DODATEK K ŠTUDIJSKEMU GRADIVU ZA PREDMET TRDNOST V ŠTUDIJSKEM LETU 2010-2011

PRISPEVEK STRIŽNIH NAPETOSTI K DOPOLNILNEMU VIRTUALNEMU DELU NOTRANJIH SIL

$$\sigma_{xy}(x,y) = -N_y(x)\frac{S_z^*(y)}{h^*(y)I_{zz}} - N_z(x)\frac{S_y^*(y)}{h^*(y)I_{yy}}$$

$$\sigma_{xz}(x,z) = -N_y(x)\frac{S_z^*(z)}{b^*(z)I_{zz}} - N_z(x)\frac{S_y^*(z)}{b^*(z)I_{yy}}$$

Obravnavamo nosilec z dvojno simetričnim prerezom: $S_y^*(y) = S_z^*(z) = 0$

$$\sigma_{xy}(x,y) = -N_y(x)\frac{S_z^*(y)}{h^*(y)I_{zz}}$$

$$\sigma_{xz}(x,z) = -N_z(x)\frac{S_y^*(z)}{b^*(z)I_{yy}}$$

$$\delta\sigma_{xy} = -\delta N_y\frac{S_z^*}{h^*I_{zz}}$$

$$\delta\sigma_{xz} = -\delta N_z\frac{S_y^*}{b^*I_{yy}}$$

$$2\varepsilon_{xy} = \frac{\sigma_{xy}}{G} = -N_y\frac{S_z^*}{G\,h^*I_{zz}}$$

$$2\varepsilon_{xz} = \frac{\sigma_{xz}}{G} = -N_z\frac{S_y^*}{G\,b^*I_{yy}}$$

$$\delta\bar{D}^* = \int_{\mathscr{V}} \sum_i \sum_j \varepsilon_{ij}\delta\sigma_{ij}\,dV$$

linijski element $\rightarrow$

$$\boxed{\delta\bar{D}^* = \int\limits_{\mathscr{V}} \varepsilon_{xx}\delta\sigma_{xx}\, dV + \int\limits_{\mathscr{V}} \left(2\varepsilon_{xy}\,\delta\sigma_{xy} + 2\varepsilon_{xz}\,\delta\sigma_{xz}\right)\, dV}$$

$$\boxed{\delta\bar{D}^*(\delta N_x, \delta M_y, \delta M_z) = \int\limits_{\mathscr{V}} \varepsilon_{xx}\delta\sigma_{xx}\, dV = \int\limits_0^l \left(\frac{N_x\delta N_x}{EA_x} + \frac{M_y\delta M_y}{EI_y} + \frac{M_z\delta M_z}{EI_z}\right) dx}$$

$$\boxed{\delta\bar{D}^*(\delta N_y, \delta N_z) = \int\limits_{\mathscr{V}} \left(2\varepsilon_{xy}\,\delta\sigma_{xy} + 2\varepsilon_{xz}\,\delta\sigma_{xz}\right)\, dV}$$

$$\delta\bar{D}^*(\delta N_y, \delta N_z) = \int\limits_{\mathscr{V}} \left(2\varepsilon_{xy}\,\delta\sigma_{xy} + 2\varepsilon_{xz}\,\delta\sigma_{xz}\right)\, dV$$

$$\delta\bar{D}^*(\delta N_y, \delta N_z) = \int\limits_{\mathscr{V}} \left[\frac{N_y\delta N_y}{GI_z^2}\left(\frac{S_z^*}{h^*}\right)^2 + \frac{N_z\delta N_z}{GI_y^2}\left(\frac{S_y^*}{b^*}\right)^2\right] dV$$

$$\delta\bar{D}^*(\delta N_y, \delta N_z) = \int\limits_0^l \left[\frac{N_y\delta N_y}{GI_z^2}\int\limits_{\mathscr{A}_x}\left(\frac{S_z^*}{h^*}\right)^2 dA_x + \frac{N_z\delta N_z}{GI_y^2}\int\limits_{\mathscr{A}_x}\left(\frac{S_y^*}{b^*}\right)^2 dA_x\right] dx$$

$$\boxed{\kappa_y = \frac{A_x}{I_z^2}\int\limits_{\mathscr{A}_x}\left(\frac{S_z^*}{h^*}\right)^2 dA_x} \qquad \boxed{\kappa_z = \frac{A_x}{I_y^2}\int\limits_{\mathscr{A}_x}\left(\frac{S_y^*}{b^*}\right)^2 dA_x}$$

$$\boxed{\delta\bar{D}^*(\delta N_y, \delta N_z) = \int\limits_0^l \left(\kappa_y\frac{N_y\delta N_y}{GA_x} + \kappa_z\frac{N_z\delta N_z}{GA_x}\right) dx}$$

Lahko tudi:

$$\boxed{A_y = \frac{A_x}{\kappa_y} \quad \textbf{in} \quad A_z = \frac{A_x}{\kappa_z}}$$

$$\boxed{\delta \bar{D}^*(\delta N_y, \delta N_z) = \int_0^l \left(\frac{N_y \delta N_y}{G A_y} + \frac{N_z \delta N_z}{G A_z} \right) dx}$$

Linijska konstrukcija, sestavljena iz n linijskih elementov:

$$\boxed{\delta \bar{D}^* = \sum_{e=1}^{n} \int_0^{l_e} \left(\frac{N_x \delta N_x}{E A_x} + \frac{M_y \delta M_y}{E I_y} + \frac{M_z \delta M_z}{E I_z} + \kappa_y \frac{N_y \delta N_y}{G A_x} + \kappa_z \frac{N_z \delta N_z}{G A_x} \right) dx}$$

VPLIV TEMPERATURNE SPREMEMBE NA MEHANSKO STANJE LINIJSKEGA NOSILCA

$$\Delta T(x,y,z) = \Delta T_x(x) + y\,\Delta T_y(x) + z\,\Delta T_z(x)$$

$$\varepsilon_{xx} = \frac{du}{dx} - y\,\frac{d^2v}{dx^2} - z\,\frac{d^2w}{dx^2} \equiv \varepsilon^g_{xx}$$

$$\varepsilon^g_{xx} = \varepsilon^m_{xx} + \varepsilon^T_{xx} = \frac{\sigma_{xx}}{E} + \alpha_T \Delta T$$

$$\sigma_{xx} = E\left(\frac{du}{dx} - \alpha_T \Delta T_x\right) - y\,E\left(\frac{d^2v}{dx^2} + \alpha_T \Delta T_y\right) - z\,E\left(\frac{d^2w}{dx^2} + \alpha_T \Delta T_z\right)$$

$$\begin{aligned} N_x &= \int_{\mathscr{A}_x} \sigma_{xx}\, dA_x \\ M_y &= \int_{\mathscr{A}_x} z\,\sigma_{xx}\, dA_x \\ M_z &= -\int_{\mathscr{A}_x} y\,\sigma_{xx}\, dA_x \end{aligned} \qquad \rightarrow \qquad \begin{aligned} \frac{du}{dx} &= \frac{N_x}{EA_x} + \alpha_T \Delta T_x \\ \frac{d^2v}{dx^2} &= \frac{M_z}{EI_{zz}} + \alpha_T \Delta T_y \\ \frac{d^2w}{dx^2} &= -\frac{M_y}{EI_{yy}} - \alpha_T \Delta T_z \end{aligned}$$

PRISPEVEK LINEARNE SPREMEMBE TEMPERATURE K DOPOLNILNEMU VIRTUALNEMU DELU NOTRANJIH SIL

Ravninski primer $\Delta T_y = 0$ **:**

$$\boxed{\Delta T = \Delta T_x + z\Delta T_z}$$

$$\boxed{\varepsilon_{xx} = \frac{\sigma_{xx}}{E} + \alpha_T \Delta T = \frac{1}{E}\left(\frac{N_x}{A_x} + z\frac{M_y}{I_y}\right) + \alpha_T\left(\Delta T_x + z\Delta T_z\right)}$$

$$\boxed{\varepsilon_{xx} = \left(\frac{N_x}{EA_x} + \alpha_T \Delta T_x\right) + z\left(\frac{M_y}{EI_y} + \alpha_T \Delta T_z\right)}$$

$$\boxed{\sigma_{xx} = \frac{N_x}{A_x} + z\frac{M_y}{I_y}} \qquad \rightarrow \qquad \boxed{\delta\sigma_{xx} = \frac{\delta N_x}{A_x} + z\frac{\delta M_y}{I_y}}$$

Bernoullijev linijski element :

$$\boxed{\delta\bar{D}^* = \int\limits_{\mathscr{V}} \varepsilon_{xx}\delta\sigma_{xx}\, dV}$$

$$\delta\bar{D}^* = \int\limits_{\mathscr{V}} \left[\left(\frac{N_x}{EA_x} + \alpha_T \Delta T_x\right) + z\left(\frac{M_y}{EI_y} + \alpha_T \Delta T_z\right)\right]\left(\frac{\delta N_x}{A_x} + z\frac{\delta M_y}{I_y}\right) dV$$

$$\boxed{\delta\bar{D}^* = \int\limits_0^l \left(\frac{N_x\delta N_x}{EA_x} + \frac{M_y\delta M_y}{EI_y}\right) dx + \int\limits_0^l \left(\alpha_T \Delta T_x \delta N_x + \alpha_T \Delta T_z \delta M_y\right) dx}$$

Prostorski primer:

$$\boxed{\Delta T = \Delta T_x + y\,\Delta T_y + z\,\Delta T_z}$$

$$\boxed{\begin{aligned}\delta\bar{D}^* = &\int_0^l \left(\frac{N_x\delta N_x}{EA_x} + \frac{M_y\delta M_y}{EI_y} + \frac{M_z\delta M_z}{EI_z}\right) dx + \\ &\int_0^l \left(\alpha_T\Delta T_x\delta N_x - \alpha_T\Delta T_y\delta M_z + \alpha_T\Delta T_z\delta M_y\right) dx\end{aligned}}$$

Metoda sil:

$$\begin{aligned}
a_{11}X_1 + a_{12}X_2 + \ldots + a_{1n}X_n + b_1 &= \underset{\sim}{u}_1 \\
a_{21}X_1 + a_{22}X_2 + \ldots + a_{2n}X_n + b_2 &= \underset{\sim}{u}_2 \\
\vdots \qquad \vdots \qquad\qquad \vdots \quad \vdots \quad \vdots & \\
a_{n1}X_1 + a_{n2}X_2 + \ldots + a_{nn}X_n + b_n &= \underset{\sim}{u}_n
\end{aligned} \qquad \rightarrow \qquad \boxed{[a_{ij}]\,\{X_j\} + \{b_i\} = \left\{\underset{\sim}{u}_j\right\}}$$

$$a_{ij} = a_{ji} = \sum_{e=1}^{n} \int_0^{l_e} \left(\frac{\bar{N}_{xi}\bar{N}_{xj}}{EA_x} + \frac{\bar{M}_{yi}\bar{M}_{yj}}{EI_y} + \frac{\bar{M}_{zi}\bar{M}_{zj}}{EI_z}\right) dx$$

$$\begin{aligned}
b_j = &\sum_{e=1}^{n} \int_0^{l_e} \left(\frac{\bar{N}_{xj}N_{xp}}{EA_x} + \frac{\bar{M}_{yj}M_{yp}}{EI_y} + \frac{\bar{M}_{zj}M_{zp}}{EI_z}\right) dx + \\
&\sum_{e=1}^{n} \int_0^{l_e} \left(\alpha_T\Delta T_x\bar{N}_{xj} - \alpha_T\Delta T_y\bar{M}_{zj} + \alpha_T\Delta T_z\bar{M}_{yj}\right) dx
\end{aligned}$$

$$\boxed{N_x = N_{xp} + \sum_{i=1}^{n} \bar{N}_{xi}X_i} \qquad \boxed{M_y = M_{yp} + \sum_{i=1}^{n} \bar{M}_{yi}X_i}$$