite:
i) Napetostni tenzor zaradi spremembe temperature!
ii) Ob enaki spremembi temperature ploskvi z normalama $\vec{e}_z$ in $-\vec{e}_z$ obremenimo z enakomerno zvezno tlačno obtežbo p.
Kolikšna mora biti obtežba, da se volumen kvadra ne spremeni?
iii) Deformacijski tenzor za dano spremembo temperature in izračunano obtežbo p! (30%)

Podatki: $\nu = 0.2$, $E = 2\cdot 10^4\,\text{kN/cm}^2$, $\alpha = 10^{-5}\,\text{K}^{-1}$, $\Delta T = 30$K.



TRDNOST 2014/15 1. KOLOKVIJ

1. Naloga

i.) $$\hat{J} = 10^{-3}\begin{bmatrix} y & x & 0 \\ 2 & 0 & 1 \\ -1 & 2y & 0 \end{bmatrix} \qquad \hat{\varepsilon} = 10^{-3}\begin{bmatrix} y & 1+\frac{x}{2} & -\frac{1}{2} \\ \text{sim.} & 0 & y+\frac{1}{2} \\ & & 0 \end{bmatrix}$$

$$\hat{\varepsilon}_T = 10^{-3}\begin{bmatrix} -1 & \frac{3}{2} & -\frac{1}{2} \\ \frac{3}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}$$

ii) $$D_{aa} \approx 10^{-3}\begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \end{bmatrix}\begin{bmatrix} -1 & \frac{3}{2} & -\frac{1}{2} \\ \frac{3}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}\begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ 0 \end{bmatrix} = 10^{-3}\begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \end{bmatrix}\begin{bmatrix} -\frac{1}{2\sqrt{5}} \\ \frac{6}{2\sqrt{5}} \\ -\frac{3}{2\sqrt{5}} \end{bmatrix}$$

$$= 10^{-3}\left(-1+\frac{6}{10}\right) = 4\cdot 10^{-4}$$

2. Naloga

$$\vec{e}_a = \frac{4}{5}\vec{e}_x - \frac{3}{5}\vec{e}_y$$
$$\vec{e}_b = \frac{3}{5}\vec{e}_x + \frac{4}{5}\vec{e}_y$$

$$60 = \begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \end{bmatrix}\begin{bmatrix} 20 & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}\begin{bmatrix} \frac{4}{5} \\ -\frac{3}{5} \end{bmatrix}$$

$$1500 = \begin{bmatrix} 4 & -3 \end{bmatrix}\begin{bmatrix} 80 - 3\sigma_{xy} \\ 4\sigma_{xy} - 3\sigma_{yy} \end{bmatrix} = 320 + 9\sigma_{yy} - 24\sigma_{xy}$$

$$\boxed{9\sigma_{yy} - 24\sigma_{xy} = 1180}$$

$$-30 = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \end{bmatrix}\begin{bmatrix} 20 & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}\begin{bmatrix} \frac{4}{5} \\ -\frac{3}{5} \end{bmatrix}$$

$$-750 = \begin{bmatrix} 3 & 4 \end{bmatrix}\begin{bmatrix} 80 - 3\sigma_{xy} \\ 4\sigma_{xy} - 3\sigma_{yy} \end{bmatrix} = 240 - 12\sigma_{yy} + 7\sigma_{xy}$$

$$\boxed{-12\sigma_{yy} + 7\sigma_{xy} = -990}$$

$$\boxed{\sigma_{xy} = -\frac{70}{3} \qquad \sigma_{yy} = \frac{620}{9}}$$

$$\boxed{\sigma_{xy} = -23.3\ \text{MPa}}$$
$$\boxed{\sigma_{yy} = 68.9\ \text{MPa}}$$

3. Naloga

a.) Deformacije: $\varepsilon_{yy} = 1\cdot 10^{-3}$

b.) Napetosti: $\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$; $\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}\begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$

$$-\frac{\sqrt{3}}{2}\sigma_{xx} + \frac{1}{2}\sigma_{xy} = 20 \quad \cdot\sqrt{3}$$
$$-\frac{\sqrt{3}}{2}\sigma_{xy} + \frac{1}{2}\sigma_{yy} = 0 \quad +$$

$$\boxed{-3\sigma_{xx} + \sigma_{yy} = 40\sqrt{3}}$$

c.) Hooke-ov zakon

$E = 2\cdot 10^5\ \text{MPa} = 2\cdot 10^4\ \text{kN/cm}^2$

$$\varepsilon_{xx} = \frac{1+\nu}{E}\sigma_{xx} - \frac{\nu}{E}(\sigma_{xx}+\sigma_{yy})$$
$$10^{-3} = \varepsilon_{yy} = \frac{1+\nu}{E}\sigma_{yy} - \frac{\nu}{E}(\sigma_{xx}+\sigma_{yy}) \Rightarrow -\nu\sigma_{xx} + \sigma_{yy} = 10^{-3}E$$
$$\varepsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx}+\sigma_{yy})$$

$$\boxed{-0.25\sigma_{xx} + \sigma_{yy} = 20}$$

$$\boxed{\sigma_{xx} = -17.9\ \text{kN/cm}^2 \qquad \sigma_{yy} = 15.5\ \text{kN/cm}^2}$$

$$\boxed{\sigma_{xy} = 8.9\ \text{kN/cm}^2}$$

$$\hat{\sigma} = \begin{bmatrix} -17.9 & 8.9 \\ 8.9 & 15.5 \end{bmatrix}\ [\text{kN/cm}^2]$$

4. Naloga

i.) deformacije: $\varepsilon_{xx}=0$

napetosti: $[\sigma]\begin{bmatrix}0\\1\\0\end{bmatrix}=\underline{0}$ $\qquad [\sigma]\begin{bmatrix}0\\0\\1\end{bmatrix}=\underline{0}$

$\sigma_{xy}=\sigma_{yy}=\sigma_{yz}=0 \qquad \sigma_{xz}=\sigma_{yz}=\sigma_{zz}=0$

Hookov zakon

$$0=\varepsilon_{xx}=\frac{1+\nu}{E}\sigma_{xx}-\frac{\nu}{E}\sigma_{xx}+\alpha_T\Delta T \quad\Rightarrow\quad \sigma_{xx}=-E\alpha_T\Delta T=-2\cdot10^{4}\cdot10^{-5}\cdot30$$

$\sigma_{xx}=-6$ kN/cm^2

$$\boxed{\hat{\sigma}=\begin{bmatrix}-6&0&0\\0&0&0\\0&0&0\end{bmatrix}\ \text{kN/cm}^2}$$

ii) deformacije $\varepsilon_{xx}=0$; $\varepsilon_V=0 \Rightarrow \varepsilon_{yy}+\varepsilon_{zz}=0$

napetosti $[\sigma]\begin{bmatrix}0\\1\\0\end{bmatrix}=\underline{0} \qquad [\sigma]\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\-p\end{bmatrix}$

$\sigma_{xy}=\sigma_{yy}=\sigma_{yz}=0 \qquad \sigma_{xz}=\sigma_{yz}=0 \quad \sigma_{zz}=-p$

Hookov zakon

$$0=\varepsilon_{xx}=\frac{1+\nu}{E}\sigma_{xx}-\frac{\nu}{E}(\sigma_{xx}-p)+\alpha_T\Delta T$$

$$\left.\begin{aligned}\varepsilon_{yy}&=\frac{1+\nu}{E}\cdot\cancel{0}-\frac{\nu}{E}(\sigma_{xx}-p)+\alpha_T\Delta T\\ \varepsilon_{zz}&=\frac{1+\nu}{E}(-p)-\frac{\nu}{E}(\sigma_{xx}-p)+\alpha_T\Delta T\end{aligned}\right\}+$$

$$0=\varepsilon_{yy}+\varepsilon_{zz}=-\frac{1+\nu}{E}p-\frac{2\nu}{E}(\sigma_{xx}-p)+2\alpha_T\Delta T$$

$$\sigma_{xx}+\nu p=-E\alpha_T\Delta T$$
$$-2\nu\sigma_{xx}+(\nu-1)p=-2E\alpha_T\Delta T$$

$$\sigma_{xx}+0.2p=-6 \quad \cdot 0.4$$
$$-0.4\sigma_{xx}-0.8p=-12$$
$$-0.72p=-14.4 \Rightarrow \boxed{p=20\ \text{kN/cm}^2}$$

iii.) $\varepsilon_{xx}=0$

za ε_{yy} in ε_{zz} potrebujemo σ_{xx}! $\quad \sigma_{xx}=-10$ kN/cm^2

$$\varepsilon_{yy}=-\frac{\nu}{E}(-30)+\alpha_T\Delta T=6\cdot10^{-4}$$
$$\varepsilon_{zz}=-\frac{1+\nu}{E}20-\frac{\nu}{E}(-30)+\alpha_T\Delta T=-6\cdot10^{-4}$$

$$\boxed{\hat{\varepsilon}=10^{-4}\begin{bmatrix}0&0&0\\0&6&0\\0&0&-6\end{bmatrix}}$$

KONTROLA $\varepsilon_{yy}+\varepsilon_{zz}=0$ ✓