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Reliability of timber structural systems, trusses and joists

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Licentiate Thesis

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Preface

The present work has been carried out at the Division of Structural Engineering at Lund University under supervision of Professor Sven Thelandersson.

I want to express my gratitude to Professor Thelandersson for his supportive guidance and for all encouragement. I'm also very grateful for all the explanations and advices I received by Dr. Tord Isaksson. For the many interesting discussions I had with Dr. Staffan Svensson I am most grateful. Thanks to all the colleagues, both former and present, at the Division of Structural Engineering for creating a pleasant and enlightening working environment.

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Abstract

Compared to other structural materials, such as steel and concrete, structural timber has a considerably higher variability of the strength properties both within and between members. The variability within a timber member is not considered in engineering design, where each member is treated under the assumption that it is homogeneous, i.e. the strength is assumed to have a constant low value along the member. The advantage of the low probability that high stresses coincide with low strengths is however not accounted for and may give an extra safety margin. The strength of timber members will, due to their inhomogeneous nature, be dependent on both the length of the member and the type of loading. For structures with pronounced moment peaks such as roof trusses, an important increase in safety could therefore be expected.

A method, which accounts for the interaction of normal and transversal forces on a member is used to determine the degree of utilization in any section along a member. The effect of buckling is included in the interaction. The suggested equations have the advantage to be linear which minimizes the necessary calculations in this study.

Monte Carlo simulations were performed to study the utilization of a roof truss of W-type. The between and within member variation of bending strength is modelled based on stochastic variables such as distance between weak sections, length of weak sections, strength of weak sections and strength between weak sections. The results show a significant system effect due to within member variability of bending strength in timber structures such as trussed rafters. The effect is larger in less homogenous timber as e.g. Radiata pine than in Norway spruce that has less variability in the strength properties.

Furthermore, a reliability study of the roof truss was carried out and the system effect was quantified.

The other system studied is a sheathed parallel timber beam structure (e.g. a floor structure or a roof element). In engineering design a system effect factor is introduced taking probabilistic and structural load sharing into account. A parameter study of the system was done to determine the sensibility for parameters such as number of beams in the system, centre-to-centre distance of the timber beams and length of span. A calibration of the code format was performed where the code format and the probabilistic format was combined. The system effect due to the difference in coefficient of variation in the single timber beams and the system was quantified. Further studies have to be done to determine the system effect factor that can be used in the code.

Key words: Structural timber, roof truss, reliability, variability, system effect, safety index, combined stress index, Monte Carlo simulation, first order reliability method.

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1 Introduction

1.1 Background

Trees have during evolution specialized in resisting its natural environment e.g. to stand snow and wind loads. In sawn timber the conditions for the wood material differ compared to the living tree. The branches that collected the sunshine for the photosynthesis are now forming knots with locally disturbed grain directions and are seen as defects in design of timber structures.

The research concerning reliability of timber structures is fairly young. The natural defects in structural timber arise special issues that in particular must be addressed in reliability studies:

- The variability of the strength properties is large.
- Difficulties in predicting some failure modes. Strength grading of structural timber may consequently be inaccurate for some of the failure modes.
- The knowledge of the variability is not documented to the same extent as competitive materials.

The safety factors have to be set relatively high, due to these special issues that affects the competitiveness of timber compared to other building materials. Different research projects have been initiated to deal with these questions.
Distributing the defects naturally existing in wood can reduce the variability of wood products. This reduction of variability is found in products such as glulam and laminated veneer lumber (LVL).
With better predictions of the strength properties new methods for structural timber design may open. The optimisation problem in design would be more adequate and the properties of the timber could be matched to the strength needed. This may be possible e.g. for prefabricated roof-trusses with punched metal plates.
Another way is to improve the knowledge of the material e.g. how the variability influences the reliability. This is always important, as many countries continuously revise their codes taking all available information into account.

1.2 Objective

The main objective with this thesis is to increase the understanding of the variability of the strength properties on the reliability of timber structures.

1.3 Limitations

The studied species are Spruce (Picea Abies) and for comparison Radiata pine. The studied failure mode has been failure caused by bending moment combined with axial force. Failure modes such as failure of joints, shear failure and so forth have not been given any attention. For the roof structures load sharing between the trusses was not considered.

Furthermore, the uncertainty of the wood strength was accounted for but the variability of the stiffness within the members or in the connections was not considered in the studied roof truss.

1.4 Outline of the thesis

This thesis consists of, a state of art review, a short summary of the results and conclusions from the papers and three appended papers. The state of the art review is divided into eight chapters:

- Reliability in timber structures.
- Variability of the stiffness and strength properties in structural timber.
- Length and load configuration effects.
- System effect.
- Floors of sheathed parallel timber beams.
- Roof trusses.
- Probabilistic methods for structural reliability.
- The KB-method and the combined stress index.

2 State of the art review

2.1 Background

Probabilistic methods are the basis of all modern design codes. The purpose of reliability-based design is the systematic consideration of all the uncertainties involved in the design process. The uncertainties could for example be found in the expected loads, the strength estimation and the models used.

Probabilistic analyses of structures can be done at different levels. According to Thoft-Christensen and Baker (1982) the three levels of structural reliability analysis and methods of design are:

Level 3: The “exact” probability of failure is determined using full probabilistic description.

Level 2: Iterative calculations with an idealisation of the failure domain and simplified representation of the variables.

Level 1: Design methods using partial safety factors related to characteristic values of the actions and resistance variables.

The codes of today are at Level 1 where the partial safety factors are determined on the basis of a Level 2 analysis.

Compared to other building materials the following differences are typical for timber structures:

- Significant within member variability of strength and stiffness properties. This means an apparent volume effect. The high anisotropy of the material makes the effect even more delicate to describe.
- The time dependent behaviour known as duration of loads (DOL).
- The influence of moisture state on strength.

2.2 Reliability in timber structures

During the last decades the international research community has been dealing quite extensively with safety and reliability of timber structures. Much work has been initiated in connection with the introduction of limit state design in many countries around the world.
Important and extensive work was done in Canada when the code was updated to modern limit state design. A documentation of this work is found in Foschi et. al. (1989). The work is based on the conditions found in Canada. For example the wood species used as a basis are Douglas-Fir, Hemlock and the Spruce-pine-fir mix and the three strength classes used are SS, No. 2 and No. 3. As the species and the strength classes are different in Europe the results can not directly be transformed into European codes.

An international NATO workshop initiated by the United States was held in Florence 1991 (Bodig, 1992) with the title "Reliability-based design of Engineered wood structures". The objectives with the workshop were to review the state of the art on reliability-based design and define the development needed with special emphasis on internationally harmonized codes for timber structures.

A European seminar was held in Paris in 1997 (Adjanohoun et. al., 1997) with the purpose to gather scientific information and propose guidelines for the implementation of a fully probabilistic design code of timber structures.

A study by Thelandersson et. al. (1999) shows that even if the reliability index is apparently lower for the Canadian code, the safety and the reliability of the structure is the same as found in the European codes due to the differences in the load descriptions, reference time and the calibration done. Further the following was pointed out in Thelandersson et. al. (1999) as areas in which more research is needed:

- Method for reliability research and how to interpret different results. This should be done using reliability based design in a predetermined way. The target reliability index should be in unison in the European countries.
- The importance of the choice of the distribution function for the strength properties of the material.
- Strength dependency of the load history.
- The strength dependency of the moisture and the moisture history. Research where the moisture content variation and the stresses introduced by uneven moisture distribution is taken into account.
- System effects and multiple failure modes.
- Serviceability limit states where deformation and vibration are analysed with reliability based methods.

2.3 Variability of stiffness and strength in structural timber

Since the 1920's until fairly recently (about 1980) the strength of a timber member was determined from the "clear-wood" strength (Madsen, 1992). The commercial timber, however, does not have the same properties as clear wood at all. Today the strength capacity is determined from direct testing of structural timber, so called ingrade testing.

The variability between individual timber elements is significantly larger than those found for steel or reinforced concrete members. The coefficient of variation (COV) for strength properties of structural timber is 20 – 40 % where the higher value is found for brittle failure modes. The variability may also differ between wood species.

The lengthwise variation of the modulus of elasticity has been studied since the mid-sixties. The within variability of bending stiffness was modelled by Czmoch (1991, 1998) as a stationary random process. He used two models, one with random bending stiffness variation around a global mean for the whole population of beams. In the other model a variable mean was used for every beam and a stationary random process expressed the local fluctuation within a beam.

The variability of tensile strength parallel to the grain has been studied for example by Showalter et al. (1987) and Lam and Varoglu (1991a,b) with both experiments and mathematical models.

Riberholt and Madsen (1979) presented a model of the lengthwise bending strength. This model is based on a constant bending strength between defects, see Figure 1. The clear wood strength and the strength in weak zones are described by stochastic variables. A Poisson process describes the occurrence of a weak section.

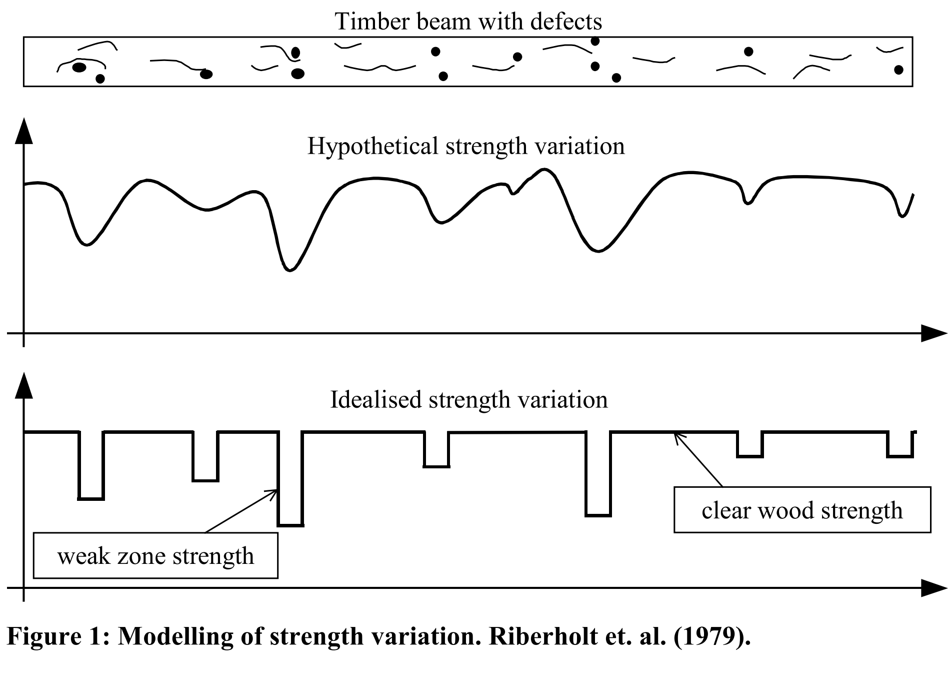

Figure 1: Modelling of strength variation. Riberholt et. al. (1979).

The model was further used by Czmoch, Thelandersson and Larsen (1991) in a study of load configuration and length effects. They suggested further investigations to get the input data to the model.

Källsner and Ditlevsen (1994) presented a study on the variability of bending strength. Weak sections were cut out from 26 beams and were finger jointed to pieces of wood with expected higher strength. In total, 197 tests were carried out. Unfortunately, only 34 percent of the failures were bending failure in the weak zone (most of the failures occurred in the finger joints). A statistical model was developed to model the bending strength variation. The model was later improved to also include the failure of the finger joint. Furthermore the experimental verification was done in three phases. The first phase contained experiments with single weak zones. Phase two was performed for long beams containing several weak zones. The third phase studied the interaction between weak zones. The verification indicates that the bending strength was lower than the predicted strength from the model. The reasons for this may be statistical uncertainty, release of stored elastic energy during the test and possible interaction between weak zones, Källsner, Ditlevsen and Salmela (1997), Källsner and Ditlevsen (1998, 1999).

In Isaksson (1999) a statistical model of the variability of the bending strength both between and within timber beams is described. The model is based on an experimental investigation of 130 timber beams of spruce (Picea Abies) where every beam was tested in 5-7 weak sections. The model is built up by the following parameters:

- Distance between weak sections.
- Width of weak sections.
- Bending strength of weak sections.
- Bending strength between weak sections.

By measuring the knot size and their position, the weak sections were identified. The distribution and length of the knot clusters determined the distance between and the width of weak sections. The length of weak sections showed good agreement with the gamma distribution. The correlation coefficient between weak sections was found to be independent of the distance between the sections.
The bending strength of weak sections within and between timber beams was written as an exponential function:

$$f_{ij} = \exp\left(\mu + \tau_i + \varepsilon_{ij}\right) \tag{1}$$

where

f_{ij} is the bending strength of beam i in section j,

μ is the logarithm mean of all weak sections,

τ_i is the difference between the logarithm mean of weak sections within one member i and μ. The mean equals zero and the standard deviation is σ_i,

ε_{ij} is the difference between weak section j in beam i and the value $\mu + \tau_i$. The mean equals zero and the standard deviation is σ_j.

The model is illustrated in Figure 2.

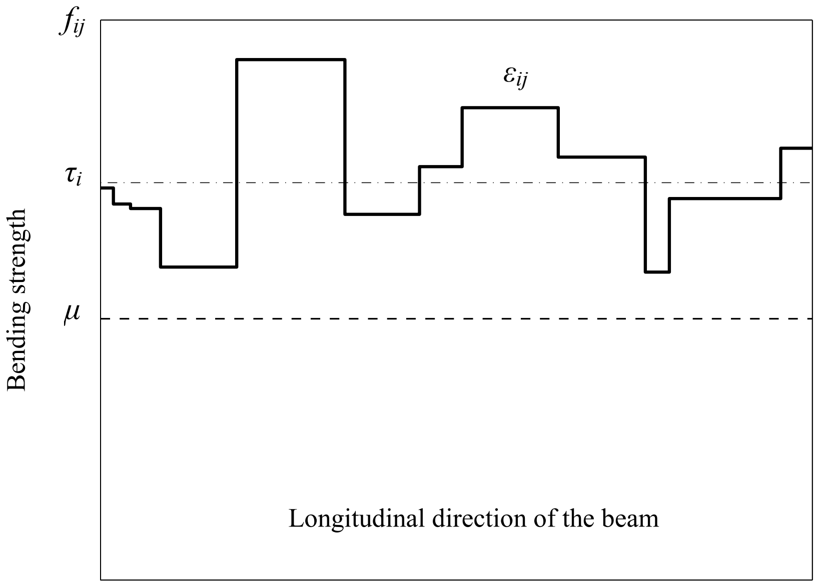

Figure 2: Modelling of longitudinal variation of bending strength. Isaksson (1999).

The input to the model is summarised in Table 1. The model for Radiata Pine was based on data found in Leicester et. al. (1996).

Table 1: Input to the model of variability in bending strength. (Isaksson 1999).

	Model		
Parameter	Norway spruce, LN50	Norway spruce, LN50Step	Radiata Pine, LNRP
Distance [mm]	Gamma(2.5445, 194.12)	Gamma(2.5445, 194.12)	Exp(1, 1300)
Width [mm]	150	-	150
Weak section strength [MPa]	LogN(4.01977, 0.247136)	LogN(4.01977, 0.247136)	LogN(3.75443, 0.38523)
Coeff. of corr. between weak section strength	0.565	0.565	0.55
Strong section strength	greatest weak section strength	-	greatest weak section strength

2.4 Length and load configuration effects

Since timber is treated as a homogeneous material by using the characteristic value in ordinary engineering design, the apparent strength of the element will be dependent on the loaded volume. The volume effect is often divided into two parts namely length and load configuration effect.

The length (or size) and load configuration effect for tension and bending failures is often described by the Weibull theory (Weibull 1939a,b). The theory claims that a chain is as strong as its weakest link. If the material is brittle and a reference volume is subjected to pure tension (stress σ) the probability of failure, p_f, can be written as:

$$p_f = P(R \le \sigma) = F(\sigma) \tag{2}$$

where *F()* is the statistical probability distribution of the strength, *R*, of the material. When the strength belongs to a three-parameter Weibull distribution the probability of failure becomes:

$$p_f = 1 - e^{-\left(\frac{\sigma - b}{a}\right)^{1/k}} \tag{3}$$

where

a is the scale parameter,

b is the location parameter,

k is the shape parameter.

It can be shown that the following relationship will apply between two volumes *V*, if the location parameter *b* is set zero.

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{V_1}{V_2}\right)^k \tag{4}$$

where σ_1 and σ_2 are the stresses causing failure for volume V_1 and V_2 respectively. There is a higher probability that a defect appears within a larger volume. This results in a lower strength for the larger volume. The shape parameter, *k*, is found in several experimental investigations to be in the interval 0.1 to 0.2 see e.g. Table 2 from Isaksson (1999).

Table 2: Length effect. Beam loaded with constant bending moment. Isaksson (1999).

Length, *L* [m]	**Mean [MPa]**	**Stand. dev. [MPa]**
2.2	49.78	11.99
3.2	47.77	11.22
4.2	46.69	10.99

Codes (e.g. Eurocode 5), however, use the depth of the timber beam as a parameter to account for size effect. This may give problems in the design procedure as the length in most cases is known and the depth has to be determined (Madsen, 1991).

The load configuration effect is similar to the length effect and refers to the fact that the apparent strength depends on the distribution of e.g. bending moment along the member. The equation for the load configuration effect will be similar to Equation (4), see for example Isaksson (1999). The more pronounced the peak of the bending moment, the higher is the effect. Table 3 based on experimental data from Isaksson (1999) shows the results of the load configuration effect for three load cases shown in Figure 3. The effect is also illustrated in Figure 4 where a beam is loaded with uniform transversal load, q, and constant bending moment, M.

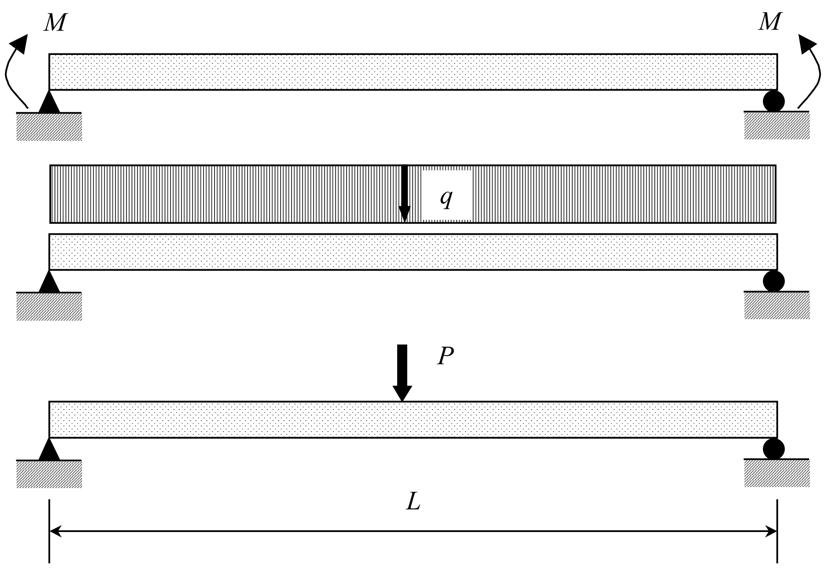

Figure 3: Load configurations. Isaksson (1999).

Table 3: Load configuration effect. (Reference length 4.2 meter). Isaksson (1999).

Load configuration (see Figure 3)	**Mean [MPa]**	**Stand. dev. [MPa]**
Constant bending moment, M	46.05	10.83
Uniform transversal load, q	50.57	12.16
Concentrated point load, P	53.74	12.30

A summary of size and load configuration effects can also be found in Madsen (1992).

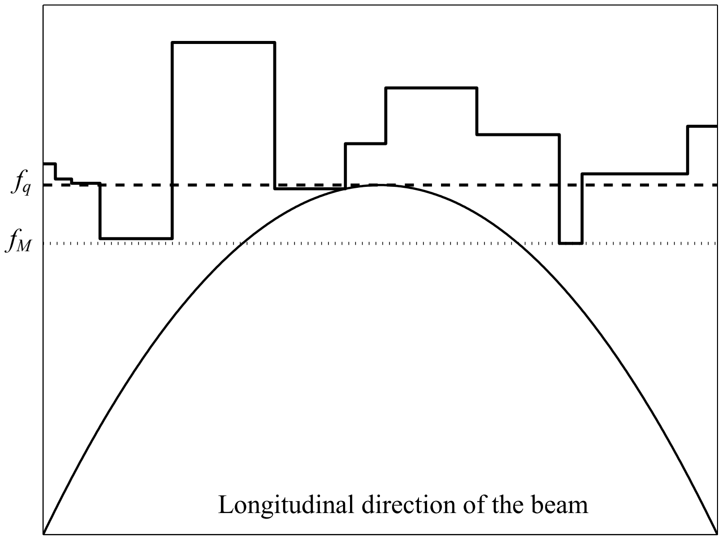

Figure 4: Evaluation of bending strength for different load configurations. *q* uniformly distributed load and *M* constant bending moment.

2.5 System effect

According to Foschi et.al. (1989), system effects can be attributed to two different causes:

- *Probabilistic load sharing* based on the smaller probability that a weak section coincides with the position of the highest stress.
- *Structural load-sharing*, due to redistribution of forces within the system.

Structures are usually redundant systems, i.e. given the first failure of some part of the system, the structure as a whole will not necessarily fail. It is often hard to establish the failure criterion/criteria of a redundant system made of structural timber. Therefore the criteria are often given in more or less vague forms, for example “a failure occurs when the building does not maintain its primary functions” (Bodig, 1992).

2.5.1 Series and parallel systems

Series systems, illustrated in Figure 5, have the good feature that the reliability is independent of the load path. A series system can be described with the weakest link theory i.e. the weakest link determines the strength of the whole system. Assuming perfect series systems give an upper bound for probability of failure. If X_i is the strength of the component i, in a system containing n elements, the system strength can be written as

$$R_n = \min_{i=1}^{n}\{X_i\} \quad (5)$$

and the probability of failure

$$p_f = P(R_n - S \le 0) \quad (6)$$

where S is the load effect.

In series systems in which the load uncertainty is much greater than the resistance uncertainty, the risk of failure mainly depends on whether a certain load level is exceeded or not whereas the number of links in the weakest-link series is unimportant (Sundararajan, 1995).

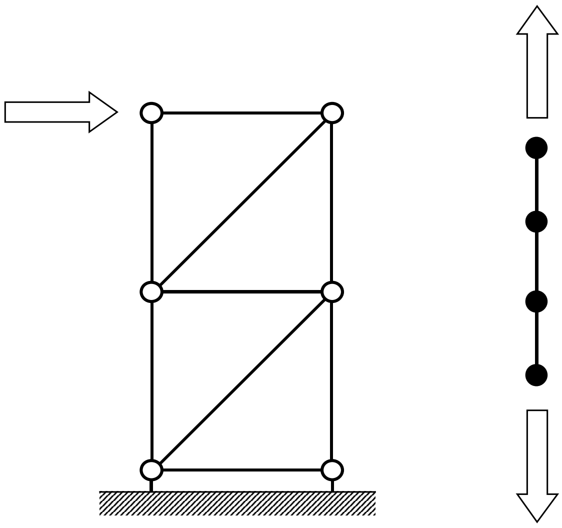

Figure 5: Examples of series systems.

More interesting, for structural purposes, but also more complicated is the parallel system. A perfect parallel system with equal load sharing and brittle failure theoretically has the following system strength (Bodig 1992):

$$R_n = \max_{i=1}^{n}\{(n-i+1)X_i\} \tag{7}$$

where X_i is the strength of the component i and $X_1 \leq \ldots \leq X_n$.
If the elements in the system are ideally elastic-plastic and the load can be shared in such a way that every element will reach its maximum strength, the system strength is given by:

$$R_n = \sum_{i=1}^{n}\{X_i\} \tag{8}$$

Nearly all structures are in between these bounds of probability of failure that the series and parallel systems create, see Figure 6. Bounds for the probability of failure are often used for structures with (potential) multiple failure modes, which is the case for most structures. To calculate the theoretical "true" probability of failure is, for most cases, a very difficult task.

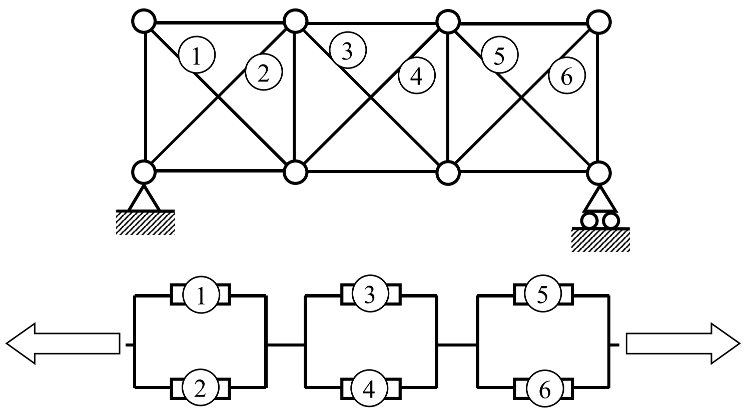

Figure 6: Example of combined series and parallel system. If only element 1-6 can fail in the truss the structure can be modeled as the figure below. Thoft-Christensen et. al. (1982)

2.6 Floors of sheathed parallel timber beams

One of the most common parallel systems is the sheathed parallel beam system used as a floor structure or a roof element. The system consists of two or more parallel timber beams with a sheathing of for example plywood board. To analyse the behaviour of a wood-joist system, McCutcheon (1977, 1984) proposed an analogy with a beam-spring model. The model also includes non-linear behaviour of the wood joists. Tremblay et. al. (1976) developed a non-linear model to take the inter layer slip into account for T-beams. The model was verified against 16 full tests and the agreement in the deflection was in general good up to and beyond the service load range. Another finite element

description has been proposed by e.g. Thompson et. al. (1975, 1977) with improvements by Foschi (1982). This linear model accounts for factors such as composite action, lateral and torsional joist deformation, interlayer slip and clearance between boards in the sheathing.
Foschi et. al. (1989) used this improved model to calibrate the system effect factor for systems used in the Canadian code. For this reliability study the statistical variables are shown in Table 4.

Table 4: Statistical input for calibration of the system effect factor. Foschi et. al. (1989).

Variable	Distribution
Dead load	Normal
Live load	Extreme Type I
Joist modulus of rupture and bending strength	2-P Weibull

An initial study showed that the reliability was insensitive to the variability of the stiffness of the fastener and the sheathing. Consequently, the variability of these properties was excluded from the analysis.
An extensive sensitivity study was performed where the following parameters were found to increase the load sharing:

- Increased stiffness in the lateral load distribution system.
- Increased variability of modulus of elasticity of the timber members.
- Higher correlation between modulus of elasticity and bending strength.

The result of the study indicates that the mean for the system effect factor is between 1.34 and 1.63 depending of the method of evaluation.

2.7 Roof trusses

Another common system is the roof truss with punched metal plate fasteners. Different structural models have been suggested for this system. The most difficult task is, however, to describe the behaviour of the joints in the roof truss. Properties as slip, clearance between members and non-linear behaviour have to be accounted for. The most common model types for two dimensional finite element models are according to Nielsen (1996):

- Plane beam elements with pinned or rigid moment joints.
- Fictitious member model.
- Spring model.
- Foschi's model.

The eccentricity of the joints that occurs when the centre lines of the timber members do not meet in the same point can be modelled by a fictitious element, see for example Riberholt (1990a,b). The spring model contains independent linear springs representing axial as well as rotational stiffnesses proportional to the contact area between nail plate and the wood. Foschi (1979) presented a model for nail-plate connections with a special beam element as joint element. Furthermore, an advanced joint model has been developed by Ellegaard and Nielsen (1999), which gives good agreement with experimental tests for serviceability load. The model is expected to be extended to include prediction of failure mode and failure load.

Zhao et. al. (1992) investigated the reliability of the truss bottom chord due to different span-length. The conclusions were that the span length and the metal plate connections were of minor importance but the truss pattern and the lumber quality have a great impact on the reliability of the bottom chord. In Hamon et. al. (1985), the influence of the correlation between timber properties was found to be not a significant factor on the reliability and accordingly no effort has to be done to determine this correlation. The calculations however indicated that a high snow load ratio gives an increased probability of failure. Bulleit and Yates (1991) analysed wood trusses using stochastic finite elements method (SFEM). Good agreement with Monte Carlo simulations was found and the along-the-length correlation of the material properties of the timber beams was shown to have marginal effect on the result.

Eurocode 5 acknowledges the probabilistic and the structural load-sharing effect for systems as roof trusses and floor systems. The system effect factor is introduced when several equally spaced similar members are laterally connected by a continuous load distribution system. In Eurocode 5 this factor is set to 1.1.

2.8 Probabilistic methods for structural reliability

For a simple strength-load (*R* - *S*) system the failure probability becomes

$$p_f = P(R - S \leq 0) = \int_{-\infty}^{\infty} F_R(x) f_s(x) dx \qquad (9)$$

where *F()* is the cumulative probability function (CDF) and *f()* is the probability density function (PDF). The probability of failure can in this case be seen as the lower (removed) part of the density function in Figure 7.
More general Equation (2) can be written as:

$$p_f = P[G(\mathbf{X}) \leq 0] = \int \ldots \int_{G(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \qquad (10)$$

where *G()* is the failure function (limit state function or performance function) and vector ***X*** represents all the basic variables.
There are different ways to calculate the probability of failure p_f, in Equation (10):

- Direct integration (only possible for special cases and is therefore not further discussed)
- Numeric integration as the Monte Carlo technique, see chapter 2.8.1.
- Transforming the integrand into a multi-normal joint probability density function as described below in the FORM/SORM chapter.

Further information in this field can be found in textbooks about general reliability in structures, e.g. Melchers (1999) and Thoft-Christensen et. al. (1982).

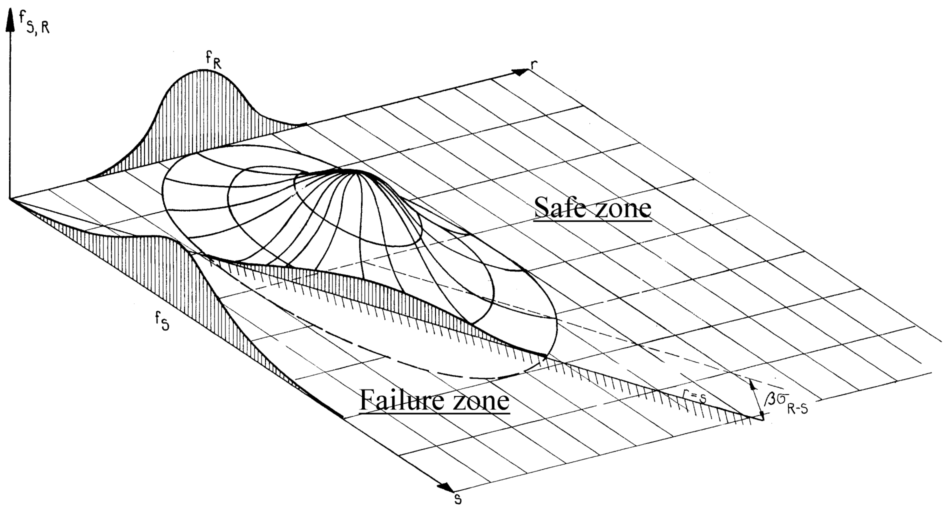

Figure 7: Density function, $f_{S,R}$ for the two-dimensional stochastic variable (S,R).

2.8.1 Monte Carlo simulation

The Monte Carlo method provides approximate solutions to a variety of mathematical problems by performing statistical sampling simulations preferably on a computer. The method is named after the city in the Monaco principality because of the roulette, a simple random number generator.

The probability in Equation (9) can be calculated by generating random values of R and S. The failure function is checked and it is recorded when a failure occurs, that is when $R - S \leq 0$. This procedure is performed until desirable accuracy is achieved. The probability of failure, p_f, is given by:

$$p_f = \frac{N(R - S \leq 0)}{N_{tot}} \tag{11}$$

where $N(R\text{-}S\leq 0)$ is the number failures and N_{tot} is the total number of simulations. If the probability of failure is small (as for constructions), a large number of calculations have to be performed for reliable results. This brutal force method may be improved in many cases e.g. by variance reduction techniques. Variance reduction can only be achieved by using *a priori* information. One example of variance reduction techniques is the importance sampling method that limits the simulation to “interesting“ regions.

2.8.2 FORM/SORM

The First Order Reliability Method is a less computer intensive way to solve the integrand by introducing a reliability index. According to Melchers (1999) this type of computation started as early as in the 1920ies but was not widely accepted until the 1960ies.

The failure function may be written as:

$$G = R - S \tag{12}$$

If R and S are normally distributed, then: $R \in$ Normal(μ_R, σ_R), $S \in$ Normal(μ_S, σ_S) and $G \in$ Normal(μ_G, σ_G).

Then the probability of failure can be written as:

$$\mathrm{p_f} = \Phi(-\beta_C) \tag{13}$$

where *Φ ( )* is the standard normal distribution function and

$$\beta_C = \frac{\mu_G}{\sigma_G} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_S^2 + \sigma_R^2}} \tag{14}$$

β_C is the reliability (or safety) index (sometimes called Cornell's reliability index after Cornell (1969)). The reliability index is the distance, given in standard deviations, by which μ_G exceeds zero and it is a measure of the safety of the system, see Figure 7. This type of calculation is often referred to as second moment method as the higher moments as skew and flatness are ignored. These formulas are only valid for linear failure functions and normally distributed, uncorrelated variables.

Hasofer and Lind (1974) proposed a method that handles arbitrary failure functions. This method has been improved and was later called first-order reliability method (FORM). All variables are transformed into their standardized form Normal(0,1). If the variables are correlated, then they have to be transformed into uncorrelated ones through an eigen-value analysis. The reliability index is then obtained as the shortest distance from origin to the failure function in the normalized system. The first-order reliability method has later been modified to also include non-normal distributed variables.

The Second Order Reliability Method (SORM) is an improved way to obtain the reliability index taking into account the non-linearity of the failure function. Even a linear failure function in the original space becomes nonlinear when transformed to the standard normal space (where the search for the design point is done) if any of the variables are non-normal. Also the transformation from correlated to uncorrelated variables might induce nonlinearity, Sundararajan (1995).

Many methods have been suggested to solve this problem. According to Melchers (1999), the most common way is to fit a parabolic or higher order surface to the actual failure surface centred on the design point. The probability has then to be sampled in some areas or approximated by first order analysis. Ditlevsen (1979a) suggested the use of a polyhedral envelop for the nonlinear limit state, consisting of tangent hyperplanes at selected points on the failure function. The failure probability is obtained through the union of failure regions defined by the individual hyperplanes. Narrow bounds was introduced to calculate the joint probability for the multiple failure regions, Ditlevsen (1979b).

2.9 The KB-method and Combined Stress Index

In the present work the strength properties are supposed to vary randomly along the element. This means that the failure criteria must be defined in any arbitrary section. Furthermore the applications in this work include interaction of normal force and bending moment in members subjected to second order buckling effects. The current codes have an interaction formula that is defined only for the most stressed section of the member. This was not suitable for this work. In the literature a method was found that fulfils this requirement called "KB-metoden" by Höglund (1968). He divided the total stress into two parts: One with bending moment stress from the transversal load (B-diagram) and one for the compression load and the stress contribution from buckling (K-diagram), see Figure 8. In Figure 8, the two first buckling modes can be seen. The first buckling mode gives a contribution to the bending moment between the supports and the second buckling mode gives an additional moment at the support.

The method approximates the deflection at failure, for an element subjected to both bending and compression, with the deflection that is found at failure when the element is under pure compression. The deflection shape for transversal load only is sometimes widely different from the one achieved with normal compression load only. However, when continuous elastic columns become instable, they will buckle in a sine wave pattern as shown in Figure 8, *superimposed* on the deflection existing at transverse loads less than the critical one (Mc Guire (1968)).

The main assumptions are:

- Linear interaction between normal force and bending moment, i.e. linear elastic behaviour is assumed.
- Sine shape of the displacement and approximation of the amplitude.
- The element is secured against buckling out of plane.
- The normal force is constant within the element.

It can be shown that for any arbitrary section in a timber element subjected to both normal force and bending moment the interaction will become:

$$\frac{N}{A \cdot f_c} + \frac{M(z)}{W \cdot f_m} + \frac{N}{A \cdot f_c}\left(\frac{1}{k_c} - 1\right)\sin\left(\frac{\pi \cdot z}{l_c}\right) = CSI \leq 1 \qquad (15)$$

For a more detailed description see Höglund (1968) and *Paper I*.

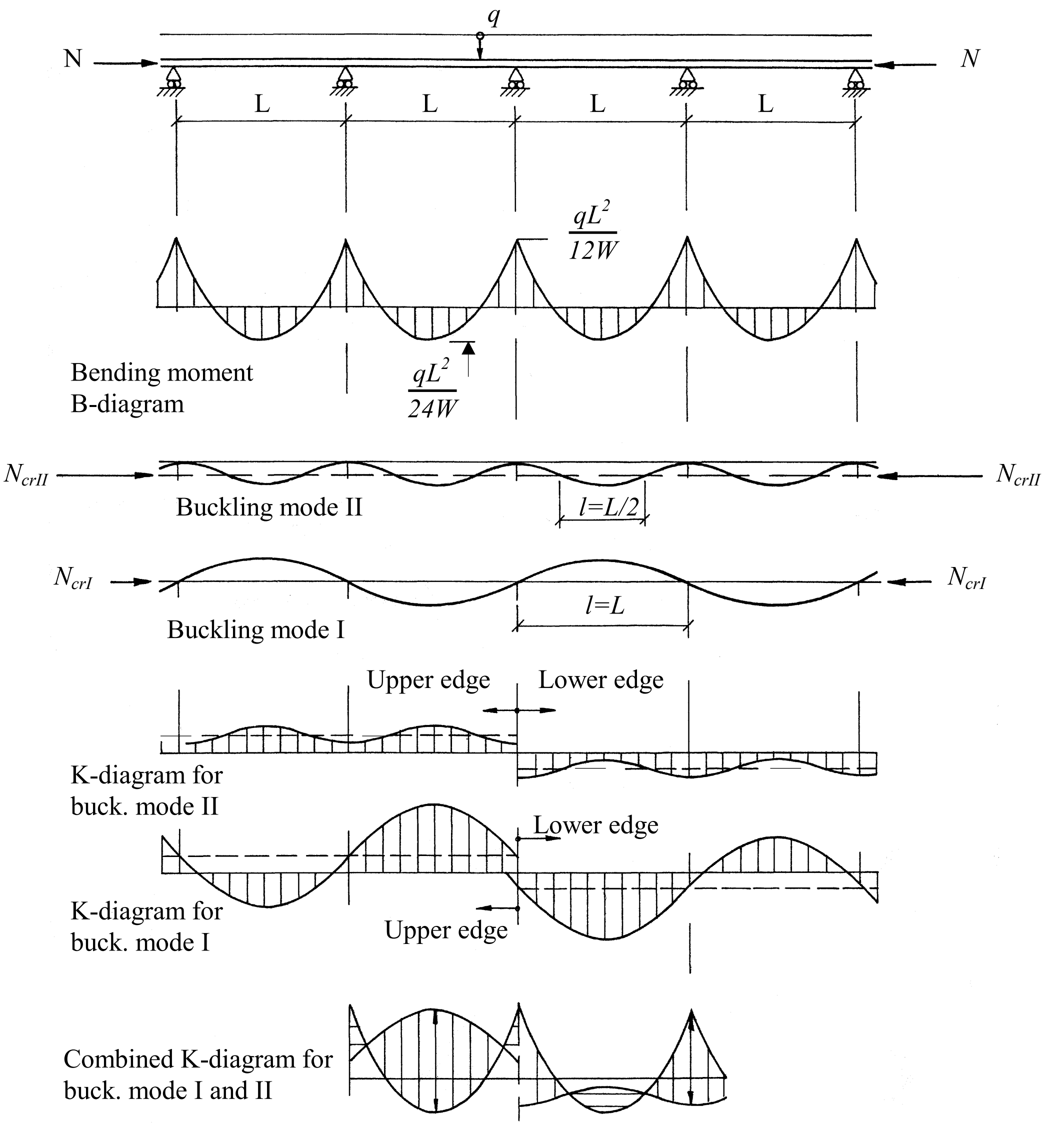

Figure 8: Continuous column with transverse load. Statens stålbyggnadskommitté (1973).

3 Results and conclusions

3.1 Roof truss

The structural system of the studied roof truss of W-shape is modelled according to Riberholt (1990a, b) with fictitious beam elements to account for the eccentricity.

In *Paper I* an interaction of normal force and bending moment was introduced. This was done by modifying work done for the Swedish steel code (Höglund, 1968), to suit structural timber and calculate the combined stress index (CSI) along the members. This approach was chosen in advance of a second order analysis because of the linearity of the formulae. The bending strength variation in the roof truss was modelled using the earlier described statistical model proposed by Isaksson (1999). Monte Carlo simulations were performed to calculate the CSI for 1000 roof trusses.

The main conclusions in *Paper I* are:

- Spruce with Scandinavian origin is a more homogeneous material and has generally higher strength compared to Radiata Pine.
- Significant statistical system effect was found in the W-shape roof truss. For the investigated W-truss the statistical system effect was found to be 12 % for spruce and 24 % for Radiata pine due to within member variation.

In *Paper II* a first order reliability study of the roof truss was performed. Two failure functions were introduced: one with statistical effects and one without. The formulae also describe the difference of variability for dead and live load. The statistical system effect is quantified. For a target reliability index equal to 4.3 the following main conclusions were found:

- The statistical system effect is presented as the possible increase of the centre-to-centre distances between trusses compared to calculations where no effect is accounted for. The increase was found to be 15 % for spruce and 30 % for Radiata pine.
- Spruce is more homogeneous which gives a 35 % higher capacity for trusses produced of spruce compared to ones of Radiata pine, with dimensions and all other factors unchanged.

3.2 Sheathed timber beam elements

In *Paper III* a sheathed parallel beam system was investigated. Monte Carlo simulations were performed for a parameter study. The calculations were done with a structural model proposed by McCutcheon (1977, 1984) and a trillinear load deflection curve for the solid timber beams. The failure criteria, given from experimental test by Håkansson and Mauritz (1999), for the system were: two neighbouring beams or any three beams.

The following parameters was studied: Rigidity between sheathing and the solid timber beam, span length of the system, the number of beams in the system, the remaining stiffness in the broken beams and the remaining bending strength in the broken beams.

Furthermore, a reliability analysis was performed where the probabilistic format was combined with the Swedish code format, BKR 99. The system effect due to the difference in coefficient of variation in the single timber beams and the system was quantified to be 1.15. Further studies have to be done to determine the system effect factor that can be used in the code.

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