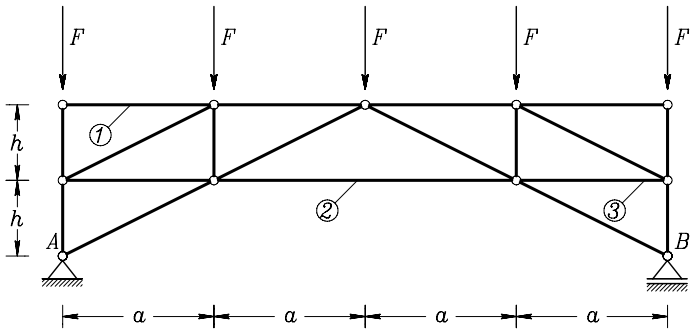
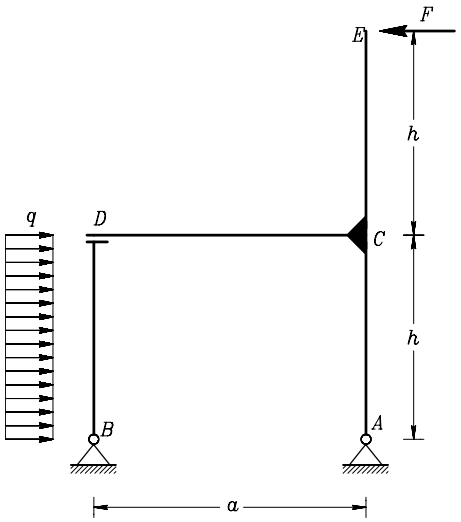
Pisni izpit iz STATIKE (Izredni študij), 25. oktober 2006

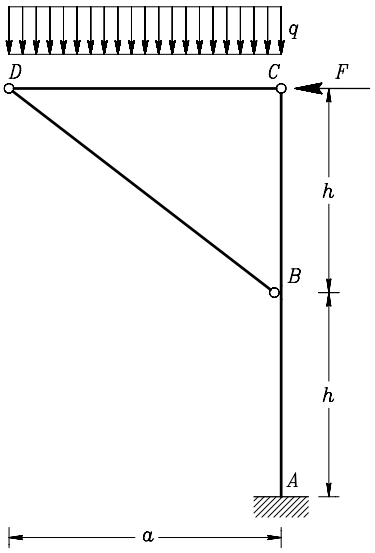
1. Ravninsko paličje na sliki je obremenjeno z navpičnimi silami F. Izračunaj računsko število prostostnih stopenj $\tilde{n}_{ps}$, reakcije ter osne sile v palicah 1, 2 in 3.
Podatki: $a = 6$ m, $h = 3$ m, $F = 10$ kN.



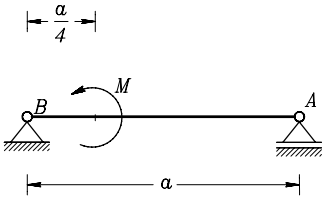
2. Ravninski okvir na sliki je obremenjen z enakomerno zvezno obtežbo q in vodoravno silo F, kot prikazuje slika. Izračunaj računsko število prostostnih stopenj $\tilde{n}_{ps}$, reakcije in sili v vezi D. V vezi D je možen samo medsebojni pomik v vodoravni smeri, medsebojni zasuk in pomik v navpični smeri pa sta preprečena.
Podatki: $a = 4$ m, $h = 4$ m, $q = 5\ \frac{\text{kN}}{\text{m}}$, $F = 10$ kN.



3. Ravninski okvir na sliki je obremenjen z zvezno obtežbo q in vodoravno silo F, kot prikazuje slika. Izračunaj računsko število prostostnih stopenj $\tilde{n}_{ps}$, reakcije, notranje sile in nariši diagrame notranjih sil.
Podatki: $a = 4$ m, $h = 4$ m, $F = 10$ kN, $q = 5\ \frac{\text{kN}}{\text{m}}$.

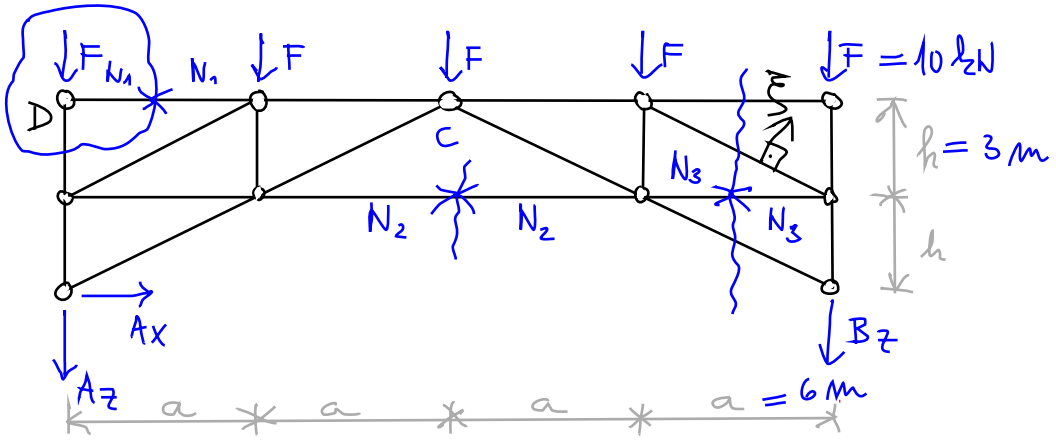


4. Z uporabo izreka o virtualnem delu izračunaj prečno silo N_z in upogibni moment M_y na sredini nosilca AB.
Podatki: $a = 8$ m, $M = 5$ kN m.



Točkovanje: 30 % + 30 % + 30 % + 30 % = 120 %

1)



$\tilde{n}_{ps} = 2 \cdot n_v - n_p - n_r = 2 \cdot 11 - 19 - 3 = 0 \Rightarrow$ SDSTT

REAKCIJE: $\Sigma X = 0$ $\quad A_x = 0$

$\Sigma z = 0, \ \Sigma M^A = 0 \Rightarrow A_z = B_z = -\frac{5F}{2} \Rightarrow$

$A_z = B_z = -25$ kN

OSNE SILE V PALICAH

$N_1 = 0$ kN, (IZREZENO VOZL. D)

N_2: $\Sigma M^C_{\ell} = 0 \quad F(2a+a) + A_z \cdot 2a + N_2 \cdot h = 0$

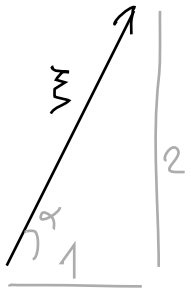
$\Rightarrow \quad N_2 = (-3aF - 2aA_z)/h$

$N_2 = (-3 \cdot 6 \cdot 10 + 2 \cdot 6 \cdot 25)/3 \Rightarrow$

$N_2 = 40$ kN

N_3: $\sum_D \xi = 0 \Rightarrow$

↑ VSOTA VSEH SIL V SMERI ξ DESNO



$\cos\alpha = \frac{1}{\sqrt{5}}$

$\sin\alpha = \frac{2}{\sqrt{5}}$

$N_3 \frac{1}{\sqrt{5}} + (F + B_z)\frac{2}{\sqrt{5}} = 0$

$N_3 = -2(F + B_z) = -2(10 - 25)$

$N_3 = 30$ kN

2)

$\tilde{n}_{ps} = 2 \cdot 3 - (2+2) - 2 = 0 \Rightarrow$ SDSTT

REAKCIJE:

① $\Sigma X = 0 \quad B_x = -qh \Rightarrow B_x = -20$ kN

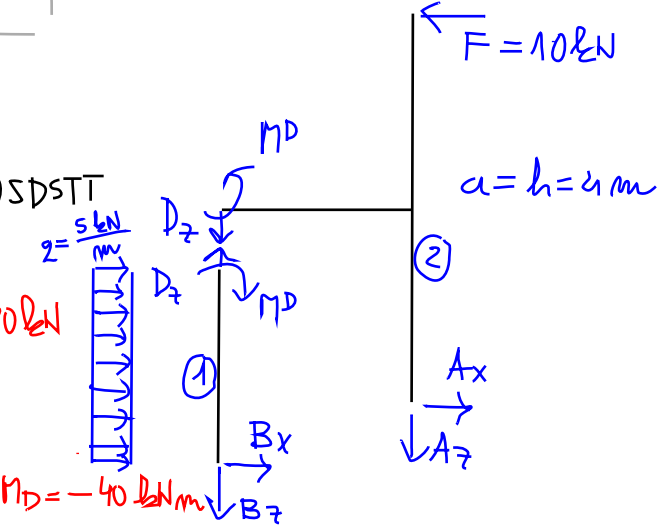
$\Sigma M^D = 0 \quad B_x h + \frac{q h^2}{2} - M_D = 0$

$\Rightarrow M_D = -20 \cdot 4 + \frac{5 \cdot 16}{2} \Rightarrow$

$M_D = -40$ kNm

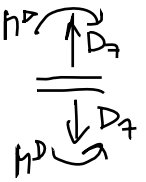
$\Sigma z = 0 \Rightarrow B_z = D_z \Rightarrow$

$B_z = -10$ kN



② $\begin{cases} \Sigma X=0 & A_x = F \Rightarrow A_x = 10\,kN \\ \Sigma z=0 & A_z + D_z = 0 \Rightarrow D_z = -A_z \Rightarrow D_z = -10\,kN \\ \Sigma M^D=0 & A_x \cdot h - A_z \cdot a + Fh + M^D = 0 \Rightarrow A_z = \frac{1}{4}(10\cdot 4 + 10\cdot 4 - 40) \Rightarrow A_z = 10\,kN \end{cases}$

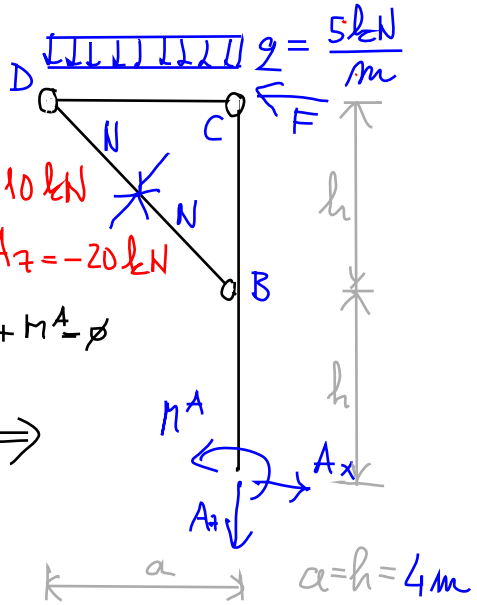
SILE V VEZI:



$D_z = -10\,kN$

$M^D = -40\,kNm$

3.) $hps = 3\cdot 3 - 3 - (2+2+2) = 0$



REAKCIJE: $\Sigma X=0 \Rightarrow A_x = F \Rightarrow A_x = 10\,kN$

$\Sigma z=0 \Rightarrow A_z = -qa \Rightarrow A_z = -20\,kN$

$\Sigma M^A = 0 \Rightarrow \frac{qa^2}{2} + F\cdot 2h + M^A = 0$

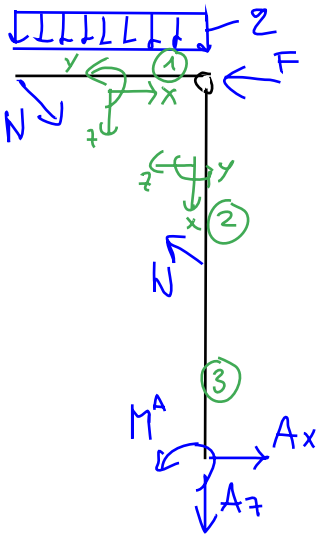
$\Rightarrow M^A = -\frac{5\cdot 16}{2} - 10\cdot 4\cdot 2 \Rightarrow$

$M^A = -120\,kNm$

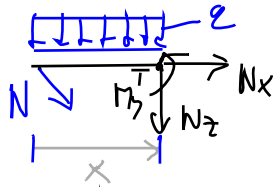
NOTRANJE SILE:

NAJPREJ DOLOČIMO OSNO SILO V PALICI BD.

$\Sigma M^C_l = 0 \Rightarrow \frac{N\sqrt{2}}{2}\cdot a + \frac{qa^2}{2} = 0 \Rightarrow N = -\frac{qa}{\sqrt{2}} = -\frac{5\cdot 4}{\sqrt{2}} \Rightarrow N = -10\sqrt{2}\,kN$



POLJE ①



$\Sigma X=0 \Rightarrow N_x = -\frac{N\sqrt{2}}{2} = +\frac{10\sqrt{2}\sqrt{2}}{2} = +10\,kN$

$\Sigma z=0 \Rightarrow N_z + qx + \frac{N\sqrt{2}}{2} = 0$

$N_z = -qx - \frac{N\sqrt{2}}{2} = -5x + 10$

$N_z(0) = 10\,kN$

$N_z(4-) = -10\,kN$

$\Sigma M^T_y = 0 \Rightarrow N\cdot\frac{\sqrt{2}}{2}\cdot x + \frac{qx^2}{2} + M_y = 0$

$M_y = -\frac{qx^2}{2} - N\frac{\sqrt{2}}{2}x$

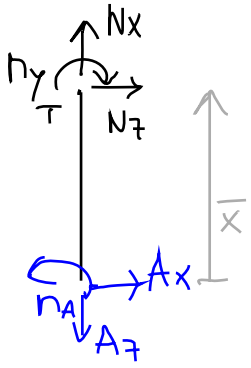
$M_y(0)=0$ ✓ $M_y(4)=\frac{-5\cdot4^2}{2}+10\sqrt{2}\frac{\sqrt{2}}{2}\cdot4$

KONTROLA $= -40+40=0$ ✓

$M_y(2)=\frac{-5\cdot2^2}{2}+10\sqrt{2}\frac{\sqrt{2}}{2}\cdot2=-10+20=$

$= 10\ kNm = \frac{qa^2}{8}=\frac{5\cdot4^2}{8}$

POLJE ③



$\Sigma X=0 \quad N_x=A_z=-20\ kN$

$\Sigma Z=0 \quad N_z=-A_x=-10\ kN$

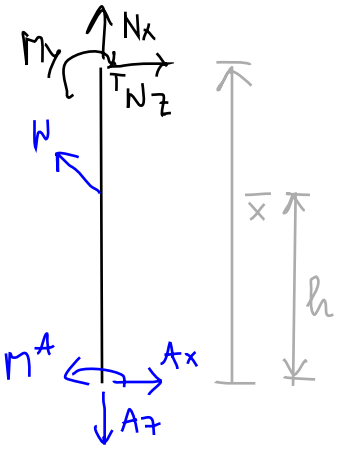
$\Sigma m_y^T=0 \quad M_y-m_A-A_x\bar{x}=0$

$M_y=m_A+A_x\bar{x}$

$M_y(0)=m_A=-120\ kNm$

$M_y(4)=m_A-A_x\cdot4=-120+40=-80\ kNm$

POLJE ②



$\Sigma X=0 \Rightarrow N_x=-N\frac{\sqrt{2}}{2}+A_z=10\sqrt{2}\frac{\sqrt{2}}{2}-20=$

$N_x=-10\ kN$

$\Sigma Z=0 \Rightarrow N_z=N\frac{\sqrt{2}}{2}-A_x=-10\sqrt{2}\frac{\sqrt{2}}{2}-10$

$N_z=-20\ kN$

$\Sigma m_y^T=0 \Rightarrow M_y+N\frac{\sqrt{2}}{2}(\bar{x}-h)-m^A-A_x\cdot\bar{x}=0$

$M_y=m^A+A_x\cdot\bar{x}-N\frac{\sqrt{2}}{2}(\bar{x}-h)$

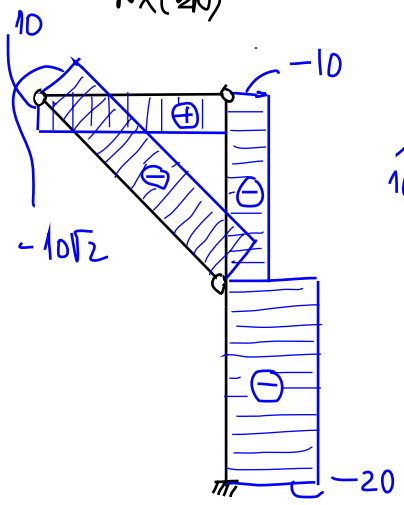
$M_y(\bar{x}=4)=-120+10\cdot4+10\sqrt{2}\cdot\frac{\sqrt{2}}{2}\cdot0=-80\ kNm$

$M_y(\bar{x}=8)=-120+10\cdot8+10\sqrt{2}\cdot\frac{\sqrt{2}}{2}\cdot4=0$ ✓

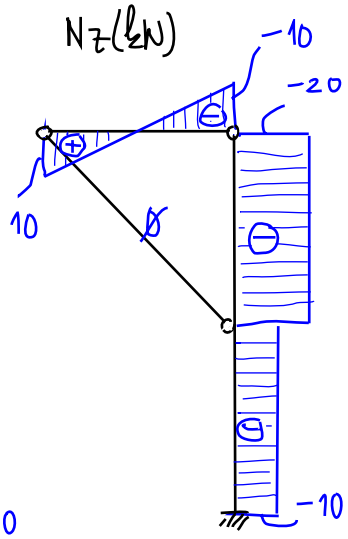
KONTROLA!

DNS:

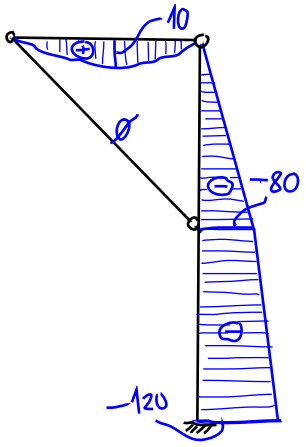
N_x (kN)



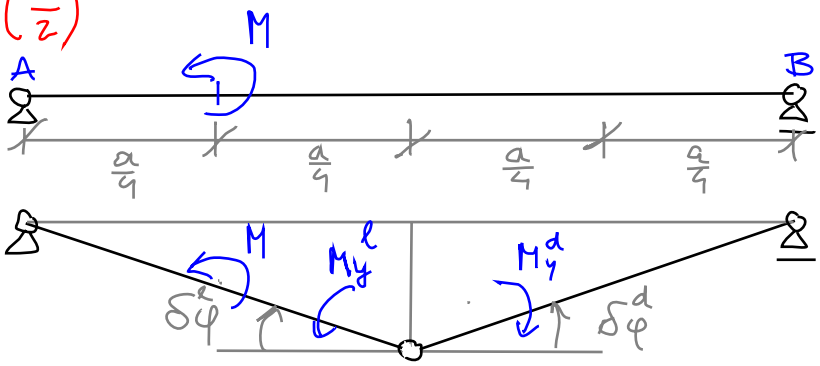
N_z (kN)



M_y (kNm)



4) $M_y\left(\frac{a}{2}\right)$



$M_y^l = M_y^d = M_y$

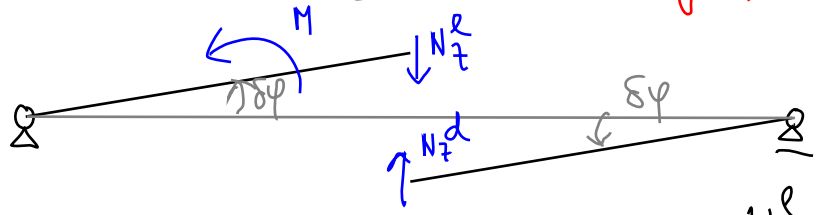
$\delta\varphi^l = \delta\varphi^d = \delta\varphi$

$\delta W = 0 \qquad \delta W = -M\,\delta\varphi^l - M_y^l\,\delta\varphi^l - M_y^d\,\delta\varphi^d = 0$

$\delta W = -M\,\delta\varphi - M_y\,\delta\varphi - M\,\delta\varphi = 0$

$\Rightarrow M_y = -\frac{M}{2} \Rightarrow M_y\left(\frac{a}{2}\right) = -\frac{M}{2} = -\frac{5}{2}\,kNm$

$N_z\left(\frac{a}{2}\right)$



$N_z^l = N_z^d = N_z$

$\delta W = 0 \qquad \delta W = M\cdot\delta\varphi - N_z^l\cdot\frac{a}{2}\cdot\delta\varphi - N_z^d\,\frac{a}{2}\,\delta\varphi = 0$

$\Rightarrow M - N_z\frac{a}{2} - N_z\frac{a}{2} = 0 \Rightarrow N_z = \frac{M}{a} \Rightarrow$

$N_z\left(\frac{a}{2}\right) = \frac{M}{a} = \frac{5}{8}\,kN$