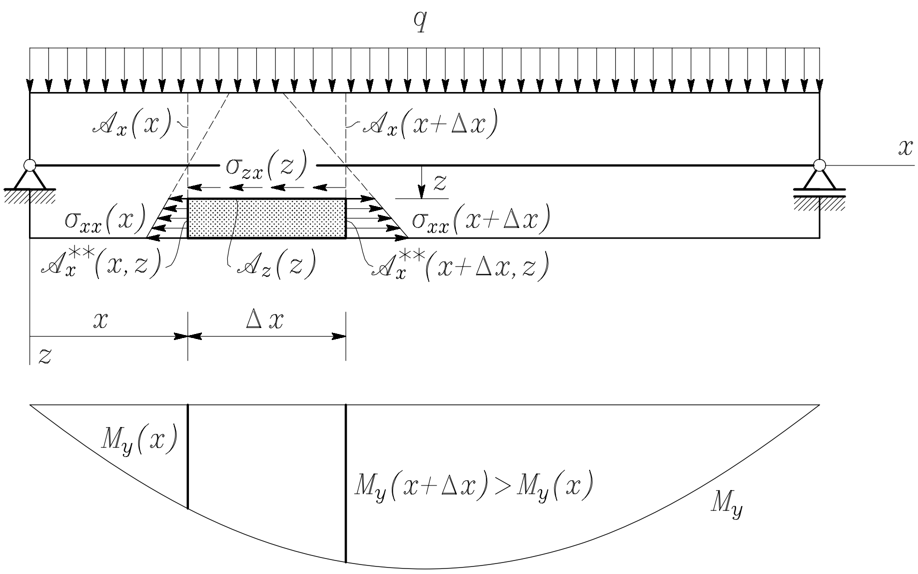
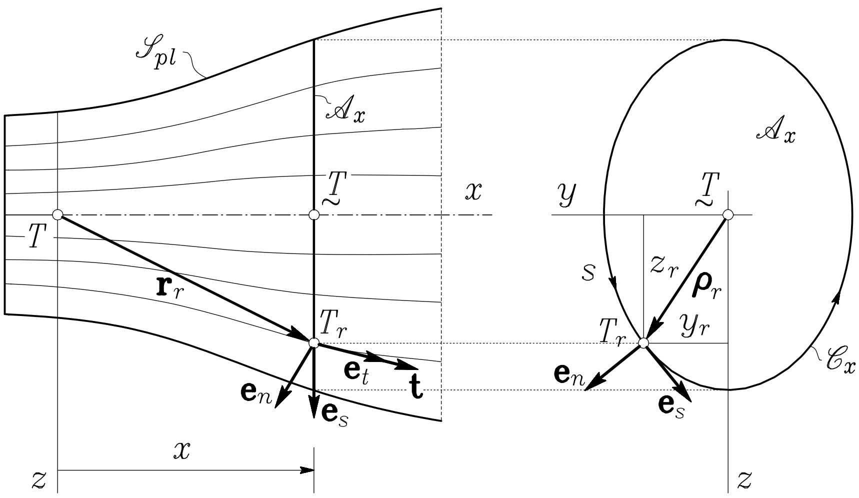
STRIŽNE IN PREČNE NORMALNE NAPETOSTI





$$T_r \ : \quad \boxed{\mathbf{r}_r = x\mathbf{e}_x + y_r\mathbf{e}_y + z_r\mathbf{e}_z} \qquad \boxed{x = konst. \quad \begin{cases} y_r = y_r(s) \\ z_r = z_r(s) \end{cases}}$$

$$\mathbf{e}_s = \left.\frac{\partial \mathbf{r}_r}{\partial s}\right|_{x=konst} \quad \rightarrow \quad \boxed{\mathbf{e}_s = \frac{dy_r}{ds}\mathbf{e}_y + \frac{dz_r}{ds}\mathbf{e}_z}$$

$$\mathbf{t} = \frac{\partial \mathbf{r}_r}{\partial x} = \mathbf{e}_x + \frac{\partial y_r}{\partial x}\mathbf{e}_y + \frac{\partial z_r}{\partial x}\mathbf{e}_z$$

$$\begin{array}{l} \dfrac{\partial y_r}{\partial x} = t_y \\ \dfrac{\partial z_r}{\partial x} = t_z \end{array} \quad \rightarrow \quad \boxed{\mathbf{t} = \mathbf{e}_x + t_y\mathbf{e}_y + t_z\mathbf{e}_z}$$

$$t = \sqrt{\mathbf{t}\mathbf{t}} = \sqrt{1 + t_y^2 + t_z^2} \quad \rightarrow \quad \boxed{\mathbf{e}_t = \frac{\mathbf{t}}{t} = \frac{1}{t}\mathbf{e}_x + \frac{t_y}{t}\mathbf{e}_y + \frac{t_z}{t}\mathbf{e}_z}$$

$$\mathbf{e}_n = \mathbf{e}_s \times \mathbf{e}_t = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ 0 & \frac{dy_r}{ds} & \frac{dz_r}{ds} \\ \frac{1}{t} & \frac{t_y}{t} & \frac{t_z}{t} \end{vmatrix}$$

$$\mathbf{e}_n = \frac{1}{t}\left(t_z\frac{dy_r}{ds} - t_y\frac{dz_r}{ds}\right)\mathbf{e}_x + \frac{1}{t}\frac{dz_r}{ds}\mathbf{e}_y - \frac{1}{t}\frac{dy_r}{ds}\mathbf{e}_z$$

$$\mathbf{e}_n = e_{nx}\mathbf{e}_x + e_{ny}\mathbf{e}_y + e_{nz}\mathbf{e}_z$$

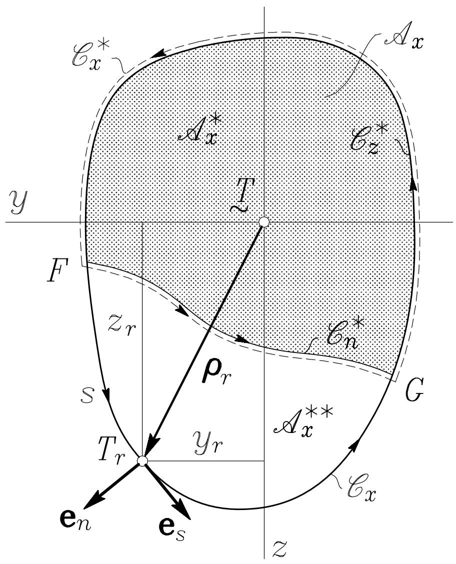
$$\text{Na} \quad \mathscr{C}_x \in \mathscr{S}_{pl} \ : \quad \boxed{y = y_r \ \text{ in } \ z = z_r}$$

$$\boxed{e_{nx} = \frac{1}{t}\left(t_z\frac{dy}{ds} - t_y\frac{dz}{ds}\right)}$$

$$e_{ny} = \frac{1}{t}\frac{dz}{ds} \qquad \boxed{dz = t\, e_{ny} ds}$$

$$e_{nz} = -\frac{1}{t}\frac{dy}{ds} \qquad \boxed{dy = -t\, e_{nz} ds}$$

$$\boxed{\begin{array}{lll} \mathscr{V} & : & \dfrac{\partial \boldsymbol{\sigma}_x}{\partial x} + \dfrac{\partial \boldsymbol{\sigma}_y}{\partial y} + \dfrac{\partial \boldsymbol{\sigma}_z}{\partial z} + \mathbf{v} = \mathbf{0} \\[2ex] \mathscr{S}_{pl} & : & \boldsymbol{\sigma}_x\, e_{nx} + \boldsymbol{\sigma}_y\, e_{ny} + \boldsymbol{\sigma}_z\, e_{nz} = \mathbf{p}_n \end{array}}$$



$$\int\limits_{\mathscr{A}_x^*} \frac{\partial \boldsymbol{\sigma}_x}{\partial x}\, dA_x + \int\limits_{\mathscr{A}_x^*} \left(\frac{\partial \boldsymbol{\sigma}_y}{\partial y} + \frac{\partial \boldsymbol{\sigma}_z}{\partial z} \right) dA_x + \int\limits_{\mathscr{A}_x^*} \mathbf{v}\, dA_x = \mathbf{0}$$

Greenov izrek : $\boxed{\oint\limits_{\mathscr{C}_x} (\mathbf{P}_y dy + \mathbf{P}_z dz) = \int\limits_{\mathscr{A}_x} \left(\dfrac{\partial \mathbf{P}_z}{\partial y} - \dfrac{\partial \mathbf{P}_y}{\partial z} \right) dA_x}$

$$\int\limits_{\mathscr{A}_x^*} \left(\frac{\partial \boldsymbol{\sigma}_y}{\partial y} + \frac{\partial \boldsymbol{\sigma}_z}{\partial z} \right) dA_x = \oint\limits_{\mathscr{C}_x^*} (-\boldsymbol{\sigma}_z dy + \boldsymbol{\sigma}_y dz) =$$

$$\int\limits_{\mathscr{C}_n^*} (-\boldsymbol{\sigma}_z dy + \boldsymbol{\sigma}_y dz) + \int\limits_{\mathscr{C}_z^*} (-\boldsymbol{\sigma}_z dy + \boldsymbol{\sigma}_y dz)$$

$$\int\limits_{\mathscr{C}_z^*} (-\boldsymbol{\sigma}_z dy + \boldsymbol{\sigma}_y dz) = \int\limits_{\mathscr{C}_z^*} t\,(\boldsymbol{\sigma}_y e_{ny} + \boldsymbol{\sigma}_z e_{nz})\;ds = \int\limits_{\mathscr{C}_z^*} t\,(\mathbf{p}_n - \boldsymbol{\sigma}_x e_{nx})\;ds$$

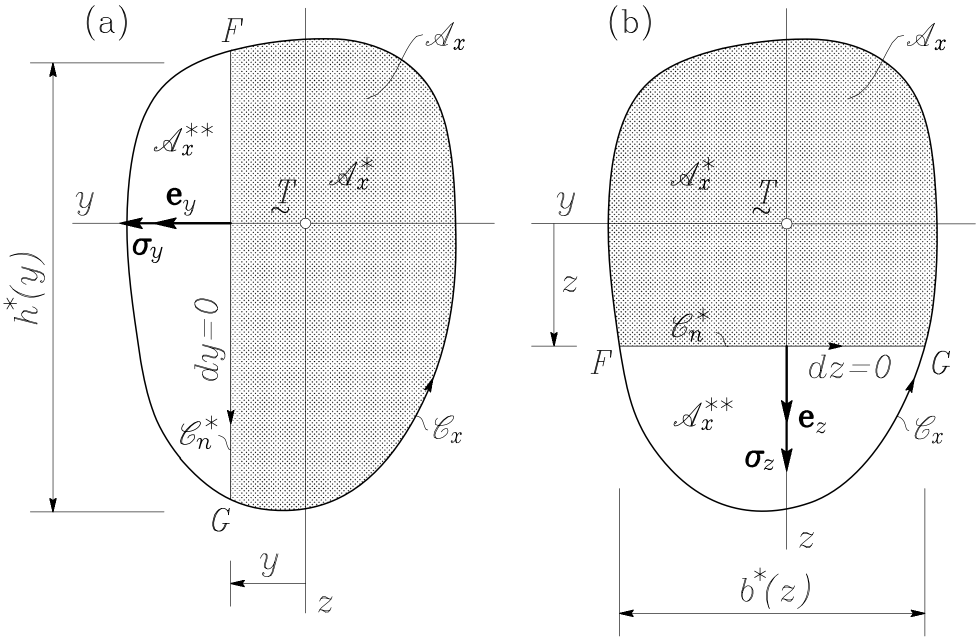
Na $\mathscr{C}_z^* \in \mathscr{S}_{pl}$: $\qquad \boldsymbol{\sigma}_y e_{ny} + \boldsymbol{\sigma}_z e_{nz} = \mathbf{p}_n - \boldsymbol{\sigma}_x e_{nx}$

$$\int\limits_{\mathscr{C}_n^*} (\boldsymbol{\sigma}_z dy - \boldsymbol{\sigma}_y dz) = \int\limits_{\mathscr{A}_x^*} \frac{\partial \boldsymbol{\sigma}_x}{\partial x}\,dA_x - \int\limits_{\mathscr{C}_z^*} t\boldsymbol{\sigma}_x e_{nx}\,ds + \underbrace{\int\limits_{\mathscr{C}_z^*} t\,\mathbf{p}_n\,ds + \int\limits_{\mathscr{A}_x^*} \mathbf{v}\,dA_x}_{\boldsymbol{\mathcal{P}}^*}$$

$$\boxed{\boldsymbol{\mathcal{P}}^* = \int\limits_{\mathscr{C}_z^*} t\,\mathbf{p}_n\,ds + \int\limits_{\mathscr{A}_x^*} \mathbf{v}\,dA_x}$$

$$\boxed{\int\limits_{\mathscr{C}_n^*} (\boldsymbol{\sigma}_z dy - \boldsymbol{\sigma}_y dz) = \int\limits_{\mathscr{A}_x^*} \frac{\partial \boldsymbol{\sigma}_x}{\partial x}\,dA_x - \int\limits_{\mathscr{C}_z^*} t\boldsymbol{\sigma}_x e_{nx}\,ds + \boldsymbol{\mathcal{P}}^*}$$

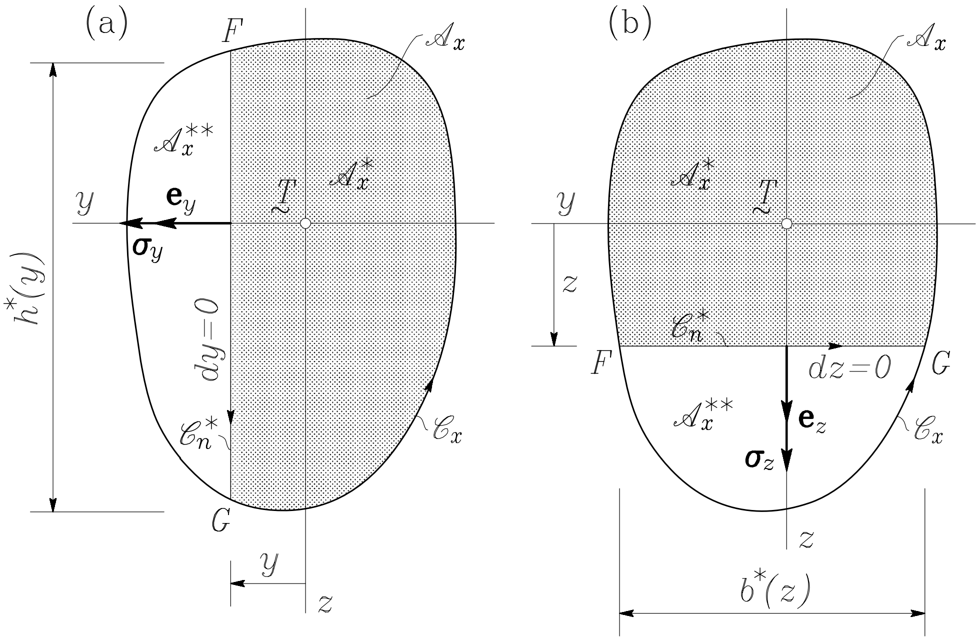
$$\boxed{\int\limits_{\mathscr{C}_n^*} (\boldsymbol{\sigma}_z dy - \boldsymbol{\sigma}_y dz) = \int\limits_{\mathscr{A}_x^*} \frac{\partial \boldsymbol{\sigma}_x}{\partial x}\, dA_x - \int\limits_{\mathscr{C}_z^*} t \boldsymbol{\sigma}_x e_{nx}\, ds + \boldsymbol{\mathcal{P}}^*}$$



$$\int\limits_{\mathscr{C}_n^*} (\boldsymbol{\sigma}_z dy - \boldsymbol{\sigma}_y dz) = -\int\limits_F^G \boldsymbol{\sigma}_y\, dz \approx -\boldsymbol{\sigma}_y \int\limits_F^G dz = -h^*(y)\, \boldsymbol{\sigma}_y$$

$$\boxed{\boldsymbol{\sigma}_y = -\frac{1}{h^*(y)} \left[\int\limits_{\mathscr{A}_x^*(y)} \frac{\partial \boldsymbol{\sigma}_x}{\partial x}\, dA_x - \int\limits_{\mathscr{C}_z^*(y)} t \boldsymbol{\sigma}_x e_{nx}\, ds + \boldsymbol{\mathcal{P}}^*(y) \right]}$$

$$\boxed{\int_{\mathscr{C}_n^*} (\boldsymbol{\sigma}_z dy - \boldsymbol{\sigma}_y dz) = \int_{\mathscr{A}_x^*} \frac{\partial \boldsymbol{\sigma}_x}{\partial x}\, dA_x - \int_{\mathscr{C}_z^*} t \boldsymbol{\sigma}_x e_{nx}\, ds + \boldsymbol{\mathcal{P}}^*}$$



$$\int_{\mathscr{C}_n^*} (\boldsymbol{\sigma}_z dy - \boldsymbol{\sigma}_y dz) = \int_F^G \boldsymbol{\sigma}_z\, dy \approx -\boldsymbol{\sigma}_z \int_G^F dz = -b^*(z)\, \boldsymbol{\sigma}_z$$

$$\boxed{\boldsymbol{\sigma}_z = -\frac{1}{b^*(z)} \left(\int_{\mathscr{A}_x^*(z)} \frac{\partial \boldsymbol{\sigma}_x}{\partial x}\, dA_x - \int_{\mathscr{C}_z^*(z)} t \boldsymbol{\sigma}_x e_{nx}\, ds + \boldsymbol{\mathcal{P}}^*(z) \right)}$$

$$\boxed{\begin{aligned}\boldsymbol{\sigma}_y \cdot \mathbf{e}_x &= \sigma_{yx} = \sigma_{xy}\\ \boldsymbol{\sigma}_z \cdot \mathbf{e}_x &= \sigma_{zx} = \sigma_{xz}\end{aligned}}$$

$$\boxed{\begin{aligned}\sigma_{xy} &= -\frac{1}{h^*(y)}\left(\int\limits_{\mathscr{A}_x^*(y)} \frac{\partial \sigma_{xx}}{\partial x}\, dA_x - \int\limits_{\mathscr{C}_z^*(y)} t\sigma_{xx} e_{nx}\, ds + \mathcal{P}_x^*(y)\right)\\ \sigma_{xz} &= -\frac{1}{b^*(z)}\left(\int\limits_{\mathscr{A}_x^*(z)} \frac{\partial \sigma_{xx}}{\partial x}\, dA_x - \int\limits_{\mathscr{C}_z^*(z)} t\sigma_{xx} e_{nx}\, ds + \mathcal{P}_x^*(z)\right)\end{aligned}}$$

$$\mathcal{P}_x^* = \int\limits_{\mathscr{C}_z^*} t\, p_{nx}\, ds + \int\limits_{\mathscr{A}_x^*} v_x\, dA_x$$

$$\boxed{\sigma_{xx} = \frac{N_x}{A_x} + z\frac{M_y}{I_y} - y\frac{M_z}{I_z}}$$

$$\boxed{\begin{aligned}&\frac{dN_x}{dx} + \mathcal{P}_x = 0\\ &\frac{dM_y}{dx} - N_z + \mathcal{M}_y = 0\\ &\frac{dM_z}{dx} + N_y + \mathcal{M}_z = 0\end{aligned}} \quad \rightarrow \quad \boxed{\begin{aligned}\frac{dN_x}{dx} &= -\mathcal{P}_x\\ \frac{dM_y}{dx} &= N_z - \mathcal{M}_y\\ \frac{dM_z}{dx} &= -N_y - \mathcal{M}_z\end{aligned}}$$

POENOSTAVLJEN PRIMER :

$$\left.\begin{aligned} \mathscr{A}_x &= konst. \\ I_y &= konst. \end{aligned}\right\} \rightarrow \boxed{\begin{aligned} e_{nx} &= 0 \\ t &= 1 \end{aligned}}$$

$$\frac{\partial \sigma_{xx}}{\partial x} = \frac{1}{A_x}\frac{dN_x}{dx} + \frac{z}{I_{yy}}\frac{dM_y}{dx} - \frac{y}{I_{zz}}\frac{dM_z}{dx}$$

$$\frac{\partial \sigma_{xx}}{\partial x} = -\frac{1}{A_x}\mathcal{P}_x + \frac{z}{I_{yy}}\left(N_z - \mathcal{M}_y\right) + \frac{y}{I_{zz}}\left(N_y + \mathcal{M}_z\right)$$

$$\boxed{\begin{aligned} \sigma_{xy} &= \frac{1}{h^*(y)}\left[\mathcal{P}_x \frac{A_x^*(y)}{A_x} - \mathcal{P}_x^*(y) - (N_z - \mathcal{M}_y)\frac{S_y^*(y)}{I_{yy}} - \right. \\ &\qquad \left. (N_y + \mathcal{M}_z)\frac{S_z^*(y)}{I_{zz}}\right] \\ \sigma_{xz} &= \frac{1}{b^*(z)}\left[\mathcal{P}_x \frac{A_x^*(z)}{A_x} - \mathcal{P}_x^*(z) - (N_z - \mathcal{M}_y)\frac{S_y^*(z)}{I_{yy}} - \right. \\ &\qquad \left. (N_y + \mathcal{M}_z)\frac{S_z^*(z)}{I_{zz}}\right]. \end{aligned}}$$

Nadaljnja poenostavitev :

$$\left.\begin{aligned} p_{nx} &= p_{nx}(x) \\ v_x &= v_x(x) \end{aligned}\right\} \rightarrow \boxed{\begin{aligned} &\mathcal{P}_x \frac{A_x^*(z)}{A_x} - \mathcal{P}_x^*(z) = 0 \\ &\mathcal{M}_y = \mathcal{M}_z = 0 \end{aligned}}$$

$$\boxed{\boxed{\begin{aligned} \sigma_{xy}(x,y) &= -N_y(x)\frac{S_z^*(y)}{h^*(y)I_{zz}} - N_z(x)\frac{S_y^*(y)}{h^*(y)I_{yy}} \\ \sigma_{xz}(x,z) &= -N_y(x)\frac{S_z^*(z)}{b^*(z)I_{zz}} - N_z(x)\frac{S_y^*(z)}{b^*(z)I_{yy}} \end{aligned}}}$$

Prečne normalne napetosti

$$\boxed{\begin{aligned} \boldsymbol{\sigma}_y \cdot \mathbf{e}_y = \sigma_{yy} &= -\frac{1}{h^*(y)} \left(\frac{\partial}{\partial x} \int\limits_{\mathscr{A}_x^*(y)} \sigma_{xy}\, dA_x + \mathcal{P}_y^*(y) \right) \\ \boldsymbol{\sigma}_z \cdot \mathbf{e}_z = \sigma_{zz} &= -\frac{1}{b^*(z)} \left(\frac{\partial}{\partial x} \int\limits_{\mathscr{A}_x^*(z)} \sigma_{xz}\, dA_x + \mathcal{P}_z^*(z) \right), \end{aligned}}$$

$$\mathcal{P}_y^* = \int\limits_{\mathscr{C}_z^*} p_{ny}\, ds + \int\limits_{\mathscr{A}_x^*} v_y\, dA_x$$

$$\mathcal{P}_z^* = \int\limits_{\mathscr{C}_z^*} p_{nz}\, ds + \int\limits_{\mathscr{A}_x^*} v_z\, dA_x$$

$$\sigma_{yy} = -\frac{1}{h^*(y)} \left[\frac{\mathcal{P}_y}{I_{zz}} \int\limits_{\mathscr{A}_x^*(y)} \frac{S_z^*(\tilde{y})}{h^*(\tilde{y})}\, dA_x + \frac{\mathcal{P}_z}{I_{yy}} \int\limits_{\mathscr{A}_x^*(y)} \frac{S_y^*(\tilde{y})}{h^*(\tilde{y})}\, dA_x + \mathcal{P}_y^*(y) \right]$$

$$\sigma_{zz} = -\frac{1}{b^*(z)} \left[\frac{\mathcal{P}_y}{I_{zz}} \int\limits_{\mathscr{A}_x^*(z)} \frac{S_z^*(\tilde{z})}{b^*(\tilde{z})}\, dA_x + \frac{\mathcal{P}_z}{I_{yy}} \int\limits_{\mathscr{A}_x^*(z)} \frac{S_y^*(\tilde{z})}{b^*(\tilde{z})}\, dA_x + \mathcal{P}_z^*(z) \right]$$

$$\chi_y^*(y) = \int\limits_{\mathscr{A}_x^*(y)} \frac{S_y^*(\tilde{y})}{h^*(\tilde{y})}\, dA_x \qquad \chi_y^*(z) = \int\limits_{\mathscr{A}_x^*(z)} \frac{S_y^*(\tilde{z})}{b^*(\tilde{z})}\, dA_x$$

$$\chi_z^*(y) = \int\limits_{\mathscr{A}_x^*(y)} \frac{S_z^*(\tilde{y})}{h^*(\tilde{y})}\, dA_x \qquad \chi_z^*(z) = \int\limits_{\mathscr{A}_x^*(z)} \frac{S_z^*(\tilde{z})}{b^*(\tilde{z})}\, dA_x$$

$$\boxed{\boxed{\begin{aligned} \sigma_{yy}(x,y) &= -\frac{1}{h^*(y)} \left[\mathcal{P}_y(x)\, \frac{\chi_z^*(y)}{I_{zz}} + \mathcal{P}_z(x)\, \frac{\chi_y^*(y)}{I_{yy}} + \mathcal{P}_y^*(x,y) \right] \\ \sigma_{zz}(x,z) &= -\frac{1}{b^*(z)} \left[\mathcal{P}_y(x)\, \frac{\chi_z^*(z)}{I_{zz}} + \mathcal{P}_z(x)\, \frac{\chi_y^*(z)}{I_{yy}} + \mathcal{P}_z^*(x,z) \right] \end{aligned}}}$$