# Transformacija komponent tenzorja napetosti

Tenzor napetosti v kartezijskem koordinatnem sistemu $(x, y, z)$ z bazo $\{e_x, e_y, e_z\}$ opišemo z matriko

$$[\sigma_{ij}] = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}.$$

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Določi komponente tenzorja napetosti $[\sigma_{\alpha\beta}]$, izražene v novi bazi $\{e_\xi, e_\eta, e_\zeta\}$. Privzemi sledeče zveze med baznimi vektorji: $e_\xi = -e_y$, $e_\eta = e_x$ in $e_\zeta = e_z$.

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Tenzor napetosti v običajni bazi $\{e_x, e_y, e_z\}$ predstavimo z matriko

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}.$$

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Napetostne vektorje zapišemo z enačbami

$$\begin{aligned} \boldsymbol{\sigma}_x &= \sigma_{xx}\, e_x + \sigma_{xy}\, e_y + \sigma_{xz}\, e_z, \\ \boldsymbol{\sigma}_y &= \sigma_{yx}\, e_x + \sigma_{yy}\, e_y + \sigma_{yz}\, e_z, \\ \boldsymbol{\sigma}_z &= \sigma_{zx}\, e_x + \sigma_{zy}\, e_y + \sigma_{zz}\, e_z. \end{aligned}$$

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Fizikalni pomen od nič različnih komponent je razviden iz slike 1.

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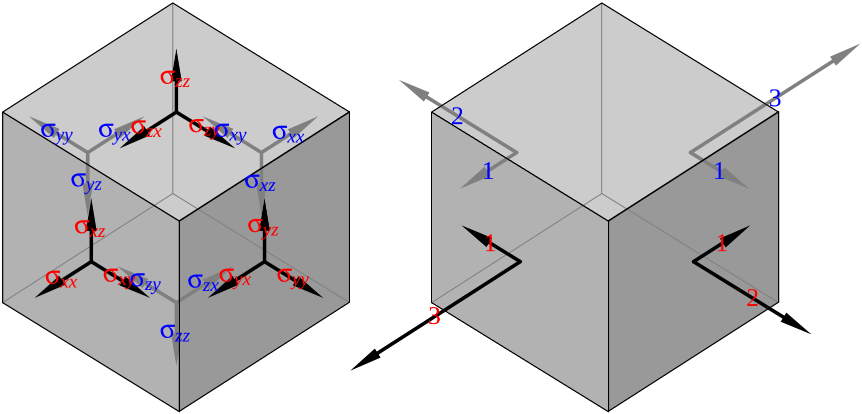
Fizikalni pomen od nič različnih komponent je razviden iz slike 1. Prikazane napetosti dejanko pripadajo točki v središču kocke, na elementarni kocki smo jih narisali le zaradi preglednosti.

Fizikalni pomen od nič različnih komponent tenzorja napetosti v bazi $\{e_x, e_y, e_z\}$. Leva slika prikazuje smeri delovanja komponent tenzorja napetosti v izbrani bazi, desna pa dejansko napetostno stanje.




Slika: 1

Tenzor napetosti v novi bazi $\{e_\xi, e_\eta, e_\zeta\}$ predstavimo z matriko

$$[\sigma_{\alpha\beta}] = \begin{bmatrix} \sigma_{\xi\xi} & \sigma_{\xi\eta} & \sigma_{\xi\zeta} \\ \sigma_{\eta\xi} & \sigma_{\eta\eta} & \sigma_{\eta\zeta} \\ \sigma_{\zeta\xi} & \sigma_{\zeta\eta} & \sigma_{\zeta\zeta} \end{bmatrix}.$$

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Napetostne vektorje zapišemo z enačbami

$$\begin{aligned} \boldsymbol{\sigma}_\xi &= \sigma_{\xi\xi}\, e_\xi + \sigma_{\xi\eta}\, e_\eta + \sigma_{\xi\zeta}\, e_\zeta, \\ \boldsymbol{\sigma}_\eta &= \sigma_{\eta\xi}\, e_\xi + \sigma_{\eta\eta}\, e_\eta + \sigma_{\eta\zeta}\, e_\zeta, \\ \boldsymbol{\sigma}_\zeta &= \sigma_{\zeta\xi}\, e_\xi + \sigma_{\zeta\eta}\, e_\eta + \sigma_{\zeta\zeta}\, e_\zeta. \end{aligned}$$

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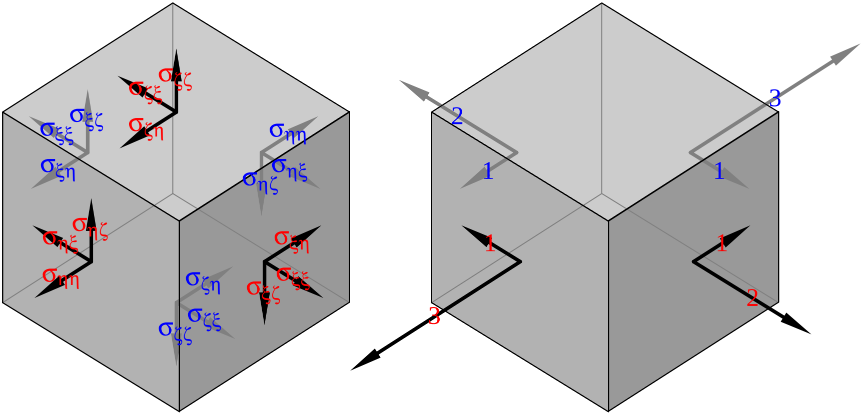
Napetostne vektorje zapišemo z enačbami

$$\begin{aligned} \boldsymbol{\sigma}_\xi &= \sigma_{\xi\xi}\, e_\xi + \sigma_{\xi\eta}\, e_\eta + \sigma_{\xi\zeta}\, e_\zeta, \\ \boldsymbol{\sigma}_\eta &= \sigma_{\eta\xi}\, e_\xi + \sigma_{\eta\eta}\, e_\eta + \sigma_{\eta\zeta}\, e_\zeta, \\ \boldsymbol{\sigma}_\zeta &= \sigma_{\zeta\xi}\, e_\xi + \sigma_{\zeta\eta}\, e_\eta + \sigma_{\zeta\zeta}\, e_\zeta. \end{aligned}$$

Fizikalni pomen od nič različnih komponent je tako razviden iz slike 2.

Fizikalni pomen od nič različnih komponent tenzorja napetosti v bazi $\{e_\xi, e_\eta, e_\zeta\}$. Leva slika prikazuje smeri delovanja komponent tenzorja napetosti v izbrani bazi, desna pa dejansko napetostno stanje.




Slika: 2

Iz slike 2 lahko s primerjavo leve in desne kocke preberemo komponente tenzorja napetosti v novi bazi.

$$[\sigma_{\alpha\beta}] = \begin{bmatrix} \sigma_{\xi\xi} & \sigma_{\xi\eta} & \sigma_{\xi\zeta} \\ \sigma_{\eta\xi} & \sigma_{\eta\eta} & \sigma_{\eta\zeta} \\ \sigma_{\zeta\xi} & \sigma_{\zeta\eta} & \sigma_{\zeta\zeta} \end{bmatrix}$$

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Napetostne vektorje potem zapišemo z enačbami

$$\begin{aligned} \boldsymbol{\sigma}_\xi &= 2\mathrm{MPa}\, \boldsymbol{e}_\xi + 1\mathrm{MPa}\, \boldsymbol{e}_\eta, \\ \boldsymbol{\sigma}_\eta &= 1\mathrm{MPa}\, \boldsymbol{e}_\xi + 3\mathrm{MPa}\, \boldsymbol{e}_\eta, \\ \boldsymbol{\sigma}_\zeta &= \mathbf{0}. \end{aligned}$$

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Zaradi enostavnih zvez med baznimi vektorji, smo lahko rezultat preprosto prebrali iz slike 2.

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V nadaljevanju bomo do istega rezultata prišli še z izračunom.

Komponente tenzorja napetosti v novi bazi so s komponentami tenzorja napetosti v stari bazi povezane z enačbami

$$\sigma_{\alpha\beta} = \sum_{i\in\{x,y,z\}} \sum_{j\in\{x,y,z\}} e_{\alpha\, i}\, e_{\beta\, j}\, \sigma_{ij}, \quad \alpha, \beta \in \{\xi, \eta, \zeta\}. \tag{1}$$

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e_{\xi\, z} &= e_{z\,\xi} = e_\xi \cdot e_z = -e_y \cdot e_z = 0, \\
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e_{\eta\, x} &= e_{x\,\eta} = e_\eta \cdot e_x = e_x \cdot e_x = 1, \\
e_{\eta\, y} &= e_{y\,\eta} = e_\eta \cdot e_y = e_x \cdot e_y = 0, \\
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e_{\eta x} &= e_{x\eta} = e_\eta \cdot e_x = e_x \cdot e_x = 1,\\
e_{\eta y} &= e_{y\eta} = e_\eta \cdot e_y = e_x \cdot e_y = 0,\\
e_{\eta z} &= e_{z\eta} = e_\eta \cdot e_z = e_x \cdot e_z = 0,\\
e_{\zeta x} &= e_{x\zeta} = e_\zeta \cdot e_x = e_z \cdot e_x = 0,\\
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e_{\eta\, y} &= e_{y\,\eta} = e_\eta \cdot e_y = e_x \cdot e_y = 0,\\
e_{\eta\, z} &= e_{z\,\eta} = e_\eta \cdot e_z = e_x \cdot e_z = 0,\\
e_{\zeta\, x} &= e_{x\,\zeta} = e_\zeta \cdot e_x = e_z \cdot e_x = 0,\\
e_{\zeta\, y} &= e_{y\,\zeta} = e_\zeta \cdot e_y = e_z \cdot e_y = 0,\\
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\end{aligned}$$

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$$\sigma_{\alpha\beta} = \sum_{i\in\{x,y,z\}} \sum_{j\in\{x,y,z\}} e_{\alpha\, i} e_{\beta\, j} \sigma_{ij}, \quad \alpha, \beta \in \{\xi, \eta, \zeta\}. \tag{1}$$

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e_{\eta\, x} &= e_{x\,\eta} = e_\eta \cdot e_x = e_x \cdot e_x = 1, \\
e_{\eta\, y} &= e_{y\,\eta} = e_\eta \cdot e_y = e_x \cdot e_y = 0, \\
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e_{\zeta\, x} &= e_{x\,\zeta} = e_\zeta \cdot e_x = e_z \cdot e_x = 0, \\
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$$\begin{aligned} \sigma_{\xi\,\xi} &= e_{\xi\,x}^2\,\sigma_{xx} + e_{\xi\,y}^2\,\sigma_{yy} + e_{\xi\,z}^2\,\sigma_{zz} + \\ &+ 2\,e_{\xi\,x}\,e_{\xi\,y}\,\sigma_{xy} + 2\,e_{\xi\,x}\,e_{\xi\,z}\,\sigma_{xz} + 2\,e_{\xi\,y}\,e_{\xi\,z}\,\sigma_{yz} = \end{aligned}$$

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&+ 2\,e_{\xi\,x}\,e_{\xi\,y}\,\sigma_{xy} + 2\,e_{\xi\,x}\,e_{\xi\,z}\,\sigma_{xz} + 2\,e_{\xi\,y}\,e_{\xi\,z}\,\sigma_{yz} = \\
&= e_{\xi\,y}^2\,\sigma_{yy} = 2\mathrm{MPa},
\end{aligned}$$

Potem pa še komponente tenzorja $\sigma_{\alpha\beta}$.
Najprej diagonalne.

$$\begin{aligned}
\sigma_{\xi\,\xi} &= e_{\xi\,x}^2\,\sigma_{xx} + e_{\xi\,y}^2\,\sigma_{yy} + e_{\xi\,z}^2\,\sigma_{zz} + \\
&+ 2\,e_{\xi\,x}\,e_{\xi\,y}\,\sigma_{xy} + 2\,e_{\xi\,x}\,e_{\xi\,z}\,\sigma_{xz} + 2\,e_{\xi\,y}\,e_{\xi\,z}\,\sigma_{yz} = \\
&= e_{\xi\,y}^2\,\sigma_{yy} = 2\mathrm{MPa}, \\
\sigma_{\eta\,\eta} &= e_{\eta\,x}^2\,\sigma_{xx} + e_{\eta\,y}^2\,\sigma_{yy} + e_{\eta\,z}^2\,\sigma_{zz} + \\
&+ 2\,e_{\eta\,x}\,e_{\eta\,y}\,\sigma_{xy} + 2\,e_{\eta\,x}\,e_{\eta\,z}\,\sigma_{xz} + 2\,e_{\eta\,y}\,e_{\eta\,z}\,\sigma_{yz} =
\end{aligned}$$

Potem pa še komponente tenzorja $\sigma_{\alpha\beta}$.
Najprej diagonalne.

$$\begin{aligned}
\sigma_{\xi\xi} &= e_{\xi x}^2\sigma_{xx} + e_{\xi y}^2\sigma_{yy} + e_{\xi z}^2\sigma_{zz} + \\
&+ 2e_{\xi x}e_{\xi y}\sigma_{xy} + 2e_{\xi x}e_{\xi z}\sigma_{xz} + 2e_{\xi y}e_{\xi z}\sigma_{yz} = \\
&= e_{\xi y}^2\sigma_{yy} = 2\text{MPa}, \\
\sigma_{\eta\eta} &= e_{\eta x}^2\sigma_{xx} + e_{\eta y}^2\sigma_{yy} + e_{\eta z}^2\sigma_{zz} + \\
&+ 2e_{\eta x}e_{\eta y}\sigma_{xy} + 2e_{\eta x}e_{\eta z}\sigma_{xz} + 2e_{\eta y}e_{\eta z}\sigma_{yz} = \\
&= e_{\eta x}^2\sigma_{xx} = 3\text{MPa},
\end{aligned}$$

Potem pa še komponente tenzorja $\sigma_{\alpha\beta}$.
Najprej diagonalne.

$$
\begin{aligned}
\sigma_{\xi\,\xi} &= e_{\xi\,x}^2\,\sigma_{xx} + e_{\xi\,y}^2\,\sigma_{yy} + e_{\xi\,z}^2\,\sigma_{zz} + \\
&+ 2\,e_{\xi\,x}\,e_{\xi\,y}\,\sigma_{xy} + 2\,e_{\xi\,x}\,e_{\xi\,z}\,\sigma_{xz} + 2\,e_{\xi\,y}\,e_{\xi\,z}\,\sigma_{yz} = \\
&= e_{\xi\,y}^2\,\sigma_{yy} = 2\text{MPa}, \\
\sigma_{\eta\,\eta} &= e_{\eta\,x}^2\,\sigma_{xx} + e_{\eta\,y}^2\,\sigma_{yy} + e_{\eta\,z}^2\,\sigma_{zz} + \\
&+ 2\,e_{\eta\,x}\,e_{\eta\,y}\,\sigma_{xy} + 2\,e_{\eta\,x}\,e_{\eta\,z}\,\sigma_{xz} + 2\,e_{\eta\,y}\,e_{\eta\,z}\,\sigma_{yz} = \\
&= e_{\eta\,x}^2\,\sigma_{xx} = 3\text{MPa}, \\
\sigma_{\zeta\,\zeta} &= e_{\zeta\,x}^2\,\sigma_{xx} + e_{\zeta\,y}^2\,\sigma_{yy} + e_{\zeta\,z}^2\,\sigma_{zz} + \\
&+ 2\,e_{\zeta\,x}\,e_{\zeta\,y}\,\sigma_{xy} + 2\,e_{\zeta\,x}\,e_{\zeta\,z}\,\sigma_{xz} + 2\,e_{\zeta\,y}\,e_{\zeta\,z}\,\sigma_{yz} =
\end{aligned}
$$

Potem pa še komponente tenzorja $\sigma_{\alpha\beta}$.
Najprej diagonalne.

$$\begin{aligned}
\sigma_{\xi\,\xi} &= e^2_{\xi\,x}\sigma_{xx} + e^2_{\xi\,y}\sigma_{yy} + e^2_{\xi\,z}\sigma_{zz} + \\
&+ 2\,e_{\xi\,x}e_{\xi\,y}\sigma_{xy} + 2\,e_{\xi\,x}e_{\xi\,z}\sigma_{xz} + 2\,e_{\xi\,y}e_{\xi\,z}\sigma_{yz} = \\
&= e^2_{\xi\,y}\sigma_{yy} = 2\text{MPa}, \\
\sigma_{\eta\,\eta} &= e^2_{\eta\,x}\sigma_{xx} + e^2_{\eta\,y}\sigma_{yy} + e^2_{\eta\,z}\sigma_{zz} + \\
&+ 2\,e_{\eta\,x}e_{\eta\,y}\sigma_{xy} + 2\,e_{\eta\,x}e_{\eta\,z}\sigma_{xz} + 2\,e_{\eta\,y}e_{\eta\,z}\sigma_{yz} = \\
&= e^2_{\eta\,x}\sigma_{xx} = 3\text{MPa}, \\
\sigma_{\zeta\,\zeta} &= e^2_{\zeta\,x}\sigma_{xx} + e^2_{\zeta\,y}\sigma_{yy} + e^2_{\zeta\,z}\sigma_{zz} + \\
&+ 2\,e_{\zeta\,x}e_{\zeta\,y}\sigma_{xy} + 2\,e_{\zeta\,x}e_{\zeta\,z}\sigma_{xz} + 2\,e_{\zeta\,y}e_{\zeta\,z}\sigma_{yz} = \\
&= e^2_{\zeta\,z}\sigma_{zz} = 0.
\end{aligned}$$

Nato izven diagonalne.

$$\begin{aligned}\sigma_{\xi\eta} &= e_{\xi x}e_{\eta x}\sigma_{xx}+e_{\xi x}e_{\eta y}\sigma_{xy}+e_{\xi x}e_{\eta z}\sigma_{xz}+\\ &+ e_{\xi y}e_{\eta x}\sigma_{yx}+e_{\xi y}e_{\eta y}\sigma_{yy}+e_{\xi y}e_{\eta z}\sigma_{yz}+\\ &+ e_{\xi z}e_{\eta x}\sigma_{zx}+e_{\xi z}e_{\eta y}\sigma_{zy}+e_{\xi z}e_{\eta z}\sigma_{zz}=\end{aligned}$$

Nato izven diagonalne.

$$\begin{aligned}
\sigma_{\xi\eta} &= e_{\xi x} e_{\eta x} \sigma_{xx} + e_{\xi x} e_{\eta y} \sigma_{xy} + e_{\xi x} e_{\eta z} \sigma_{xz} + \\
&+ e_{\xi y} e_{\eta x} \sigma_{yx} + e_{\xi y} e_{\eta y} \sigma_{yy} + e_{\xi y} e_{\eta z} \sigma_{yz} + \\
&+ e_{\xi z} e_{\eta x} \sigma_{zx} + e_{\xi z} e_{\eta y} \sigma_{zy} + e_{\xi z} e_{\eta z} \sigma_{zz} = \\
&= e_{\xi y} e_{\eta x} \sigma_{yx} = 1\mathrm{MPa},
\end{aligned}$$

Nato izven diagonalne.

$$\begin{aligned}
\sigma_{\xi\,\eta} &= e_{\xi\,x}e_{\eta\,x}\sigma_{xx} + e_{\xi\,x}e_{\eta\,y}\sigma_{xy} + e_{\xi\,x}e_{\eta\,z}\sigma_{xz} + \\
&+ e_{\xi\,y}e_{\eta\,x}\sigma_{yx} + e_{\xi\,y}e_{\eta\,y}\sigma_{yy} + e_{\xi\,y}e_{\eta\,z}\sigma_{yz} + \\
&+ e_{\xi\,z}e_{\eta\,x}\sigma_{zx} + e_{\xi\,z}e_{\eta\,y}\sigma_{zy} + e_{\xi\,z}e_{\eta\,z}\sigma_{zz} = \\
&= e_{\xi\,y}e_{\eta\,x}\sigma_{yx} = 1\text{MPa}, \\
\sigma_{\xi\,\zeta} &= e_{\xi\,x}e_{\zeta\,x}\sigma_{xx} + e_{\xi\,x}e_{\zeta\,y}\sigma_{xy} + e_{\xi\,x}e_{\zeta\,z}\sigma_{xz} + \\
&+ e_{\xi\,y}e_{\zeta\,x}\sigma_{yx} + e_{\xi\,y}e_{\zeta\,y}\sigma_{yy} + e_{\xi\,y}e_{\zeta\,z}\sigma_{yz} + \\
&+ e_{\xi\,z}e_{\zeta\,x}\sigma_{zx} + e_{\xi\,z}e_{\zeta\,y}\sigma_{zy} + e_{\xi\,z}e_{\zeta\,z}\sigma_{zz} =
\end{aligned}$$

Nato izven diagonalne.

$$\begin{aligned}
\sigma_{\xi\eta} &= e_{\xi x} e_{\eta x} \sigma_{xx} + e_{\xi x} e_{\eta y} \sigma_{xy} + e_{\xi x} e_{\eta z} \sigma_{xz} + \\
&+ e_{\xi y} e_{\eta x} \sigma_{yx} + e_{\xi y} e_{\eta y} \sigma_{yy} + e_{\xi y} e_{\eta z} \sigma_{yz} + \\
&+ e_{\xi z} e_{\eta x} \sigma_{zx} + e_{\xi z} e_{\eta y} \sigma_{zy} + e_{\xi z} e_{\eta z} \sigma_{zz} = \\
&= e_{\xi y} e_{\eta x} \sigma_{yx} = 1\mathrm{MPa}, \\
\sigma_{\xi\zeta} &= e_{\xi x} e_{\zeta x} \sigma_{xx} + e_{\xi x} e_{\zeta y} \sigma_{xy} + e_{\xi x} e_{\zeta z} \sigma_{xz} + \\
&+ e_{\xi y} e_{\zeta x} \sigma_{yx} + e_{\xi y} e_{\zeta y} \sigma_{yy} + e_{\xi y} e_{\zeta z} \sigma_{yz} + \\
&+ e_{\xi z} e_{\zeta x} \sigma_{zx} + e_{\xi z} e_{\zeta y} \sigma_{zy} + e_{\xi z} e_{\zeta z} \sigma_{zz} = \\
&= 0,
\end{aligned}$$

Nato izven diagonalne.

$$\begin{aligned}
\sigma_{\xi\eta} &= e_{\xi x} e_{\eta x} \sigma_{xx} + e_{\xi x} e_{\eta y} \sigma_{xy} + e_{\xi x} e_{\eta z} \sigma_{xz} + \\
&+ e_{\xi y} e_{\eta x} \sigma_{yx} + e_{\xi y} e_{\eta y} \sigma_{yy} + e_{\xi y} e_{\eta z} \sigma_{yz} + \\
&+ e_{\xi z} e_{\eta x} \sigma_{zx} + e_{\xi z} e_{\eta y} \sigma_{zy} + e_{\xi z} e_{\eta z} \sigma_{zz} = \\
&= e_{\xi y} e_{\eta x} \sigma_{yx} = 1\mathrm{MPa}, \\
\sigma_{\xi\zeta} &= e_{\xi x} e_{\zeta x} \sigma_{xx} + e_{\xi x} e_{\zeta y} \sigma_{xy} + e_{\xi x} e_{\zeta z} \sigma_{xz} + \\
&+ e_{\xi y} e_{\zeta x} \sigma_{yx} + e_{\xi y} e_{\zeta y} \sigma_{yy} + e_{\xi y} e_{\zeta z} \sigma_{yz} + \\
&+ e_{\xi z} e_{\zeta x} \sigma_{zx} + e_{\xi z} e_{\zeta y} \sigma_{zy} + e_{\xi z} e_{\zeta z} \sigma_{zz} = \\
&= 0, \\
\sigma_{\eta\zeta} &= e_{\eta x} e_{\zeta x} \sigma_{xx} + e_{\eta x} e_{\zeta y} \sigma_{xy} + e_{\eta x} e_{\zeta z} \sigma_{xz} + \\
&+ e_{\eta y} e_{\zeta x} \sigma_{yx} + e_{\eta y} e_{\zeta y} \sigma_{yy} + e_{\eta y} e_{\zeta z} \sigma_{yz} + \\
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\end{aligned}$$

Nato izven diagonalne.

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\sigma_{\xi\eta} &= e_{\xi x} e_{\eta x} \sigma_{xx} + e_{\xi x} e_{\eta y} \sigma_{xy} + e_{\xi x} e_{\eta z} \sigma_{xz} + \\
&+ e_{\xi y} e_{\eta x} \sigma_{yx} + e_{\xi y} e_{\eta y} \sigma_{yy} + e_{\xi y} e_{\eta z} \sigma_{yz} + \\
&+ e_{\xi z} e_{\eta x} \sigma_{zx} + e_{\xi z} e_{\eta y} \sigma_{zy} + e_{\xi z} e_{\eta z} \sigma_{zz} = \\
&= e_{\xi y} e_{\eta x} \sigma_{yx} = 1\mathrm{MPa}, \\
\sigma_{\xi\zeta} &= e_{\xi x} e_{\zeta x} \sigma_{xx} + e_{\xi x} e_{\zeta y} \sigma_{xy} + e_{\xi x} e_{\zeta z} \sigma_{xz} + \\
&+ e_{\xi y} e_{\zeta x} \sigma_{yx} + e_{\xi y} e_{\zeta y} \sigma_{yy} + e_{\xi y} e_{\zeta z} \sigma_{yz} + \\
&+ e_{\xi z} e_{\zeta x} \sigma_{zx} + e_{\xi z} e_{\zeta y} \sigma_{zy} + e_{\xi z} e_{\zeta z} \sigma_{zz} = \\
&= 0, \\
\sigma_{\eta\zeta} &= e_{\eta x} e_{\zeta x} \sigma_{xx} + e_{\eta x} e_{\zeta y} \sigma_{xy} + e_{\eta x} e_{\zeta z} \sigma_{xz} + \\
&+ e_{\eta y} e_{\zeta x} \sigma_{yx} + e_{\eta y} e_{\zeta y} \sigma_{yy} + e_{\eta y} e_{\zeta z} \sigma_{yz} + \\
&+ e_{\eta z} e_{\zeta x} \sigma_{zx} + e_{\eta z} e_{\zeta y} \sigma_{zy} + e_{\eta z} e_{\zeta z} \sigma_{zz} = \\
&= 0.
\end{aligned}$$

Upoštevamo simetrijo tenzorja in dobljene rezultate povežemo v končni rezultat

$$[\sigma_{\alpha\beta}] = \begin{bmatrix} \sigma_{\xi\xi} & \sigma_{\xi\eta} & \sigma_{\xi\zeta} \\ \sigma_{\eta\xi} & \sigma_{\eta\eta} & \sigma_{\eta\zeta} \\ \sigma_{\zeta\xi} & \sigma_{\zeta\eta} & \sigma_{\zeta\zeta} \end{bmatrix}$$

Upoštevamo simetrijo tenzorja in dobljene rezultate povežemo v končni rezultat

$$[\sigma_{\alpha\beta}] = \begin{bmatrix} \sigma_{\xi\xi} & \sigma_{\xi\eta} & \sigma_{\xi\zeta} \\ \sigma_{\eta\xi} & \sigma_{\eta\eta} & \sigma_{\eta\zeta} \\ \sigma_{\zeta\xi} & \sigma_{\zeta\eta} & \sigma_{\zeta\zeta} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}.$$

Do istega rezultata lahko pridemo tudi na drug način.

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Tvorimo transformacijsko matriko smernih kosinusov

$$[T] = \begin{bmatrix} e_{\xi x} & e_{\eta x} & e_{\zeta x} \\ e_{\xi y} & e_{\eta y} & e_{\zeta y} \\ e_{\xi z} & e_{\eta z} & e_{\zeta z} \end{bmatrix} = \begin{bmatrix} e_{\xi} & e_{\eta} & e_{\zeta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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Uporabimo dogovor o seštevanju in enačbe (1) prepišemo v obliko

$$[\sigma_{ij}] = [T]\,[\sigma_{\alpha\beta}]\,[T]^T, \qquad [\sigma_{\alpha\beta}] = [T]^T\,[\sigma_{ij}]\,[T],$$

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$$[T] = \begin{bmatrix} e_{\xi\,x} & e_{\eta\,x} & e_{\zeta\,x} \\ e_{\xi\,y} & e_{\eta\,y} & e_{\zeta\,y} \\ e_{\xi\,z} & e_{\eta\,z} & e_{\zeta\,z} \end{bmatrix} = \begin{bmatrix} e_\xi & e_\eta & e_\zeta \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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Kratek izračun da

$$[\sigma_{\alpha\beta}] = [T]^T\,[\sigma_{ij}]\,[T] =$$

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$$[T] = \begin{bmatrix} e_{\xi x} & e_{\eta x} & e_{\zeta x} \\ e_{\xi y} & e_{\eta y} & e_{\zeta y} \\ e_{\xi z} & e_{\eta z} & e_{\zeta z} \end{bmatrix} = \begin{bmatrix} e_\xi & e_\eta & e_\zeta \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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$$[\sigma_{\alpha\beta}] = [T]^T\,[\sigma_{ij}]\,[T] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

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$$[T] = \begin{bmatrix} e_{\xi\, x} & e_{\eta\, x} & e_{\zeta\, x} \\ e_{\xi\, y} & e_{\eta\, y} & e_{\zeta\, y} \\ e_{\xi\, z} & e_{\eta\, z} & e_{\zeta\, z} \end{bmatrix} = \begin{bmatrix} e_{\xi} & e_{\eta} & e_{\zeta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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